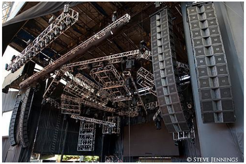
4.4 Antenna and arrays

H. Lissek

November 14, 2018

Introduction



Antenna and arrays in acoustics

- counteract intrinsic directivity of sources/sensors
- control the directivity of sources/sensors

The combination of multiple sources/sensors allows such custom directivities.

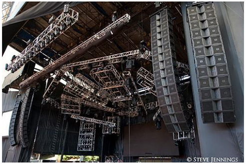
Examples

- line array sound system for controlling speech intelligibility
- microphones arrays for focused sensing

Introduction

Antenna and arrays in acoustics

- counteract intrinsic directivity of sources/sensors
- control the directivity of sources/sensors



The combination of multiple sources/sensors allows such custom directivities.

Examples

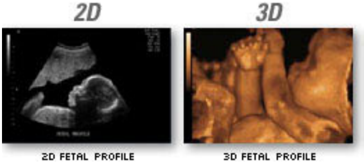
- line array sound system for controlling speech intelligibility
- microphones arrays for focused sensing

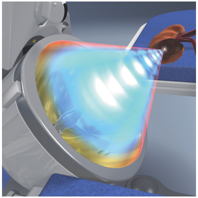
Note

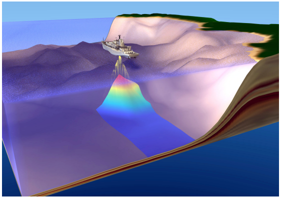
In this lecture, we will distinguish antenna and arrays:

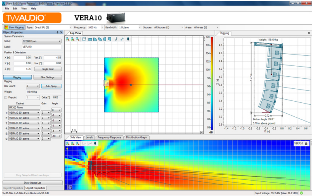
- antenna will refer to continuous arrangements of acoustic sources/sensors
- arrays will refer to discrete arrangements of acoustic sources/sensors

Applications of acoustic arrays/antennas





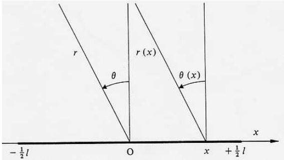


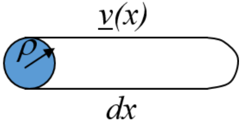


Principle of linear antenna

Linear antenna

Continuum on a segment l of small pulsating sources of flow velocity dq.





In the far field

- $\theta(x) \approx \theta$
- $r(x) \approx r + x.\sin\theta \approx r$

Radiation

Each element $dx \approx$ cylindrical monopole, with
$dq(x) = 2\pi\rho v(x)dx = 2\pi\rho a(x)v_0 dx$
$\rightarrow$ weighting function $a(x)$

Then $p(r,\theta)$ can be computed by integrating between $-l/2$ and $l/2$ the monopoles fields $dp = jZ_c k \underbrace{2\pi\rho a(x) v_0 dx}_{dq(x)} \dfrac{\exp(-jk(r + x\sin\theta))}{4\pi r}$

Field of linear antenna

Sound pressure

$$p(r,\theta) = \int_{x=-l/2}^{+l/2} jZ_c k 2\pi\rho v_0 a(x) \frac{\exp(-jk(r + x\sin\theta))}{4\pi r} dx$$

$$= \underbrace{jZ_c k \underbrace{2\pi\rho l v_0}_{q_0} \frac{\exp(-jkr)}{4\pi r}}_{\text{monopole field}} \underbrace{\frac{1}{l} \int_{x=-l/2}^{+l/2} a(x)\exp(-jk_\theta x)dx}_{D_0(\theta)}, \text{ with } k_\theta = k\sin\theta$$

Fourier transform

Since $a(x) = 0$ outside segment l, we can integrate over $]-\infty;+\infty[$:

$$\int_{-\infty}^{+\infty} a(x)\exp(-jk_\theta x)dx = F_x|a(x)|$$

The directivity can be computed as a Fourier transform - over space quantities - of the distribution $a(x)$.

Application

This example yields two different perspectives for using antenna theory:

- Analysis : from $a(x)$ deriving the directivity $D_0(\theta)$
- Synthesis: specifying a directivity $D_0(\theta)$ to identify the target velocity distribution $a(x)$

However the second scenario is impractical: D_0 only specified for k_θ such as $|\sin\theta| < 1$ (incomplete information).
$a(x)$ would be defined from $-\infty$ to ∞ but we need limited dimensions in practice.

