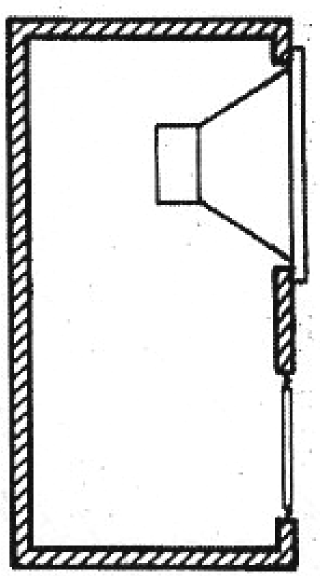
5.3 Loudspeaker systems

Definitions

- Loudspeaker system: constituted of a loudspeaker and an enclosure:
  - Closed-box enclosure
  - Vented-box enclosure
  - Enclosure with passive radiator



enceinte ouverte à radiateur passif

LF properties

At LF, enclosures are smaller than λ : equivalent schemes are valid

Radiating openings = rigid circular pistons

= semi-monopoles, flow velocities $\underline{q}_i$

distances between openings smaller than λ,

➔Coincident openings: $\underline{q}_r = \sum_i \underline{q}_i$

➔Radiated power: $P_a = R_{ar}\tilde{q}_r^2 = R_{ar}\tilde{q}_b^2$

then the power can be computed after $\underline{q}_b$

$\underline{q}_b = -\underline{q}_r$

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Values of R_{ar}

<u>Reference</u> radiation resistance (for simple comparisons):

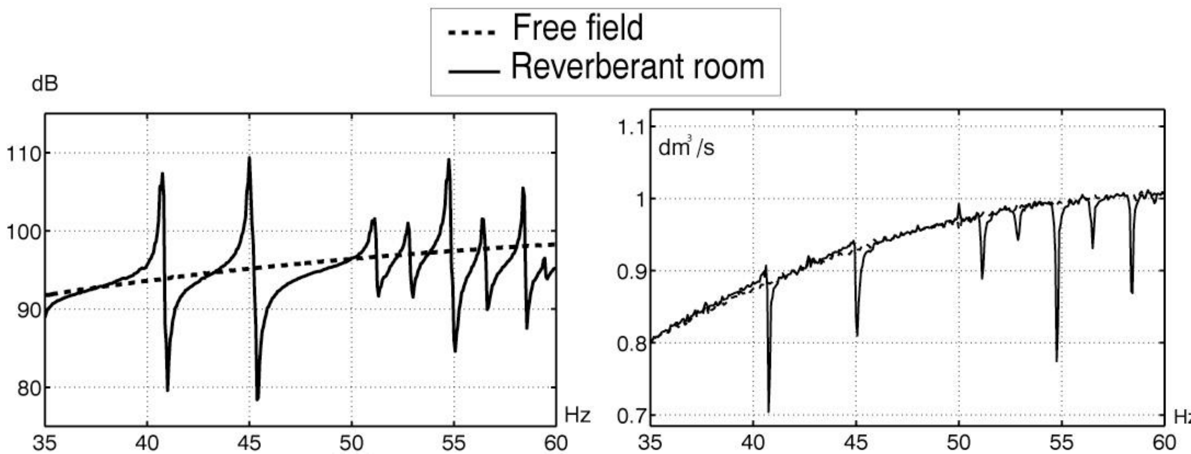
small pulsating source close to ideally rigid screen

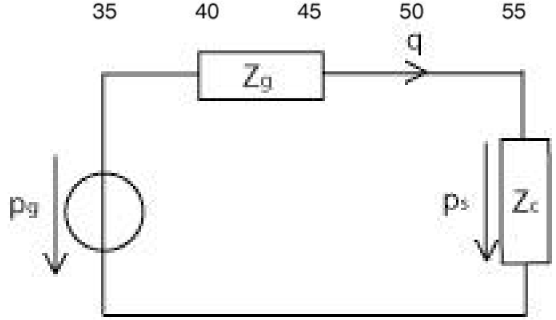
$$R_{ar} = 2\pi\rho \frac{f^2}{c}$$

It is always possible to adapt to a particular case

Modal domain

Validity of radiation impedance: above Schroeder frequency
Below: modal domain ➔ influence of the room on the radiation impedance





$$q_c = \frac{p_g}{Z_g + Z_c}$$

-p_g depends on the loudspeaker voltage

- Z_g depends on TS parameters and enclosure dimensions

- p_s and q dependant of Z_c

- Z_c =total load impedance

Analysis and synthesis

- Analysis: establish the system properties: equivalent schemes
- Synthesis: determine required specifications for the driver and the enclosure for given performances
  - Response curve at LF
  - Volume, given space
  - Efficiency inside *B*
  - Max. acoustic power for a given distortion

Closed-box enclosure system

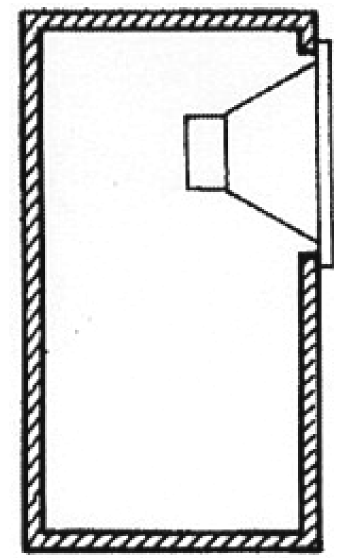
At LF: enclosure=compliance $$C_{ab} = \frac{\beta V_b}{\rho c^2}$$

+ acoustic resistance R_{ab}

+ equivalent mass m_{ab}

(depending on the filling)

Based on the scheme of a loudspeaker on infinite screen, with $\underline{Z}_{ar2}$=enclosure



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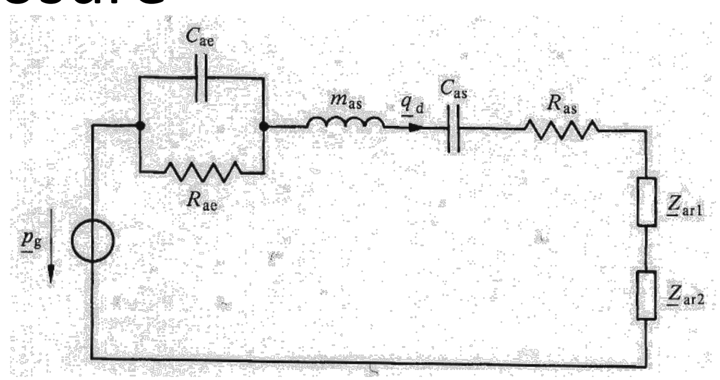

loudspeaker on infinite screen

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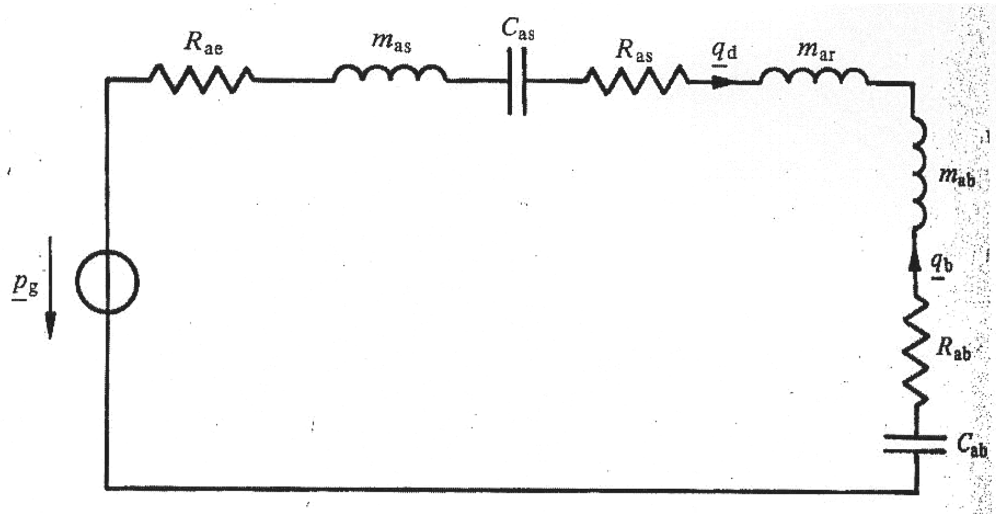
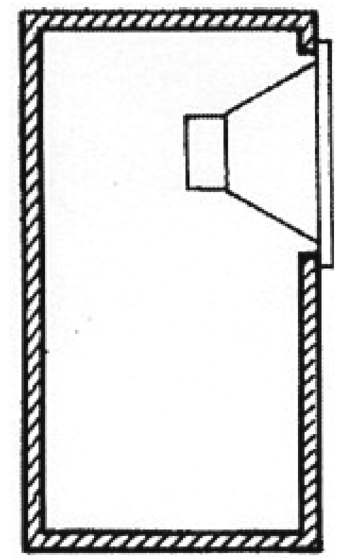
Closed-box enclosure system





