

(* This is the second problem of the week *)

In[48]:= **idealToBeTested** = {x z - y^2, x^3 - z^2}

Out[48]= $\{-y^2 + xz, x^3 - z^2\}$

In[49]:= **gb** = GroebnerBasis[idealToBeTested, {x, y, z}, MonomialOrder → Lexicographic]

Out[49]= $\{y^6 - z^5, -y^2 + xz, xy^4 - z^4, x^2y^2 - z^3, x^3 - z^2\}$

In[51]:= **testPoly** = -4 x^2 y^2 z^2 + y^6 + 3 z^5

Out[51]= $y^6 - 4x^2y^2z^2 + 3z^5$

In[53]:= **red** = PolynomialReduce[{testPoly}, gb, {x, y, z}][[1]]

Out[53]= $\{\{-3, -4y^4 - 4xy^2z, 0, 0, 0\}, 0\}$

(* This means that testPoly belongs to the ideal idealToBeTested *)

(* It has zero remainder with elements of the ideal given as ... *)

red[[1, 1]] * gb[[1]] + red[[1, 2]] * gb[[2]] - testPoly

Out[56]= $-y^6 + 4x^2y^2z^2 - 3z^5 + (-y^2 + xz)(-4y^4 - 4xy^2z) - 3(y^6 - z^5)$

In[57]:= **red[[1, 1]] * gb[[1]] + red[[1, 2]] * gb[[2]] - testPoly // Simplify**

Out[57]= 0