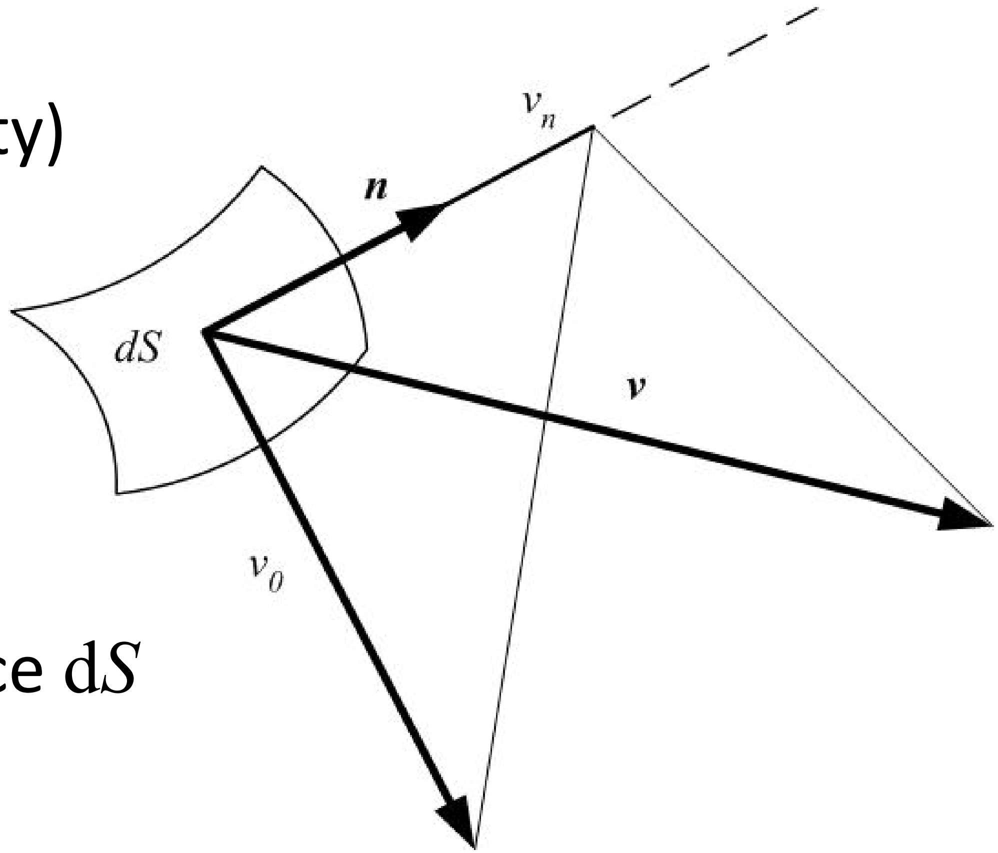
4.3 Sound radiation: Mechanical – acoustical coupling

Fundamental hypothesis

- No disbonding
- No turbulence (no vorticity)



Projecting velocity on surface $\mathrm{d}S$

$$\vec{v}_0.\vec{n} = \vec{v}.\vec{n} = v_n$$

EPFL

Definition: volume flow velocity

- Volume flow velocity : it is the volume displaced, by time unit, by the *speaker*
- For the element dS:

$$dq = v_n . dS = \boldsymbol{v} . \boldsymbol{n} . dS \qquad \mathrm{m^3/s}$$

- For the whole speaker's face:

$$q = \int_S v_n . dS \qquad \mathrm{m^3/s}$$

Vibrating sphere

Vibrating sphere, with radial movement: $\underline{v}_r(\theta,\phi)$

General case: no symmetry: computation of $\underline{p}(r,\theta,\phi)$ after d'Alembert equations

In the following, we consider revolution symmetry

➔ only dependance on θ

Vibrating sphere

In the general case, $\underline{p}(r,\theta)$ can be splitted after:

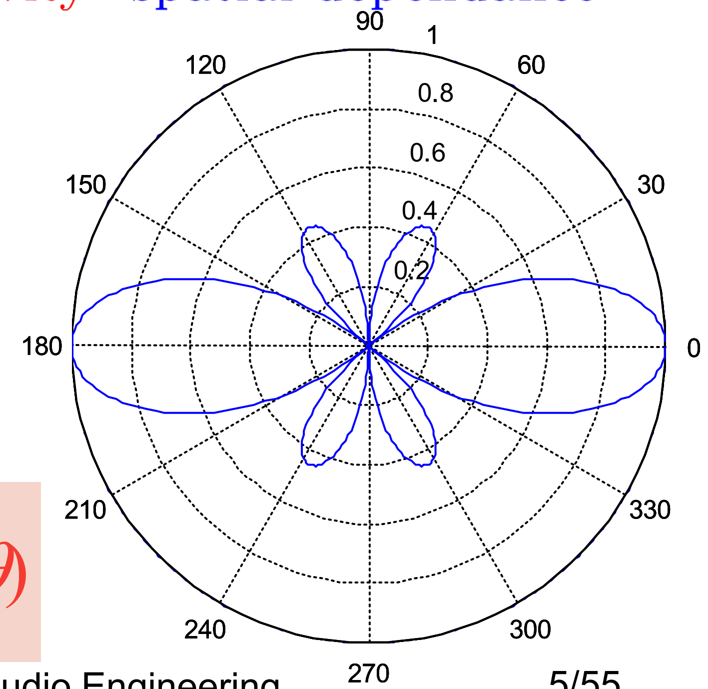
$$p(r,\theta) = \sum_{m=0}^{\infty} \alpha_m \underbrace{\mathcal{P}_m(\cos\theta)}_{\text{directivity}}. \quad \underbrace{h_m^{(2)}(kr)}_{\text{spatial dependance}}$$