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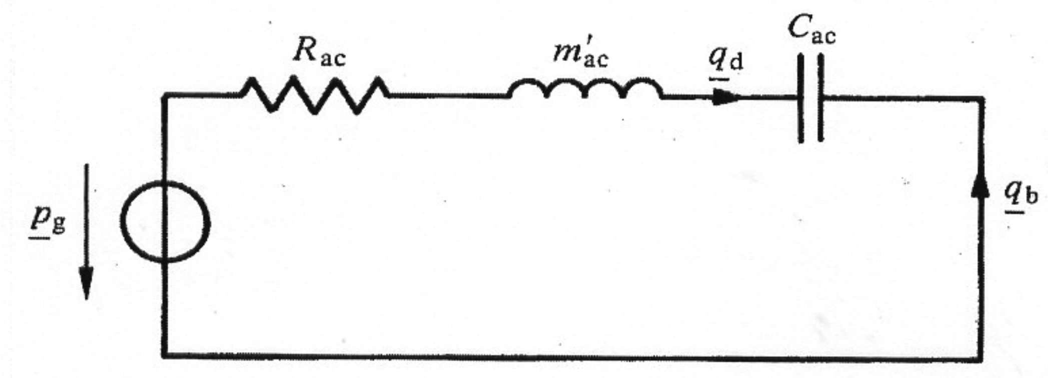
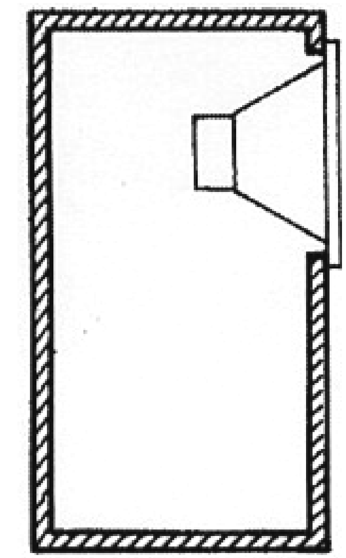
Closed-box enclosure system





enceinte close

$$R_{ac} = R_{ae} + R_{as} + R_{ab}$$

$$m'_{ac} = m_{as} + m_{ar} + m_{ab}$$

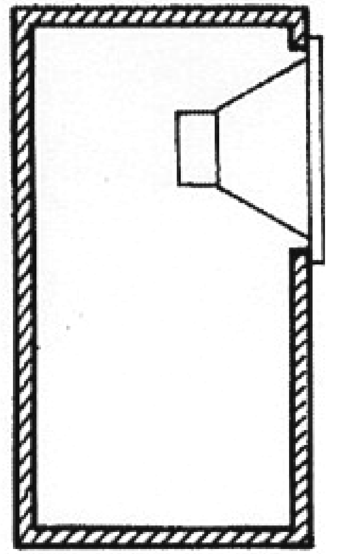
$$C_{ac} = \frac{C_{as} C_{ab}}{C_{as} + C_{ab}}$$

Closed-box enclosure system

$$C'_c = \frac{m'_{ac} S_d^2}{(Bl)^2}$$

$$R_c = \frac{(Bl)^2}{S_d^2 R_{ac}}$$

$$L_c = \frac{C_{ac}(Bl)^2}{S_d^2}$$



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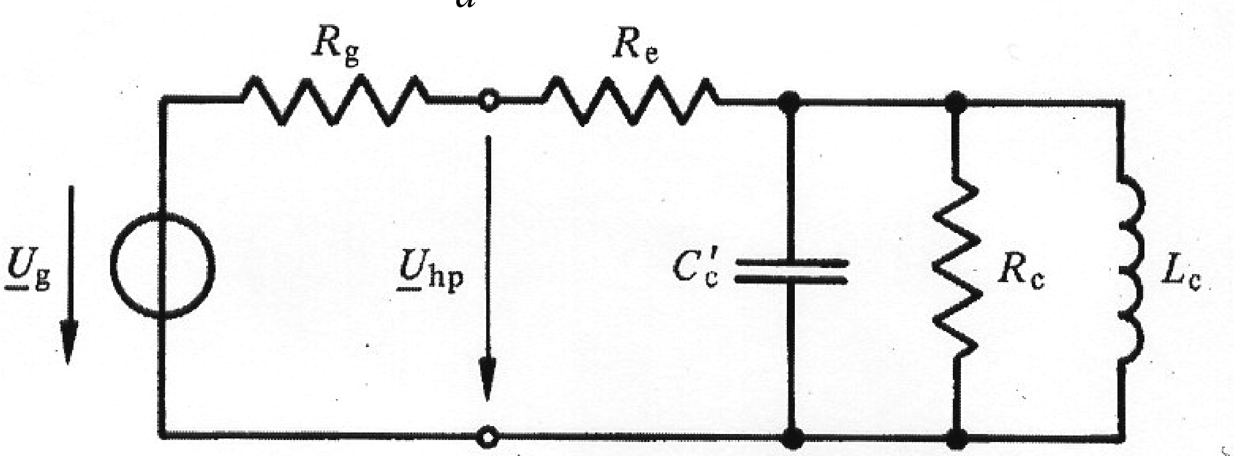


Small signals parameters

- *Resonance frequency* $$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{m'_{ac} C_{ac}}} = \frac{1}{2\pi\sqrt{C'_c L_c}}$$
- *Electrical quality factor @ ω_c* $$Q_{ec0} = \omega_c C'_c R_e$$
- *Mechanical quality factor @ ω_c* $$Q_{mc} = \omega_c C'_c R_c$$
- *Total quality factor @ ω_c* $$Q_{tc0} = \frac{Q_{ec0} Q_{mc}}{Q_{ec0} + Q_{mc}}$$
- *Compliance factor* $$\alpha = \frac{C_{as}}{C_{ab}}$$

if $R_g \neq 0$, different electrical and total quality factors

Acoustic suspension

compliance factor implies that the enclosure behaves as a suspension in parallel, thus a higher resonance frequency $f_c > f_s$

For α high enough, $C_{ac} \sim C_{ab}$

➔compliance of driver = the enclosure's one

➔acoustic suspension

α of the order of magnitude of 3-10

Ratios of the quality factors

$$\frac{Q_{ec}}{Q_{es}} \cong \frac{f_c}{f_s} \cong \sqrt{1+\alpha}$$

$$\frac{Q_{mc}}{Q_{ms}} \cong \frac{f_c}{f_s} \cong \sqrt{1+\alpha}$$

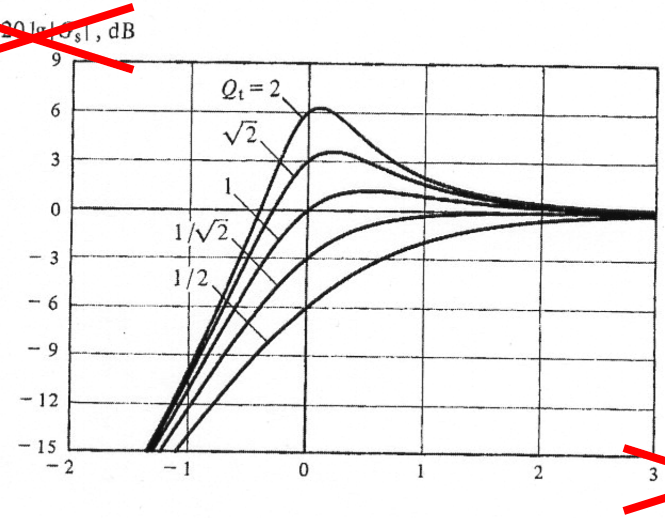
$$\frac{Q_{tc}}{Q_{ts}} \cong \frac{f_c}{f_s} \cong \sqrt{1+\alpha}$$

Response curves

Given the transfer function $G_s(j\omega/\omega_s)$, replacing with the SSP in closed-box (subscript *c*)

$$G_c(j\omega/\omega_c) = \frac{(j\omega/\omega_c)^2}{(j\omega/\omega_c)^2 + Q_{tc}^{-1}(j\omega/\omega_c) + 1}$$

Response curves in P_a and p



replace
-*fs* with *fc*
-*Qt* with *Qtc*

Response curves

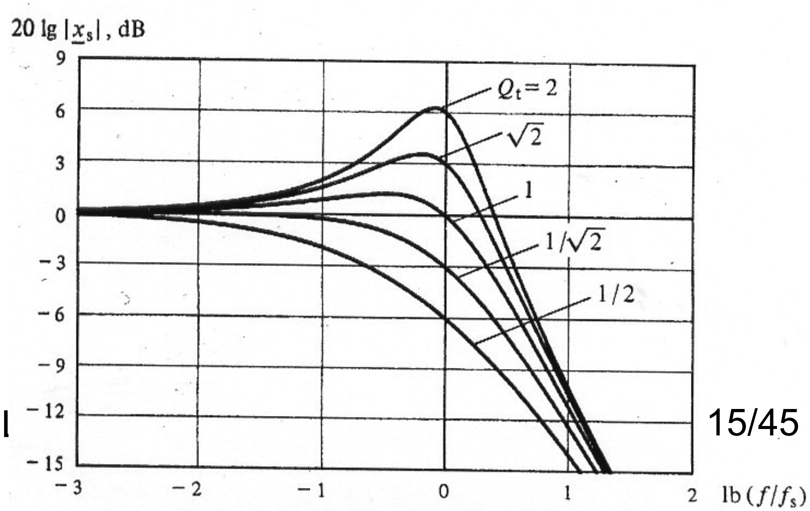
Given the transfer function $G_s(j\omega/\omega_s)$, replacing with the SSP in closed-box (subscript *c*)

$$G_c(j\omega/\omega_c) = \frac{(j\omega/\omega_c)^2}{(j\omega/\omega_c)^2 + Q_{tc}^{-1}(j\omega/\omega_c) + 1}$$