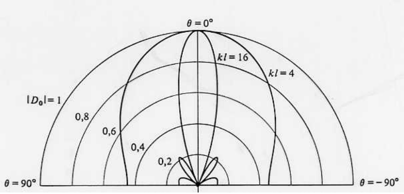
Particular cases

1. Symmetric antenna:
   $a(x)$ pair function $\rightarrow$ $D(\theta)$ also pair function.
   $D_0(\theta) = \frac{1}{l}\int_{-l/2}^{+l/2} a(x)\cos(-k_\theta x)dx$
2. Uniform Linear Antenna (ULA): $a(x) = 1,\ \forall x \in [-\frac{l}{2}; \frac{l}{2}]$
   $D_0(\theta) = \frac{1}{l}\int_{-l/2}^{+l/2} \cos(-k_\theta x)dx = \frac{\sin(\frac{k_\theta l}{2})}{\frac{k_\theta l}{2}} = \mathrm{sinc}(\frac{k_\theta l}{2})$

ULA representations

Polar representation

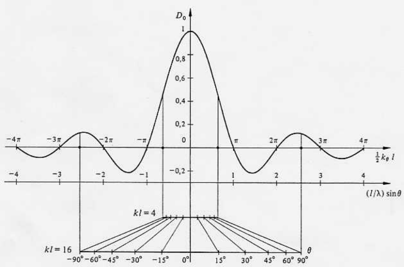
Directivity diagram in polar coordinates for a single antenna
Highlight a principal lateral lobe (normal to the antenna)
+ eventual secondary lobes, if $kl > 2\pi$



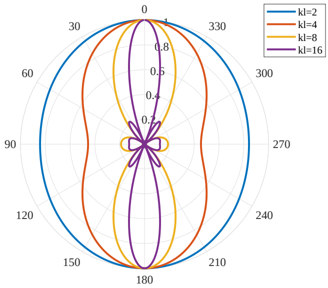
Cartesian representation

Cartesian representation of $D(\theta)$ as a function of $k_\theta l$.
Allows comparing:

- 2 antennas of different lengths, at same frequency
- 2 antennas of same length, for different frequency ranges



ULA properties



- Main lobe is normal to the antenna axis $\rightarrow$ **"broadside"** antenna
- **Directivity factor Δ**: represents the ratio between a useful signal (targeted direction) and the noise (diffuse field).

$$\Delta = \frac{4\pi}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} D_0^2(\theta,\phi)\sin(\theta)d\theta d\phi}$$

- **Radiation width θ_{-3}** (half-power):

  Defined by $\mathrm{sinc}(\frac{kl}{2}\sin\theta_{-3}) = \frac{1}{\sqrt{2}}$

  Here, for $\theta_{-3} < 30^\circ$, $\theta_{-3} \approx 51\frac{l}{\lambda}$

Phased linear antenna

Now if the distribution $a(x)$ is a phase weighting:

$$a(x) = \exp(-j\beta x), \text{ with } \beta \in \mathbb{R}$$

According to the former theory (Fourier transform), the directivity reads:

$$D_0(\theta) = \frac{\sin\left(\frac{l(k_\theta + \beta)}{2}\right)}{\frac{l(k_\theta + \beta)}{2}} = \text{sinc}\left(\frac{l(k_\theta + \beta)}{2}\right)$$

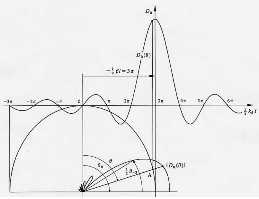
The phased linear antenna allows controlling the orientation of the main lobe of a uniform antenna:

Polar representation

rotated by $\theta_0 = -\text{asin}(\frac{\beta}{k})$ in polar representation

Cartesian representation

shifted by $-\frac{\beta l}{2}$ in the cartesian representation



directivity of a phased linear antenna with $\beta = -k$ and $kl = 6\pi$

Note: not only the orientation of the main lobe is rotated, but also a deformation occurs:

Example: for $\theta_0 = \frac{\pi}{2}$ (or $\beta = -k$), we have $\theta_{-3dB} \approx 110\frac{\sqrt{l}}{\lambda}$

Interference microphone

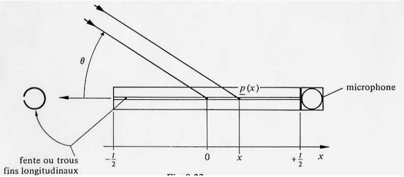
This is an example of an actual antenna concept used as a microphone.
It is mainly used as highly focusing microphone for the film/radio industry.

Same as phased linear antenna with longitudinal lobe (lineic phase shift $\beta = -k$). But how to achieve this?

