Chapter 1 - Generalities on acoustics

Hervé Lissek

Audio Engineering (MA1)

1 Sound propagation in an (infinite) waveguide

Let's consider the propagation of an acoustic wave in a one-dimensional, cylindric duct of cross-section S, directed towards an axis Ox (acoustic waveguide), filled with a fluid of mass density ρ_0. We suppose the fluid is compressible and the compressibility is *isentropic*, characterized by the isentropic coefficient Γ_0 (constant in the medium), linking the fluid volume and the applied pressure forces after $P_{tot}.V^{\Gamma_0} = C_0$ (where $P_{tot} = p_s + p$, with p_s is the atmospheric pressure and p is the acoustic pressure, and C_0 is a constant of the fluid).
We look at a fluid particle comprised between the cross-sections at x and $x + dx$. We denote $\xi(x)$ the displacement of the side of the fluid section at abscissa x (which should differ a priori from $\xi(x + dx)$).

- what is the expression of the mass of fluid in the volume of thickness dx ?
- assuming the only applied forces are due to the acoustic pressure $p(x)$ in x and $p(x + dx)$ in $x + dx$, what is the local expression of the Newton law applied to the fluid section of thickness dx ?
- derive the law of adiabatic compression of the fluid, in order to obtain a relationship between acoustic pressure p and the volume variation of the fluid δV.
- express the volume variation δV of the fluid section dx as a function of the partial derivative $\frac{\partial \xi}{\partial x}$. Conclude that $p = -\Gamma_0 p_s \frac{\partial \xi}{\partial x}$
- at last, write the acoustic wave propagation equation $\frac{\partial^2 p}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = 0$. Express sound celerity c_0 as a function of ρ_0, Γ_0 and p_s.

2 $\lambda/4$ resonator

Let's consider a cylindrical duct of length L=2.5 cm and cross-section S=38 mm^2, one extremity being closed, the other one being open. The duct is filled with air at 20°. We designate the axis of symmetry as Ox and consider the closed termination is at the origin of abscissaes at x=0.

1. write the sound wave propagation in 1D. What are the general solutions $p(x, t)$ of this equation ?
2. remind the Newton's law, linking a partial derivative of the particle velocity $v(x, t)$ and a partial derivative of the acoustic pressure $p(x, t)$. Assuming p and v are harmonic disturbances at angular frequency ω ($\underline{p}(x, t) = p_0 e^{j\phi(x)} e^{j\omega t}$), express $\underline{v}$ as a function of $\underline{p}$.
3. deduce then the boundary condition at x=0.
4. the boundary condition at $x = L$ reads $p(L, t) = 0$ (the pressure at the opening is balanced with the atmospheric pressure). Deduce the eigenfrequencies (resonance frequencies) of this problem. Why is it called a $\lambda/4$ resonator ?
5. calculate the first resonance frequency of the tube of 2.5 cm.
6. write the expression of the pressure $p(x, t)$ all along the duct. On Matlab, draw the pressure as a function of the distance along the axis x for the first 4 resonance frequencies.

3 Sound absorption

Let's consider an acoustic waveguide (eg. a cylindrical duct) of length L. At one termination ($x = 0$), a loudspeaker generates a pure tone at frequency f, and the other termination ($x = L$) is closed with a given material. We assume plane-wave field within the duct. The sound pressure inside the duct follows the general solution for plane waves, as expressed in the former exercise. We will discard here the dependency on time t in the following, and only consider the spatial variation of pressure along the duct.
At each position x along the duct, we can define the specific acoustic impedance $Z_s(x)$ as

$$Z_s(x) = \frac{p(x)}{v(x)}$$

.

1. give the general expression of $Z_s(x)$ as a function of amplitudes p_+ and p_- of the propagating pressures along increasing and decreasing x, the wavenumber k and the position x, and the characteristic impedance of the medium $Z_c = \rho c$, where ρ is the medium mass density and c is the sound celerity.

In the same way, we can define, at each position x, the reflection coefficient $r(x)$ as the ratio of the pressure propagated along decreasing x over the pressure propagated along increasing x. This quantity represents the amount of incident pressure (along increasing x) that is reflected back (along decreasing x) at a given position x.

2. give the general expression of $r(x)$ at each position x inside the duct,
3. show that $Z_s(x) = Z_c \dfrac{1 + r(x)}{1 - r(x)}$,
4. show that the boundary condition $r(x = L) = 0$ yields $Z_s(x = L) = Z_c$.