Response curves in P_a and p

same for elongation:

with $\underline{\xi}_c = \dfrac{\underline{U}_g}{\omega_c Q_{ec}(Bl)}$



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Reference efficiency

- As for loudspeaker in infinite screen

$$\eta_c = 100.\left\{\frac{4\pi^2}{c^3}\right\}\left\{\frac{f_c^3 V_{ac}}{Q_{ec}}\right\}$$

$$\text{where } V_{ac} = \rho c^2 C_{ac}$$

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Power limited by the elongation

One observes: $$\frac{\underline{\xi}_c}{\underline{\xi}_s} = \frac{\underline{U}_g}{\omega_c Q_{ec}(Bl)} \cdot \frac{\omega_s Q_{es}(Bl)}{\underline{U}_g} \cong \frac{1}{1+\alpha}$$

The displacement is lower with a cabinet than in infinite screen (for same $\underline{U}g$)

➔ $$P_{a\xi c} = (1+\alpha)^2 P_{a\xi} = \left\{\frac{4\pi^3\rho}{c}\right\}\left\{f_c^4 \hat{V}_d^2\right\}\frac{1}{\underbrace{x_{\max}^2}_{\max|\underline{x}_c|}}$$

then: $$P_{e\xi c} = \frac{P_{a\xi c}}{\eta_c} = \left\{\pi\rho c^2\right\}\left\{\frac{Q_{ec0} f_c \hat{V}_d^2}{V_{ac}}\right\} \cdot \frac{1}{x_{\max}^2}$$

Non-linearities of the enclosure

- Linear hypothesis in the enclosure
- Not true for high elongations of diaphragm

➔ effects on compressions and releases of air

$$(p_b + p_s)(V_b - \underbrace{\underbrace{\xi_b}_{-\xi_d} S_d}_{V_d})^{\Gamma} = p_s V_b^{\Gamma}$$

$$p_b = p_s \left\{ (1 - \underbrace{\varepsilon}_{\xi_b S_d / V_b})^{-\Gamma} - 1 \right\} = p_s \left\{ \Gamma \varepsilon - \frac{1}{2} \Gamma (\Gamma + 1) \varepsilon^2 + \ldots \right\}$$

➔ Numeric resolution

➔ practically: enclosure distortion < driver distortion

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Behavior at HF

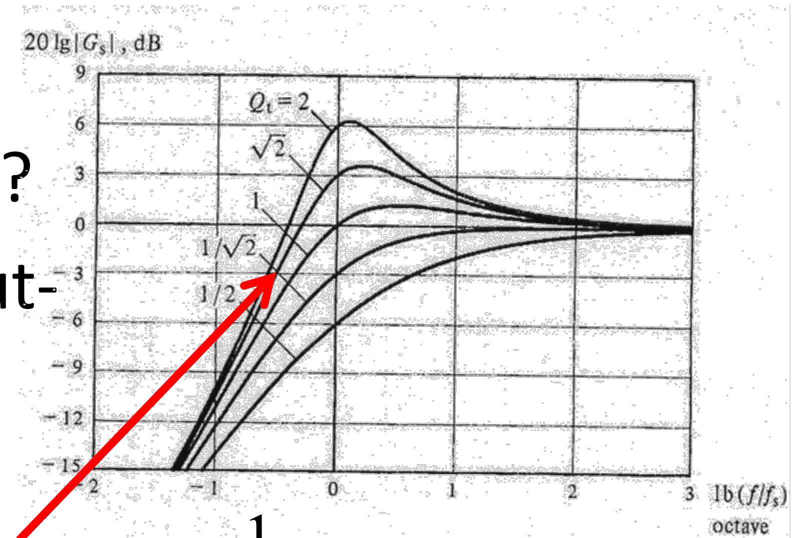
- See eigenmodes
- Role of filling is also to lower the influence of enclosure modes

Response curves

curves vary after Q_t

→ best Q_{tc} for a given application?

→ one can define a half-power cut-off frequency (-3dB): f_3

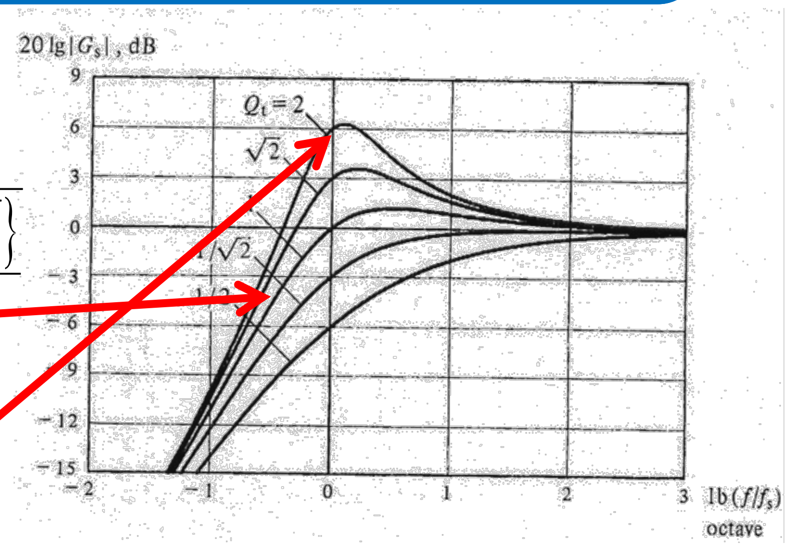


Note: amplitude peak $|G_c|_{max}$ for $Q_{tc} > \frac{1}{\sqrt{2}}$

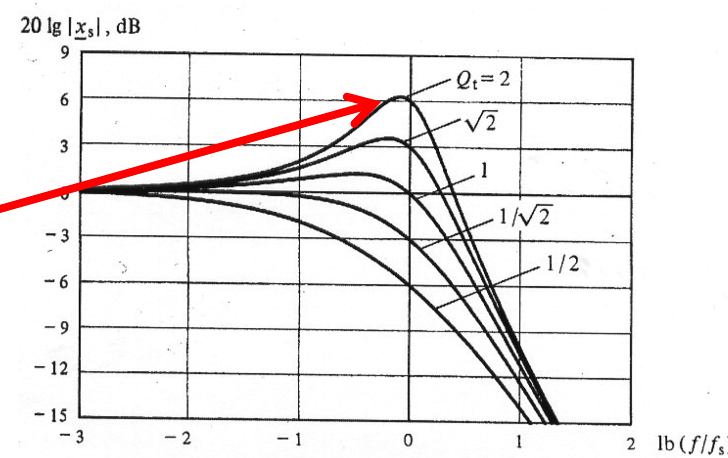
$$\left|G_c(j\omega_3 / j\omega_c)\right|^2 = \frac{1}{2} \Rightarrow \frac{f_3}{f_c} = \sqrt{\frac{\left\{Q_{tc}^{-2} - 2 + \sqrt{\left(Q_{tc}^{-2} - 2\right)^2 + 4}\right\}}{2}}$$

Response curves

$$\left|G_c(j\omega_3 / j\omega_c)\right|^2 = \frac{1}{2} \quad \Rightarrow \quad \frac{f_3}{f_c} = \sqrt{\frac{\left\{Q_{tc}^{-2} - 2 + \sqrt{(Q_{tc}^{-2} - 2)^2 + 4}\right\}}{2}}$$



$$\frac{d}{d\omega}\left|G_c(j\omega / j\omega_c)\right|^2\Bigg|_{f=f_{\max}} = 0 \quad \Rightarrow \quad \frac{f_{\max}}{f_c} = \frac{1}{\sqrt{1 - \frac{1}{2}Q_{tc}^{-2}}}$$



$$\frac{d}{d\omega}\frac{G_c(j\omega / j\omega_c)}{(j\omega / j\omega_c)^2}\Bigg|_{f=f_\xi} = 0 \quad \Rightarrow \quad \frac{f_\xi}{f_c} = \sqrt{1 - \frac{1}{2}Q_{tc}^{-2}}$$