Interference microphone

This is an example of an actual antenna concept used as a microphone.
It is mainly used as highly focusing microphone for the film/radio industry.

Same as phased linear antenna with longitudinal lobe (lineic phase shift $\beta = -k$). But how to achieve this?



Cylindrical tube (plane mode) + narrow holes (or continuous slot), with a microphone at the termination (sensitivity M_p)
→ Pressure p(x) created in x by far field sources induces a progressive plane wave in the tube up to the microphone

Interference microphone

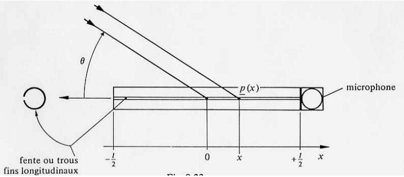
Microphone output voltage = accumulation of infinitesimal plane waves.
Each plane wave contributes to the global sensitivity (voltage) of the microphone:

$M'(x) = M'_p \exp\left[-jk\left(\frac{l}{2} - x\right)\right] = M'_0 \exp(jkx)$, M'_p= lineic sensitivity $\frac{M_p}{l}$

It is then a phased antenna with $\beta = -k$.

Then the total sensitivity becomes: $\frac{M(\theta)}{M_p} = \text{sinc}\left(\frac{kl(1-\cos\theta)}{2}\right)$

(here θ is defined wrt the axis of the microphone $\rightarrow \cos\theta$)



Cylindrical tube (plane mode) + narrow holes (or continuous slot), with a microphone at the termination (sensitivity M_p)
$\rightarrow$ Pressure p(x) created in x by far field sources induces a progressive plane wave in the tube up to the microphone

Superdirective antenna

A particular case of phased linear antenna: $-\beta > k$:

- lobe centered at $\theta_0 > \dfrac{\pi}{2}$:
  a part of the lobe becomes "virtual" ($|\sin\theta| > 1$!)
- logitudinal real lobe ("endfire"):
  radiation angle narrower than the phased linear antenna

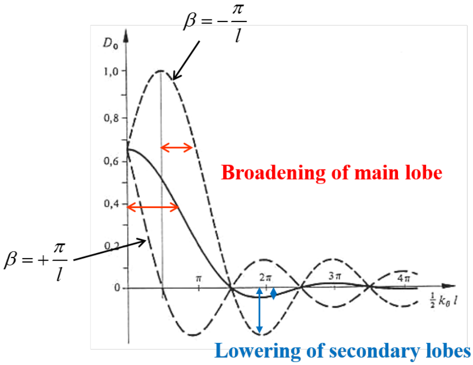
Cosinus weighting

Let's assume $a(x) = \cos\frac{\pi x}{l}$

We can decompose into exponentials

$\rightarrow$ sum of 2 phased linear antennas with $\beta = \pm\frac{\pi}{l}$

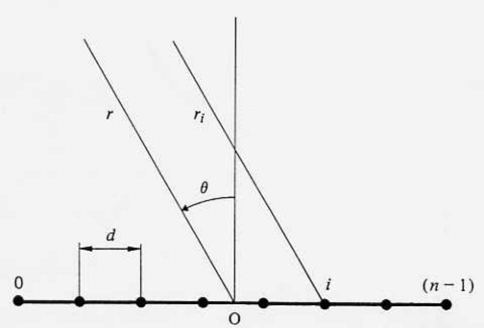
$$D_0(\theta) = \frac{1}{2}\left[\operatorname{sinc}\left(\frac{k_\theta l - \pi}{2}\right) + \operatorname{sinc}\left(\frac{k_\theta l + \pi}{2}\right)\right]$$



Linear array

Now, we will consider n sources/sensors, regularly arranged on a line with distance d from each other (total length $l = (n-1)d$).

- uniform linear array: small pulsating sources with same flow q
- phased linear array: small pulsating sources with flow $q = q_0 \exp(-ji\psi)$, $i \in [0; n-1]$



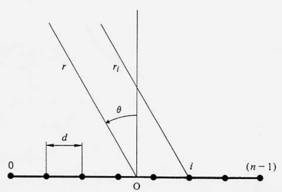
Uniform linear arrays

Each source presents the same flow $\rightarrow a(i) = 1,\ i \in [0; n-1]$.

