

What is  $p \cdot n$ ?

$$p \cdot n = N_c \cdot N_v \cdot \exp \left( \frac{-E_c + E_F - E_F + E_v}{kT} \right)$$

$$= N_c \cdot N_v \cdot \exp \left( \frac{E_v - E_c}{kT} \right) = n_i^2 \quad \text{law of mass-action}$$

thus:  $p \cdot n = n_i^2$  it is a constant!  $E_g/kT$

we can rearrange the terms:

$$n = n_i \cdot \left( \frac{E_F - E_i}{kT} \right)$$

$$p = n_i \cdot \left( \frac{E_i - E_F}{kT} \right)$$

if degenerate

$$p \cdot n \approx n_i^2$$

back to slides

Doping:

Donor  $\begin{cases} \text{neutral} \\ \text{positive} \end{cases}$

Acceptor  $\begin{cases} \text{neutral} \\ \text{negative} \end{cases}$

Not all dopants are necessarily ionized

$$N_D^+ = \frac{N_D}{1 + g_D \cdot \exp \left( \frac{E_D - E_F}{kT} \right)}$$

degeneracy of donor level = 2 (spin of electron)

$$N_A^- = \frac{N_A}{1 + g_A \cdot \exp \left( \frac{E_A - E_F}{kT} \right)}$$

degeneracy of acceptor level = 4  $\begin{matrix} 2 \text{ spin} \\ 2 \end{matrix} \times \text{degeneracy of band at } k=0$



in most semiconductors there are 2 bands degenerate at  $k=0$  (3)

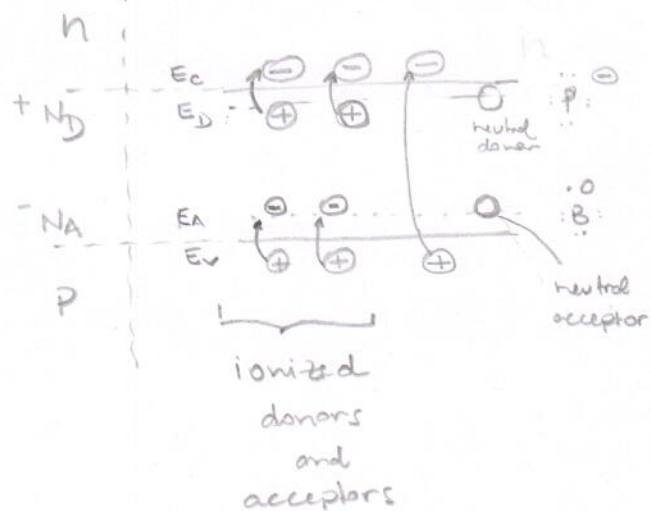
Due to charge neutrality:

$$n + N_A^- = p + N_D^+$$

the law of mass-action

$$n \cdot p = n_i^2$$

still applies! (until degeneracy)



How to calculate the Fermi level?

for n-type Si:

$$n = p + N_D^+$$

$$\Rightarrow n \approx N_D^+$$

$p$ : intrinsic  $\sim 10^{10} \text{ cm}^{-3}$

$N_D$ : doping  $\sim 10^{18} \text{ cm}^{-3}$

$$N_C \cdot \exp\left(-\frac{E_C - E_F}{kT}\right) \approx N_D \cdot \frac{1}{1 + 2 \exp\left(-\frac{E_D - E_F}{kT}\right)}$$

For a given  $N_C$ ,  $N_D$ ,  $T$ ,  $E_D \Rightarrow$  we can calculate  $E_F$  and then  $n$ .

$\Rightarrow$  use graphical method.

or

$$\left\{ \begin{array}{l} \text{n-type} \\ N_D \gg N_A : \quad n \approx \sqrt{\frac{N_D N_C}{2}} \cdot \exp\left[-\frac{(E_C - E_D)}{2kT}\right] \\ \text{-if } N_A \text{ is non-negligible:} \\ N_D \gg N_A : \quad n \approx \left(\frac{N_D - N_A}{2N_A}\right) \cdot N_C \cdot \exp\left[-\frac{(E_C - E_D)}{kT}\right] \end{array} \right.$$

At relatively high temperatures, all donors and acceptors are

ionized:

$$n + N_A = p + N_D$$

and since  $n \cdot p = n_i^2 \Rightarrow n = \frac{1}{2} \left[ (N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2} \right]$

$$n^2 - (N_D - N_A)n - n_i^2 = 0$$

in n-type semiconductor:  $N_D > N_A$

$$n_{no} = \frac{1}{2} \left[ (N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2} \right] \approx N_D$$

if  $|N_D - N_A| \gg n_i$

$$p_{no} = \frac{n_i^2}{n_{no}} \approx \frac{n_i^2}{N_D}$$

or  $N_D \gg N_A$

Fermi level:

$$n_{no} = N_D = N_C \cdot \exp\left(-\frac{E_C - E_F}{kT}\right) = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

We can solve numerically or graphically.

for p-type semiconductor

$$p = \frac{1}{2} \left[ (N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4n_i^2} \right]$$

$$\approx N_A$$

$$\left\{ \begin{array}{l} \text{if } |N_A - N_D| \gg n_i \\ \text{or } N_A \gg N_D \end{array} \right.$$

$$n = \frac{n_i^2}{N_A}$$

• first + exercises

• complete 1st list of exercises.

Fermi-level:

$$p = N_A = N_V \cdot \exp\left(-\frac{E_F - E_V}{kT}\right) = n_i \cdot \exp\left(\frac{E_i - E_F}{kT}\right)$$