Response curves

$$\left|G_c(j\omega_3 / j\omega_c)\right|^2 = \frac{1}{2} \Rightarrow \frac{f_3}{f_c} = \sqrt{\frac{\left\{Q_{tc}^{-2} - 2 + \sqrt{(Q_{tc}^{-2} - 2)^2 + 4}\right\}}{2}}$$

$$\left.\frac{d}{d\omega}\left|G_c(j\omega / j\omega_c)\right|^2\right|_{f=f_{\max}} = 0 \Rightarrow \frac{f_{\max}}{f_c} = \frac{1}{\sqrt{1 - \frac{1}{2}Q_{tc}^{-2}}}$$

$$\left.\frac{d}{d\omega}\frac{G_c(j\omega / j\omega_c)}{(j\omega / j\omega_c)^2}\right|_{f=f_\xi} = 0 \Rightarrow \frac{f_\xi}{f_c} = \sqrt{1 - \frac{1}{2}Q_{tc}^{-2}}$$

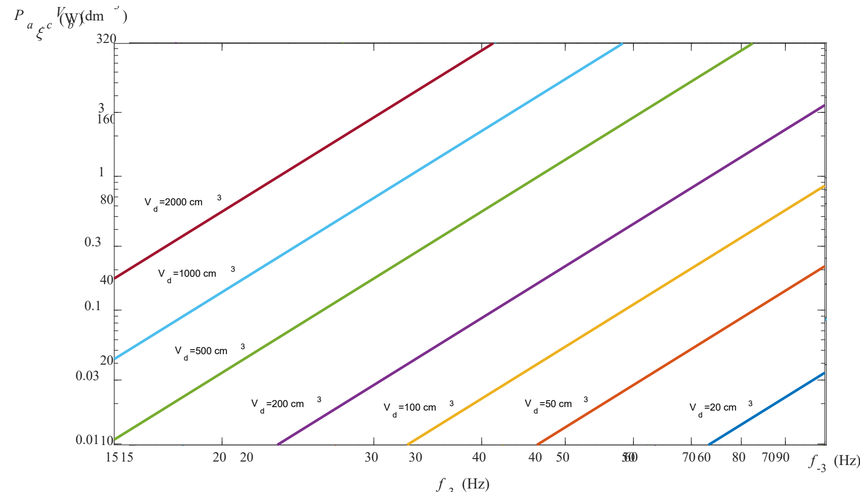
$$\left|G_c(f_{\max} / f_c)\right| = x_{\max} = \sqrt{\frac{Q_{tc}^4}{Q_{tc}^2 - \frac{1}{4}}}$$

Merit factors

Factor defined w resp. to the *efficiency* and the *acoustic power* limited by elongation

$$\eta_c = 100.\left\{\frac{4\pi^2}{c^3}\right\}\left\{\frac{f_c^3 V_{ac}}{Q_{ec}}\right\} = k_\eta f_3^3 V_b$$

$$P_{a\xi c} = \left\{\frac{4\pi^3\rho}{c}\right\}\left\{f_c^4 \hat{V}_d^2\right\}\frac{1}{x_{\max}^2} = k_P f_3^4 \hat{V}_d^2$$



Merit factor in efficiency:

➔acoustic suspension + filling

$$k_\eta = \underbrace{\underbrace{\left[\frac{Q_{tc}}{Q_{ec}}\right]}_{0,5-0,9}\underbrace{\left[\frac{V_{ac}}{V_b}\right]}_{\beta\alpha/(1+\alpha)}}_{\cong 1}\underbrace{\left[\frac{4\pi^2}{c^3}\cdot\frac{1}{(f_3/f_c)^3 Q_{tc}}\right]}_{\max \cong 2.10^{-6} \text{ for } Q_{tc}\cong 1.1}$$

Merit factor in power:

➔also max (≈0.85) for Q_{tc}≈1.1

$$k_P = \frac{4\pi^3\rho}{c}\cdot\underbrace{\frac{1}{(f_3/f_c)^4}}_{f^\circ(Q_{tc})} x_{\max}^{-2}$$

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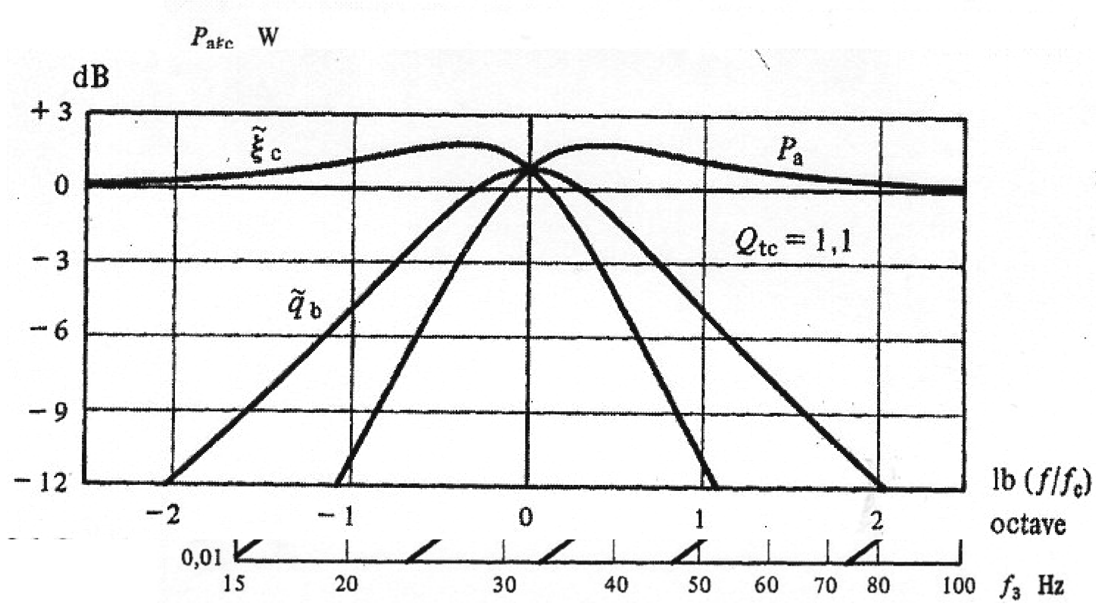
Optimal synthesis

one can show that k_η and k_P are functions of only Q_{tc}, and max for Q_{tc} =1,1

➔ The goal is to obtain Q_{tc} =1,1 then:

$$\eta_c(\max) \cong 2.10^{-6} f_3^3 V_b$$

$$P_{a\xi c}(\max) \cong 0,85 f_3^4 \hat{V}_d^2$$



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Synthesis with a given driver

Chosing the driver removes 1 degree of freedom : $\frac{f_c}{Q_{tc}} \cong \frac{f_s}{Q_{ts}}$

$f_c > f_s \rightarrow Q_{tc} > Q_{ts}$

One sets $Q_{tc} \rightarrow f_c, f_3$ and $x_{\max}$

Yields: $$\alpha \cong \left(\frac{Q_{tc}}{Q_{ts}}\right)^2 - 1$$

Note: empty enclosure: $$V_{ab} = V_{as} / \alpha$$

enclosure + absorbing material: $$V_b = V_{ab} / \beta$$

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Global synthesis

The application ➔ the driver and enclosure specifications

Generally: f_3, Q_{tc}, $P_{\mathrm{a\xi c}}$ and V_{b} are imposed

Requires to estimate *a priori* Q_{mc}, α and β

- Q_{mc} :
  - Empty enclosure: Q_{mc} between 2-10,
  - With filling: Q_{mc} between 2-5
- α from 3 to 10 (see maximization of k_η by an acoustic suspension)
- β=1,2 for a good enclosure

Hypothesis on R_g: $Q_{tc}=Q_{tc0}$

Constraint: V_b imposed

Global synthesis

Constraint: V_b is imposed

1. f_c defined after f_3 and Q_{tc}

$$f_c = \frac{f_3}{\sqrt{\dfrac{\left\{Q_{tc}^{-2} - 2 + \sqrt{(Q_{tc}^{-2} - 2)^2 + 4}\right\}}{2}}}$$

2. $f_s = f_c \Big/ \sqrt{1+\alpha}$

3. Q_{tc} and Q_{mc} ➔ $Q_{ec} = \dfrac{Q_{tc} Q_{mc}}{(Q_{mc} - Q_{tc})}$

4. $V_{as} = \beta\alpha V_b$

5. $V_{ac} = V_{as}/(1+\alpha)$

6. η_c, either by k_η, either by $\eta_c = \left\{\dfrac{4\pi^2}{c^3}\right\}\left\{\dfrac{f_c^3 V_{ac}}{Q_{ec}}\right\}$

7. $P_{e\xi c} = P_{a\xi c} / \ \eta_c$

8. V_d either by k_P, either by $P_{e\xi c} = \left\{\pi\rho c^2\right\}\left\{\dfrac{Q_{ec} f_c \hat{V}_d^2}{V_{ac}}\right\} \cdot \dfrac{1}{\underbrace{x_{\max}^2}_{\sqrt{Q_{tc}^4 / Q_{tc}^2 - \frac{1}{4}}}}$

9. $Q_{es} = \dfrac{Q_{ec}}{\sqrt{1+\alpha}}$

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Input impedance

- See loudspeaker on infinite screen

Measurement of Small Signal Parameters

these parameters can be assessed with electrical measurements (input impedance)