5. If we define now the absorption coefficient at the termination $x = L$, under normal incidence $\alpha = 1 - |r(x = L)|^2$, what is the value of α in this case ? how would you qualify a material with a specific acoustic impedance of Z_c ?
6. can you simplify the general expression of the pressure inside the duct in this case ?

4 Sound levels

A source produces a sound power of 3 W. Compute :

— the power level of the source,

— the sound intensity at 5 m from the source, then 10 m,

— the sound intensity level at 5 m and at 10 m.

Data : $I_0 = 10^{-12}\text{Wm}^{-2}$, $P_0 = 10^{-12}$ W.

5 Sound levels and acoustic quantities

We know that sound pressure level corresponding to a pressure of p=1 Pa is $L_p(p = 1$ Pa)=94 dB.

(a) without employing a calculator, deduce the sound pressure level corresponding to p'=2 Pa, 0.1 Pa and 10 Pa.

(b) what is the sound intensity I corresponding to 1 Pa in the air (ρ_0=1.2 kg.m^{-3} and c=340 m.s^{-1}) ?

(c) what is the sound intensity level if we double the sound intensity $I' = 2I$?

6 Addition and substraction of decibels

Two cars are producing individual sound pressure levels of 77 dB and 80 dB measured at the border of a road. What is the resulting sound pressure level when the pass together ?

A motorcycle passes at the same time, and we measure a total sound pressure level of $L_p(car_1 + car_2 + motorcycle)$=84 dB. What would be the sound pressure level measured if the motorcycle has passed by alone ?

7 Beats

Let's consider two pure tones of frequency f_1 and $f_2 = f_1 + \Delta f$, played simultaneously (ie. at time $t = 0$ s, sound pressures $p_1(t = 0) = p_2(t = 0) = 0$) with the same amplitude. Show that the resulting pressure reads :
$p(t) = 2sin(2\pi(f + \frac{\Delta f}{2})t).cos\pi\Delta ft$

If we consider $\Delta f \ll f$, what would the sound pressure waveform look like ? You may consider, as an example on Matlab, two pure tones of same amplitude and 1 s duration at $f_1 = 440$ Hz and $f_2 = 450$ Hz to figure out the result.

What happens if the two frequencies are significantly distant ?

8 Pythagore's scale

The most used musical scale (in western cultures) is the "equal-temperament" scale, which split one octave into 12 equal intervals, called half-tones, that we can enumerate from the Do (C) :
Do (f_0) - Do# (f_1) - Ré (f_2) - Ré# (f_3) - Mi (f_4) - Fa (f_5) - Fa# (f_6) - Sol (f_7) - Sol# (f_8) - La (f_9) - La# (f_{10}) - Si (f_{11}).

- Knowing that the next note after the Si is a Do, one octave higher than the first Do (f_0) in the list, and that all intervals $\dfrac{f_{i+1}}{f_i}$ (where i is the i^{th} note of the octave) shall be equal, what is the value of the half-tone interval $\dfrac{f_{i+1}}{f_i}$?
- What is the value of $\frac{f_7}{f_0}$, that is usually called *quint* (7 half-tones) ?

— Calculate the values of $\frac{f_i}{f_1}$ and the corresponding frequencies f_i of the equal-temperament scale (fill the values in the table below).

Pythagore, the famous greek mathematician, also introduced a musical scale, also base on 12 half-tones. However, in this case, the scale is build upon successive quints, so that : $f_{i+7} = \dfrac{3}{2} f_i$.
Ex : the quint above the Do (f_0) is the Sol ($f_7 = \frac{3}{2} f_1 = 1.5 f_0$).
Note that the quint of the Sol is a Ré (f_2), but one octave higher ($f_{14} = \frac{9}{4} f_0 = 2.25 f_0 > 2 f_0$), which is then outside the initial octave. In order to find the equivalent in the initial octave, one should just divide this last value by 2 so that $f_2 = \frac{9}{8} f_0 = 1.125 f_0$.

— Calculate the values of f_i/f_0 and f_i in the Pythagore scale (fill the table below).

Note	Equal temperament scale		Pythagore scale	
	interval	frequency	interval	frequency
Do	1	262 Hz	1	$f_0 = 262$ Hz
Do#				
Ré			1.125	$f_2 = 294.75$ Hz
Ré#				
Mi				
Fa				
Fa#				
Sol			1.5	$f_7 = 393$ Hz
Sol				
Sol#				
La				
La#				
Si				