Far field

$$p(r,\theta) = p_m \sum_{i=0}^{n-1} \exp[-jk(r_i - r)]$$

Let's write $r_i = r + \left(i\,d - \frac{l}{2}\right)\sin\theta$



Pressure p builds a geometrical suite:

$$p(r,\theta) = p_m \sum_{i=0}^{n-1} \exp\left[-jk_\theta (id - \frac{l}{2})\right]$$

$$= p_m \exp(jk_\theta \frac{l}{2}) \sum_{i=0}^{n-1} \exp(-jk_\theta d)^i$$

$$= p_m \exp(jk_\theta \frac{l}{2}) . \frac{\exp(-jnk_\theta d) - 1}{\exp(-jk_\theta d) - 1}$$

Then $D(\theta) = \dfrac{p(r,\theta)}{np_m} = \dfrac{\mathrm{sinc}\left(\frac{nk_\theta d}{2}\right)}{\mathrm{sinc}\left(\frac{k_\theta d}{2}\right)} = \dfrac{\text{ULA of length } l' = nd = l + d}{\text{ULA of length } d}$

Uniform linear arrays

$$D(\theta) = \frac{p(r,\theta)}{np_m} = \frac{\mathrm{sinc}\left(\dfrac{nk_\theta d}{2}\right)}{\mathrm{sinc}\left(\dfrac{k_\theta d}{2}\right)} = \frac{\text{ULA of length } l' = nd = l + d}{\text{ULA of length } d}$$

- if $d < \dfrac{\lambda}{4}$: $D(l = d) \approx 1 \rightarrow$ array $=$ ULA of length $l + d$
- if $d > \dfrac{\lambda}{4}$: higher directivity than antenna of same dimension
- for $d = m\lambda$, with $m \in \mathbb{N}$, $D = 1$ not only for $\theta = 0$: some secondary lobes might be as powerful as the main lobe...

Phased linear arrays

Let's consider: $q_i = qexp(-ji\psi), i \in [0; n-1]$.

Then $$D_0(\theta) = \frac{\text{sinc}\left[\dfrac{n(k_\theta d + \psi)}{2}\right]}{\text{sinc}\left[\dfrac{k_\theta d + \psi}{2}\right]} = \frac{\text{PLA of length } l' = nd}{\text{PLA of length } d}$$

Once again, the phase ψ will orientate the lobe by $\theta_0 = -\text{asin}\left(\frac{\psi}{kd}\right)$

Array of directive sources

Let's consider n identical sources with intrinsic directivity $D_S(\theta)$, orientated along same direction (coincident main lobes).
The array directivity is simply derived by multiplying the directivities:

- of the array $D_0(\theta)$
- of each directive source $D_S(\theta)$

$$\underbrace{D_r(\theta)}_{\text{array with directive sources}} = \underbrace{D_S(\theta)}_{\text{directivity of sources}} \underbrace{D_0(\theta)}_{\text{array with monopoles}}$$

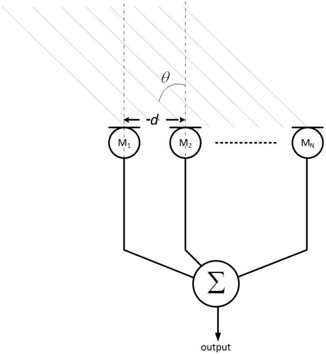
Example: uniform array of n small uniform linear antenna of length $a < d$:

$$D_R(\theta) = \underbrace{\left[\operatorname{sinc}\left(\frac{k_\theta a}{2}\right)\right]}_{\text{directivity of one antenna}} \cdot \underbrace{\left[\frac{\operatorname{sinc}\left(\frac{n k_\theta d}{2}\right)}{\operatorname{sinc}\left(\frac{k_\theta d}{2}\right)}\right]}_{\text{directivity of the array}}$$

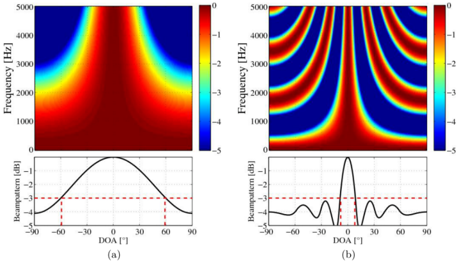
Objectives

- Control directivity
- reduce secondary lobes
- lower (or increase) θ_{-3dB}

Delay and sum beamforming



$$V_{\text{output}} = \frac{1}{N}\sum_{i=1}^{N} \exp\left[j\frac{2\pi i d \sin\theta}{c}\right]$$



Beampatterns of a delay-and-sum beamformer composed of two microphones spaced by (a) 3.5 cm and (b) 20 cm

Delay and sum beamforming

Microphone array in the Cathedral of Lausanne (M. Rossi, F. Bongard, 2003)