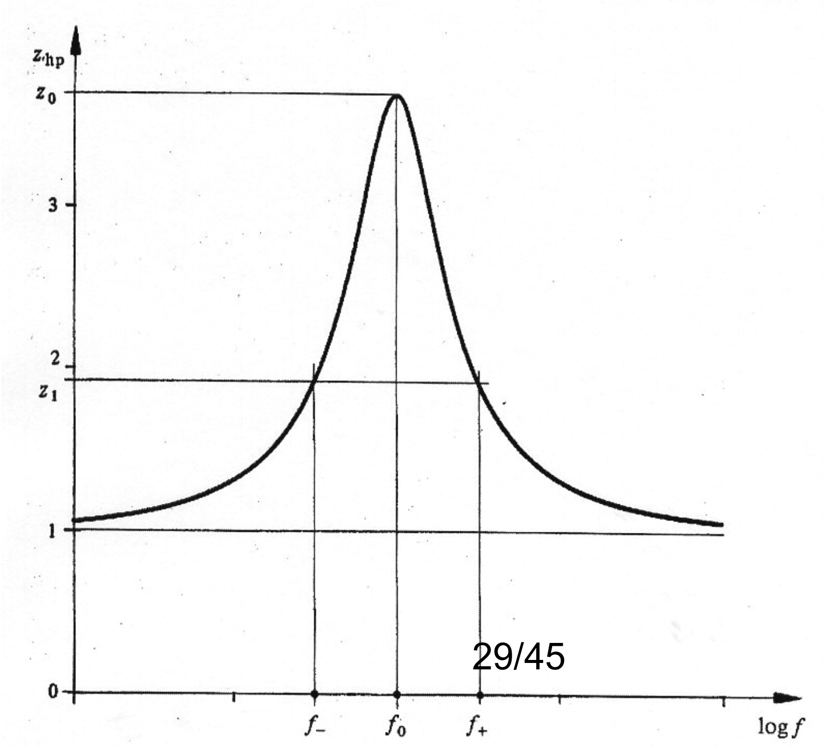
one defines, around f_0 ($z=z_0$), f_+ and f_- so that:

$$z(f_-) = z(f_+) = z_1$$

$$f_0 = \sqrt{f_+ f_-}$$

$$z_1 = \sqrt{\frac{z_0^2 + Q_m^2\left[(f_+ - f_-)/f_0\right]^2}{1 + Q_m^2\left[(f_+ - f_-)/f_0\right]^2}}$$

$$\rightarrow Q_m = \frac{f_0}{f_+ - f_-}\sqrt{\frac{z_0^2 - z_1^2}{z_1^2 - 1}}$$



Measurement of Small Signal Parameters

these parameters can be assessed with electrical measurements (input impedance)

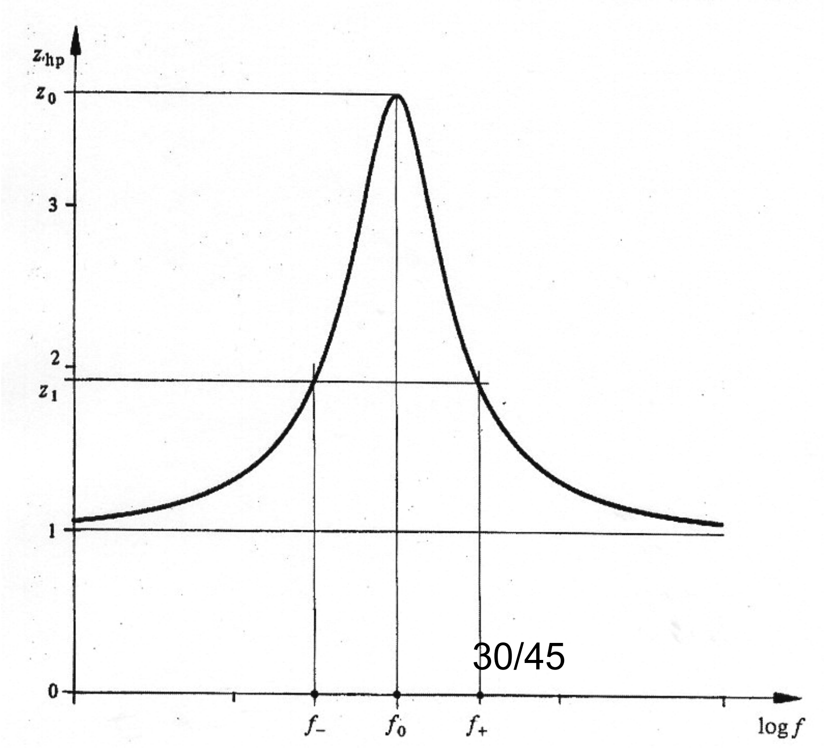
one defines, around f_0 ($z=z_0$), f_+ and f_- so that:

$$z_1 = \sqrt{z_0}$$

$$Q_m = \frac{f_0}{f_+ - f_-}\sqrt{z_0}$$

$$Q_e = Q_m / (z_0 - 1)$$

$$Q_t = Q_m / z_0$$

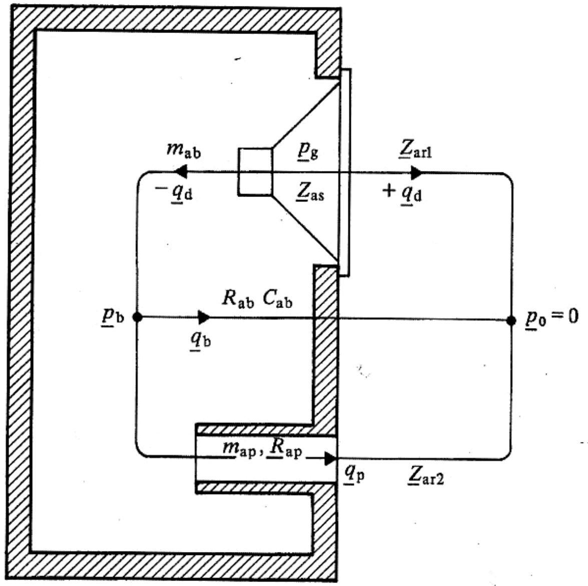


Determination of Small Signal Parameters

- 2 successive tests:
  - on screen
    - f_s
    - Q_{ms}
    - Q_{es}
    - Q_{ts}
  - on empty closed-box(β=1)
    - α
    - $V_{as}=\alpha V_b$
- Alternative method: additional mass

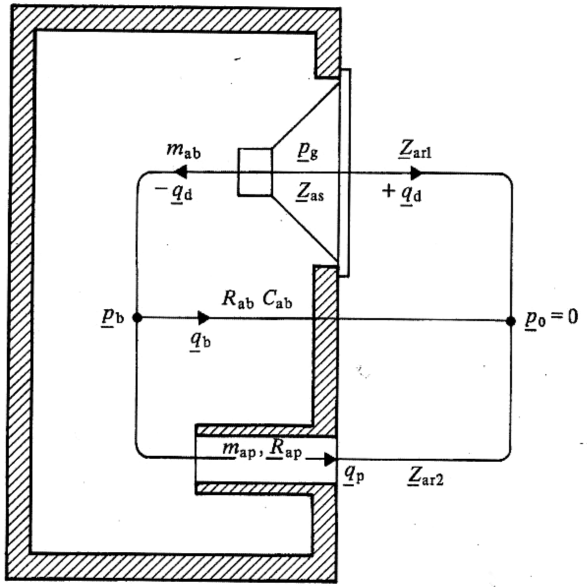
Vented-box enclosure system

- Acoustic components:
  - driver = pressure source p_g and source impedance Z_{as}
  - front radiation = Z_{ar1}
  - enclosure = C_{ab}, R_{ab}, m_{ab}
  - vent = m_{ap}, R_{ap}, radiation impedance Z_{ar2}



Vented-box enclosure system

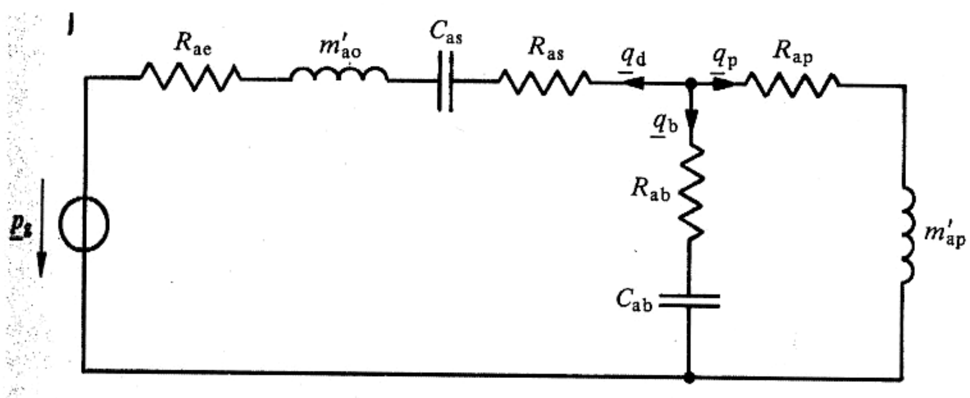
- Radiation resistances neglected
- We group acoustic masses



$$m'_{ao} = m_{as} + m_{ab} + m_{ar1}$$

$$m'_{ap} = m_{ap} + m_{ar2}$$

m_{ap} includes termination corrections



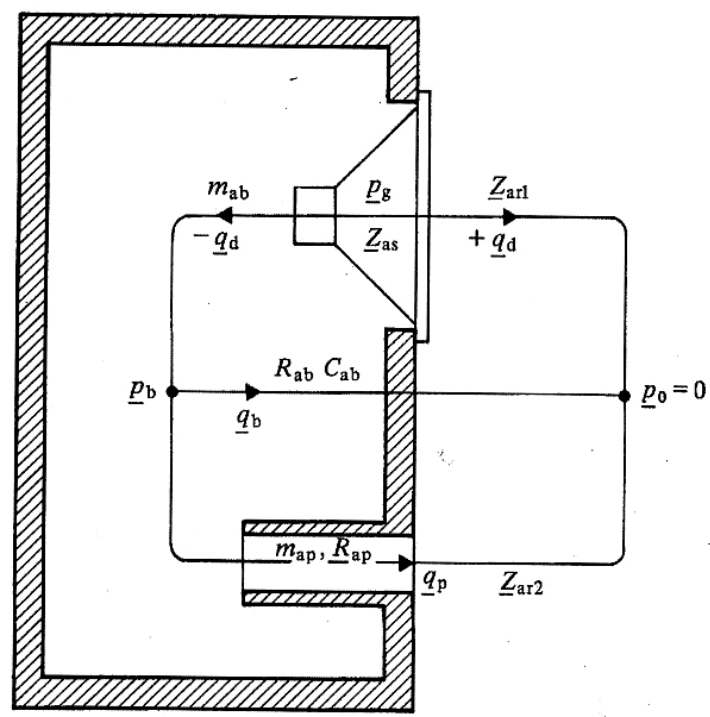
Vented-box enclosure system

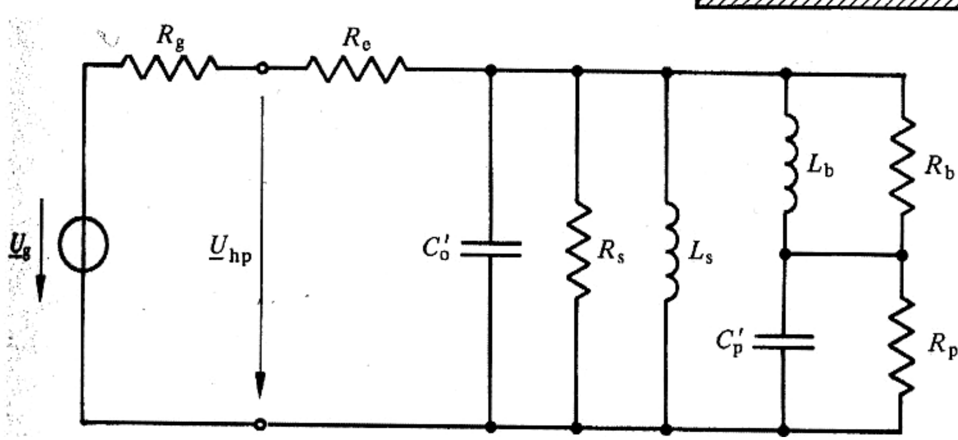
- Electrical equivalent:

$$C'_o = m'_{ao} \frac{S_d^2}{(Bl)^2} \qquad C'_p = m'_{ap} \frac{S_p^2}{(Bl)^2}$$

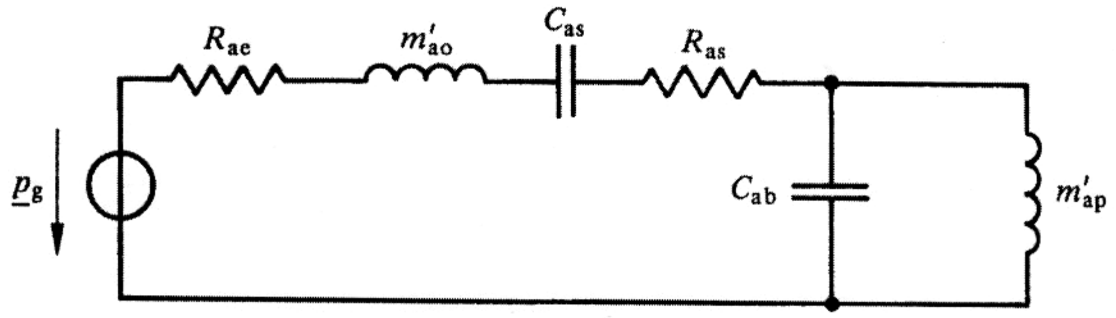
$$L_b = C_{ab} \frac{(Bl)^2}{S_d^2}$$

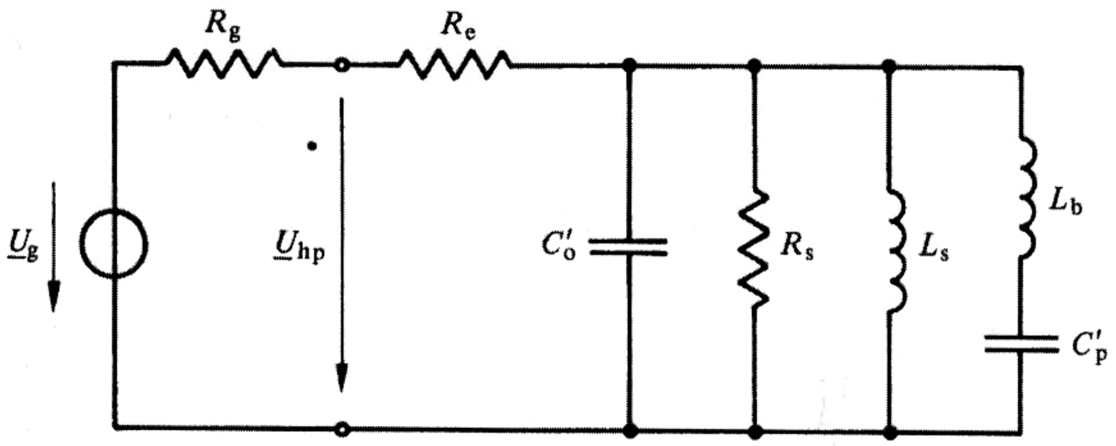
$$R_b = \frac{(Bl)^2}{S_d^2} \cdot \frac{1}{R_{ab}} \qquad R_b = \frac{(Bl)^2}{S_p^2} \cdot \frac{1}{R_{ap}}$$





Lossless enclosure





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Small signals parameters

- *resonance frequency*

$$f_b = \frac{\omega_b}{2\pi} = \frac{1}{2\pi\sqrt{m'_{ap}C_{ab}}} = \frac{1}{2\pi\sqrt{C'_p L_b}}$$

- *normalized value*

$$h = \frac{f_b}{f_s}$$

- *compliance factor*

$$\alpha = \frac{C_{as}}{C_{ab}}$$

- *total quality factor @* ω_s (lossless)

$$\frac{1}{Q_t} = \frac{1}{Q_{ms}} + \frac{1}{Q_{es}}\left[\frac{R_e}{R_e + R_g}\right]$$

Radiated acoustic power

- Knowing q_b, one finds P_a

$$P_a = P_{ao} \left| G_o(j\omega/\omega_s) \right|^2$$

$$G_o(j\omega/\omega_s)$$

$$= \frac{(j\omega/\omega_s)^4 h^{-2}}{(j\omega/\omega_s)^4 h^{-2} + (j\omega/\omega_s)^3 h^{-2} Q_t^{-1} + (j\omega/\omega_s)^2 \left\{1 + h^{-2}(1+\alpha)\right\} + (j\omega/\omega_s) Q_t^{-1} + 1}$$

➔ 4th order high-pass filter

Alignment

- Master the response curve

→ alignment of G_o to a given gauge

→ find values of h, Q_t and α

$$G_o(j\omega/\omega_o) = \frac{(j\omega/\omega_o)^4}{(j\omega/\omega_o)^4 + b_1(j\omega/\omega_o)^3 + b_2(j\omega/\omega_o)^2 + b_3(j\omega/\omega_o) + 1}$$

→

$$h = b_3/b_1$$

$$\alpha = b_2 h - h^2 - 1$$

$$Q_t = 1/\sqrt{b_1 b_3}$$

$$\omega_3/\omega_s = \sqrt{h}(\omega_3/\omega_o)$$

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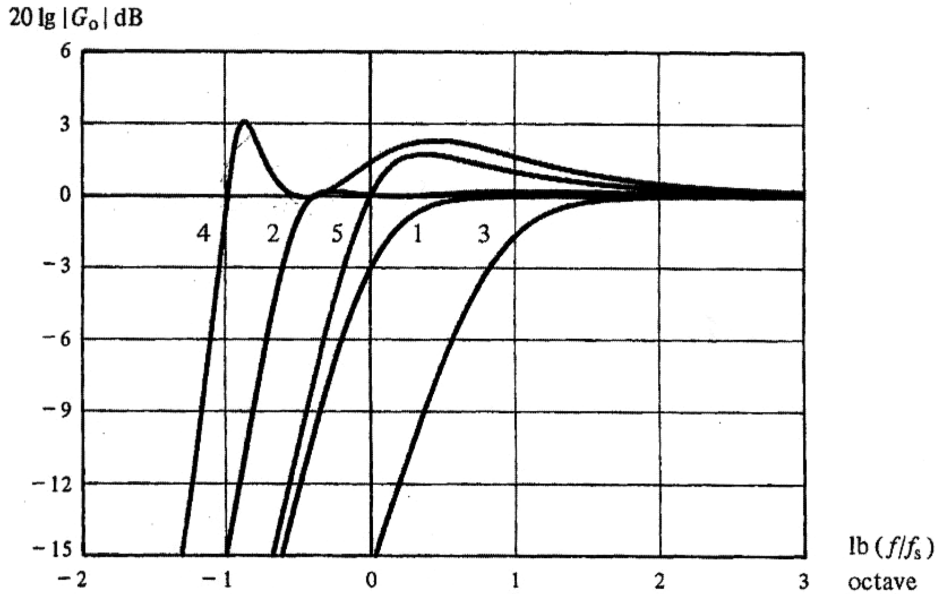
Alignment

No	Type	h	α	Q_t	f_3/f_S
1	B_4	1	1,414	0,384	1
2	$T_4\ (k = 0{,}5)$	0,770	0,589	0,505	0,65
3	$QB_3\ (B = 3{,}25)$	1,414	4,46	0,259	1,77
4	HM 39	0,6	0,3	0,8	0,46
5	HM 50	1	1	0,447	0,87

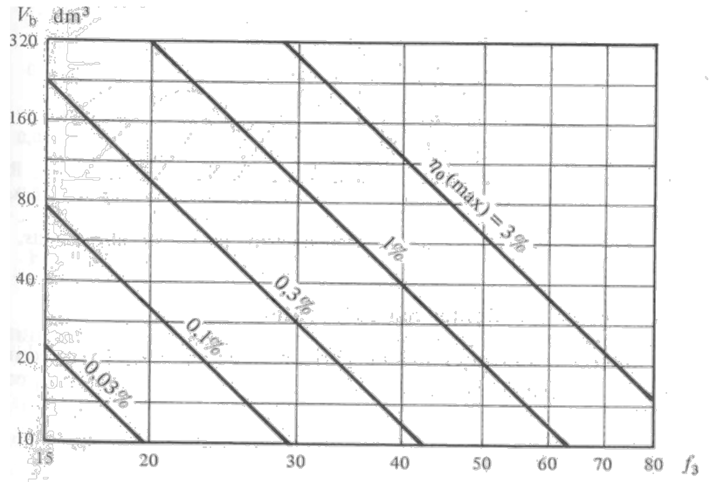


Maximization of efficiency

$$\eta_0(\max) \cong 3{,}9.10^{-6}\, f_3^3 V_b$$

$$\eta_0 = k_\eta f_3^3 V_b$$

$$k_\eta = \underbrace{\frac{Q_t}{Q_{es}}}_{0.8\text{-}0.95} \underbrace{\frac{V_{as}}{V_b}}_{\alpha} \left[\frac{4\pi^2}{c^3} \frac{1}{(f_3 / f_s)^3 Q_t} \right]$$



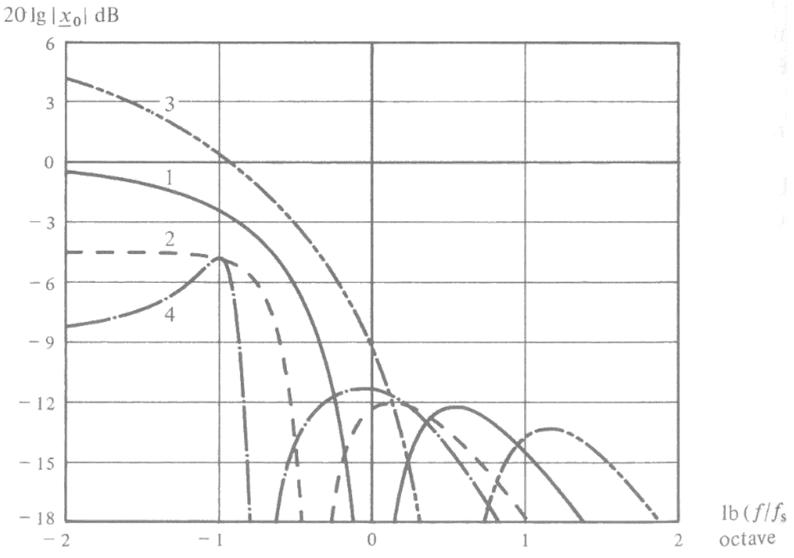
Note: $k_\eta(\max)=3{,}6.10^{-6}$ with B4 alignement

EPFL

Diaphragm elongation

$$\xi_d = \frac{q_d}{j\omega} \quad \Longrightarrow \quad \tilde{x}_0 = \frac{\tilde{\xi}_d}{\tilde{\xi}_o} = |h^2(\tfrac{f_s}{f})^2 - h^4(\tfrac{f_s}{f})^4|.|G_o(\tfrac{f}{f_s})|$$

Where $\xi_o = \dfrac{U_g}{\omega_b h Q_e B\ell}$



Note: $\tilde{x}_0 = 0$ for $f = f_b = h.f_s$

theoretically no displacement at the Helmholtz frequency f_b

In fact, we discarded the individual radiation of the port and the speaker ➔small but not null

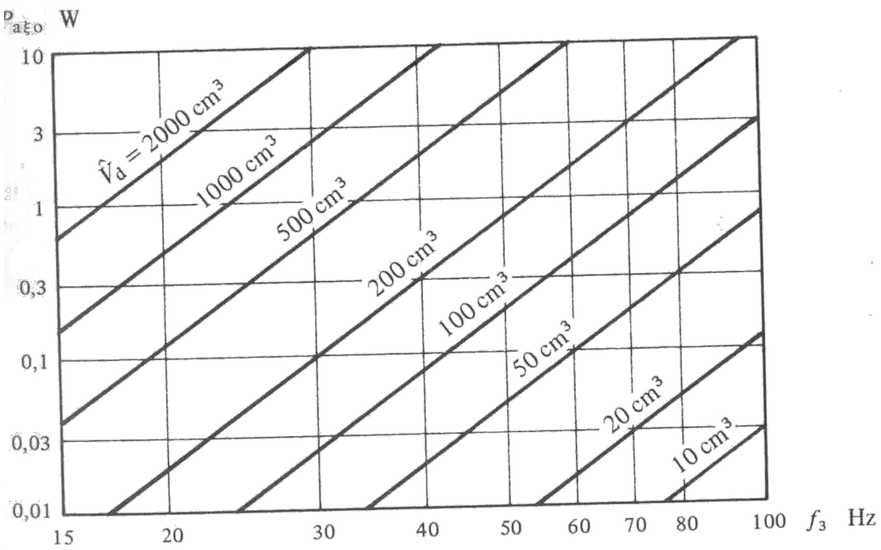
$$\text{Power limited by the elongation: } P_{a\xi_o} = \left\{\frac{4\pi^3\rho}{c}\right\}.\left\{f_b^4\hat{V}_d^2\right\}\frac{1}{x_{max}^2}$$

Maximization of power

$$P_{a\xi_0} = k_P f_3^4 \widehat{V}_d^2$$

where

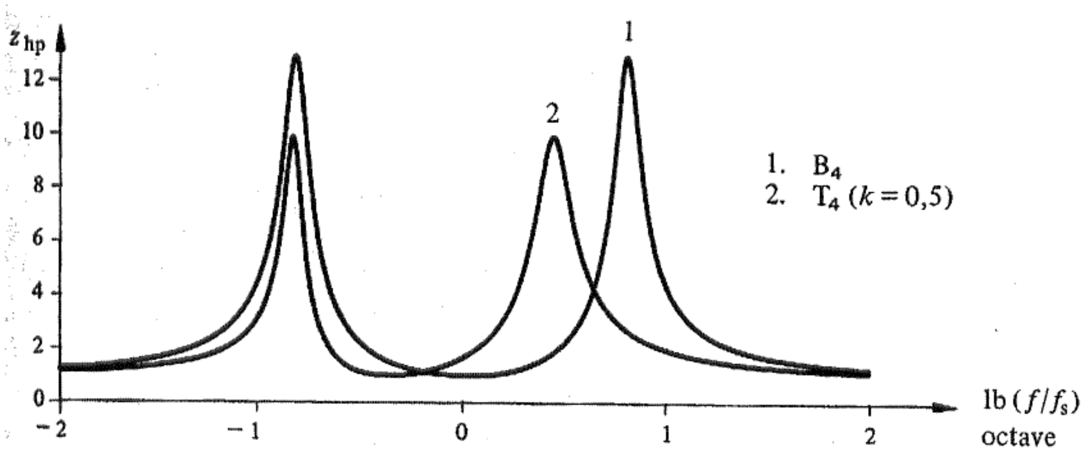
$$k_P = \left\{\frac{4\pi^3\rho}{c}\right\}\left\{\frac{f_b}{f_3}\right\}^4 \frac{1}{x_{\max}^2}$$



Input impedance of vented-box system

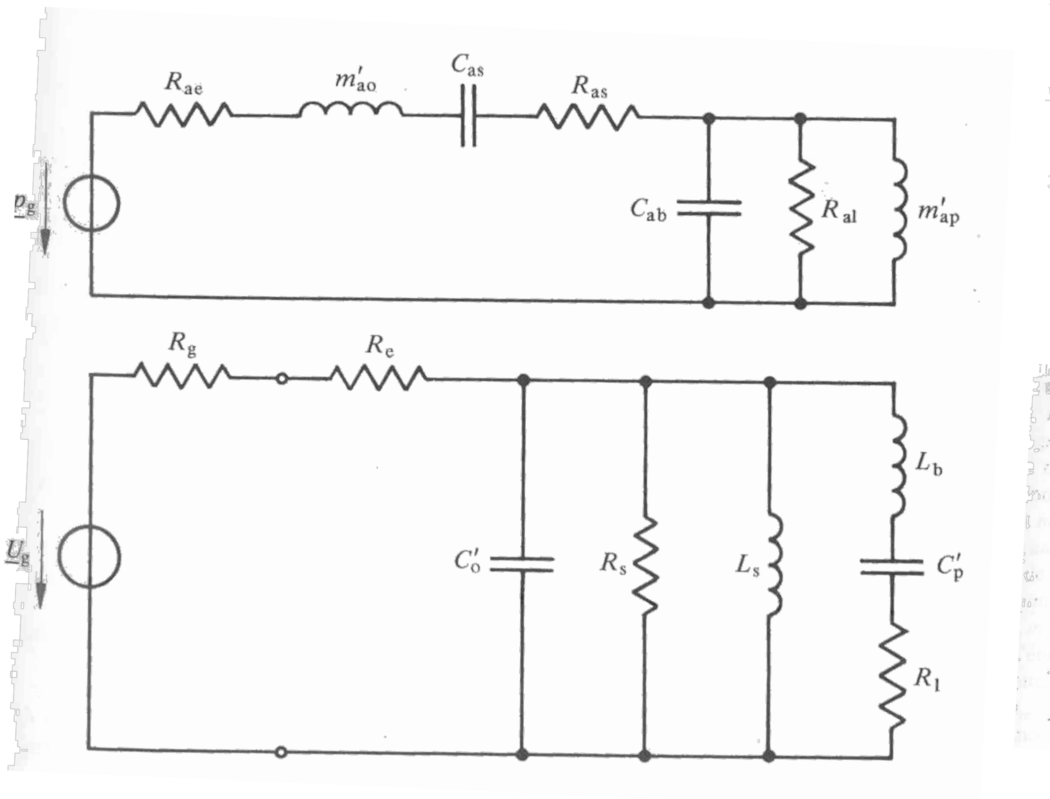
$$\underline{z}_{hp} = \frac{\underline{Z}_{hp}}{R_e}$$

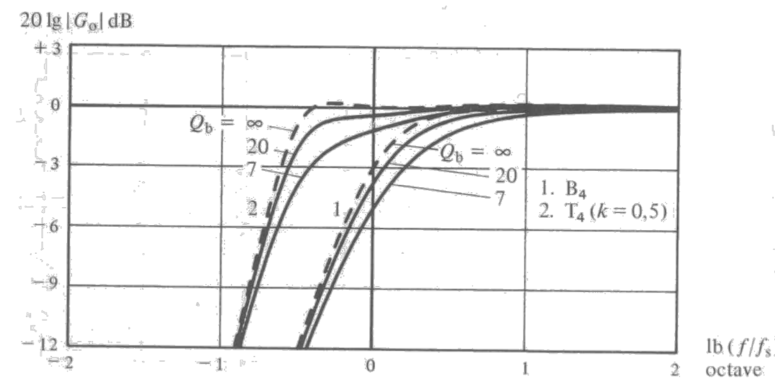
$$= 1 + \left\{ (Q_{ms} / Q_{es}) \frac{(j\omega/\omega_s) Q_{ms}^{-1} \left\{ h^{-2} (j\omega/\omega_s)^2 \right\} + 1}{(j\omega/\omega_s)^4 h^{-2} + (j\omega/\omega_s)^3 h^{-2} Q_t^{-1} + (j\omega/\omega_s)^2 \left\{ 1 + h^{-2}(1+\alpha) \right\} + (j\omega/\omega_s) Q_t^{-1} + 1} \right\}$$

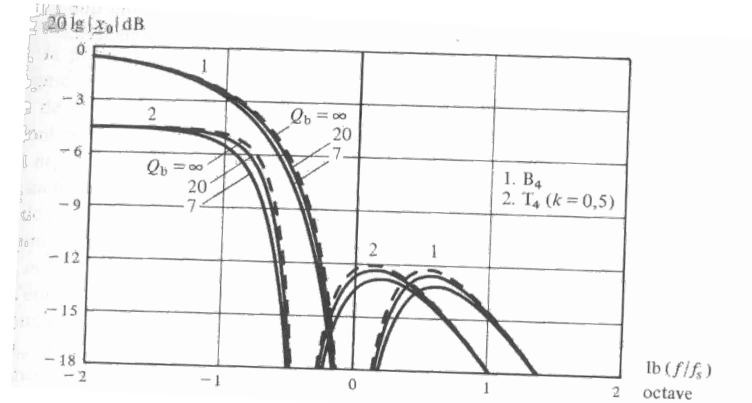


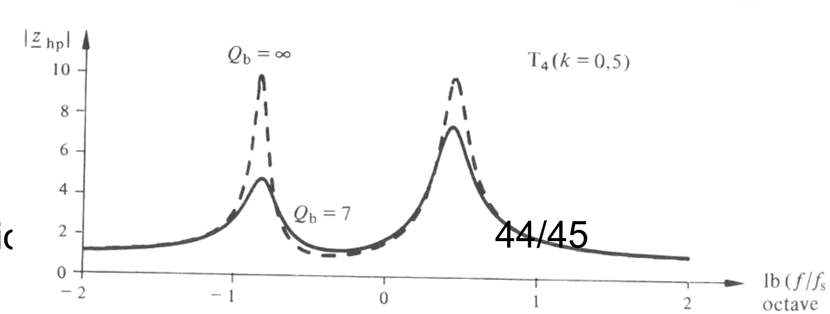
EPFL

With losses

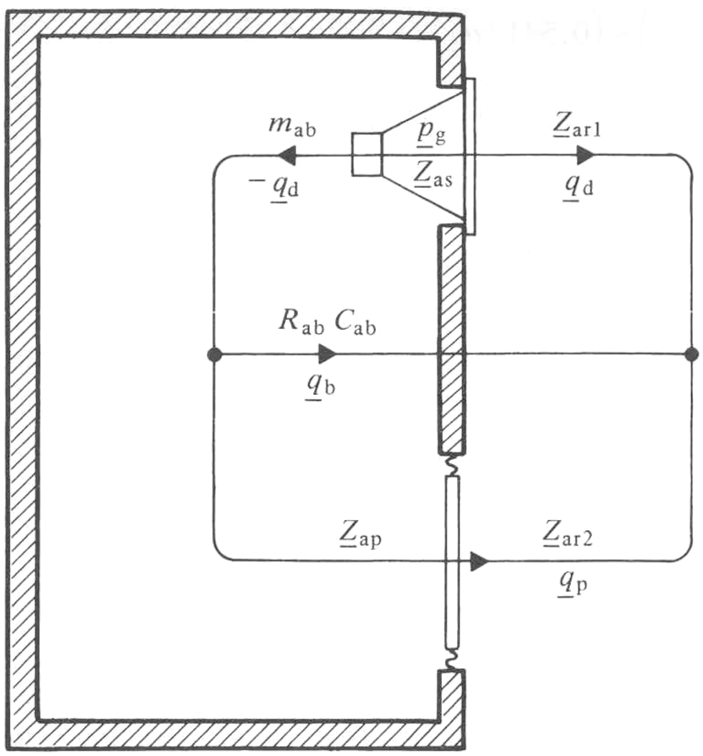

R_{ae}
m'_{ao}
C_{as}
R_{as}
p_g
C_{ab}
R_{al}
m'_{ap}
R_g
R_e
U_g
C'_o
R_s
L_s
L_b
C'_p
R_1


20 lg |G_φ| dB
$Q_b = \infty$
20
7
2
1
1. B_4
2. T_4 (k = 0,5)
lb (f/f_s)
octave


20 lg |x_0| dB
$Q_b = \infty$
20
7
1. B_4
2. T_4 (k = 0,5)
lb (f/f_s)
octave


|z_{hp}|
$Q_b = \infty$
T_4 (k = 0,5)
$Q_b = 7$
lb (f/f_s)
octave

Passive radiator



- **Exercise**: draw the equivalent acoustic circuit