$$\mathcal{P}_0(\cos\theta) = 1$$

$$\mathcal{P}_1(\cos\theta) = \cos\theta$$

$$\mathcal{P}_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

$$\mathcal{P}_3(\cos\theta) = \frac{1}{2}(5\cos^3\theta - 3\cos\theta)$$



EPFL

Vibrating sphere

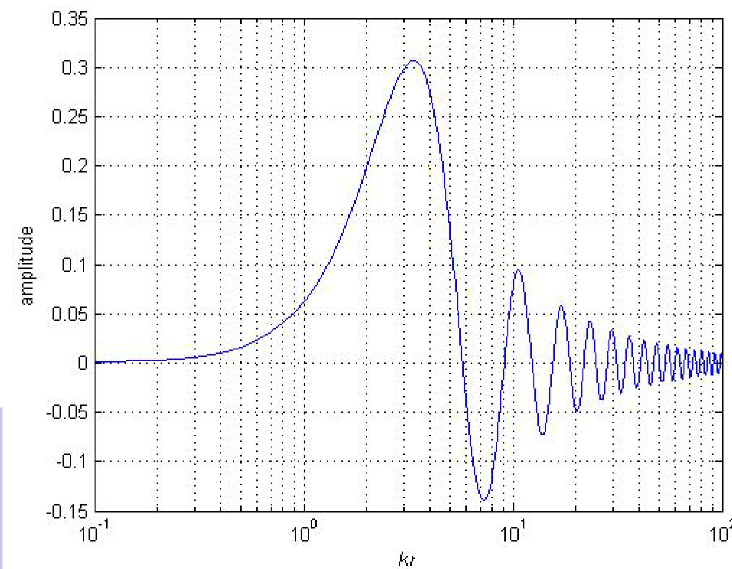
In the general case, $p(r,\theta)$ can be splitted after:

$$p(r,\theta) = \sum_{m=0}^{\infty} \alpha_m \underbrace{\mathcal{P}_m(\cos\theta)}_{\text{directivity}} . \qquad \underbrace{h_m^{(2)}(kr)}_{\text{spatial dependance}}$$

$$h_0^{(2)}(kr) = j\frac{\exp(-jkr)}{kr}$$

$$h_1^{(2)}(kr) = -\frac{\exp(-jkr)}{kr}\left[1+\frac{1}{jkr}\right]$$

$$h_2^{(2)}(kr) = -j\frac{\exp(-jkr)}{kr}\left[1+\frac{3}{jkr}+\frac{3}{(jkr)^2}\right]$$



Resolution

We want to:

- determine acoustic pressure $p(r,\theta)$
- after the velocity of the sphere $v_r(\theta) = \vec{v}(a).\vec{e}_r$
- and then apply Euler's law at sphere surface (r=a)

$$\vec{\nabla} p = -\rho \partial_t \vec{v}$$

Hypothesis: radial movement

➔decomposition into "Spherical Harmonics" (SH)

SH provide the expression of $\underline{\alpha_m}$ determining $\underline{p}(r,\theta)$.

$$v_r(\theta) = \sum_{m=0}^{M} v_{rm} \mathscr{P}_m(\cos\theta) \text{ where } v_{rm} = j\frac{\alpha_m}{kZ_c}\left[\partial_r h_m^{(2)}(kr)\right]_{r=a}$$

Far field

In Sphrical Harmonics (SH), it is possible to group terms by (negative) powers of (kr)

kr>>1, $p(r,\theta) \approx \frac{\exp(-jkr)}{kr}$ comprising at least one term in 1/kr,

➔its gradient (and then $v_r = v.e_r$) comprises at least 1 term in 1/kr,

$$\nabla \underline{p} = \underbrace{\frac{\partial p}{\partial r}}_{\substack{\text{at least one term} \\ \text{in } 1/kr}} e_r + \underbrace{\frac{1}{r}\frac{\partial p}{\partial \theta}}_{\substack{\text{1st term} \\ \text{in } 1/(kr)^2}} e_\theta + \cancel{\frac{1}{r \sin\phi}\frac{\partial p}{\partial \phi}} e_\phi = -\rho \frac{\partial v}{\partial t}$$

(though v_θ has at least terms in 1/(kr)^2)

If v_{r0}≠0, $\underline{v}(r,\theta)$ is radial in far field (kr>>1) and $v \cong v_r = j\frac{1}{kZ_c}\partial_r p \cong \frac{p}{Z_c}$

➔ as spherical waves: progressive plane waves **in the far field**

Far field - Directivity

Regrouping by powers of kr: $p(r,\theta) \propto Z_c \underline{v}_r(\theta) \dfrac{\exp(-jkr)}{kr}$

$$\rightarrow \quad p(r,\theta) = \underline{D}(\theta) . \underline{p}_{m0}$$

proportionality depends on $\cos(\theta)$ after the SH:

directivity $D(\theta)$

Intensity: $I \cong \tilde{p}^2 / Z_c = \underbrace{D^2(\theta)}_{\text{directivity}} \quad \underbrace{I_{m0}}_{\text{monopole of same flow velocity}}$

Same properties as pulsating sphere except directivity (direction-dependent amplitude)

Properties

- Directivity involves $\underline{\alpha}_m$ ➔ depends on ka
- Limiting the series of $\underline{v}_r$ to M terms, we show for ka>>M:

$$\underline{D}(\theta) = \underline{v}_r(\theta)\frac{\exp(-jka)}{jka\underline{v}_{r0}} \quad \text{and} \quad \underline{p}(r,\theta) = Z_c\underline{v}_r(\theta)a\frac{\exp\left[-jk(r-a)\right]}{r}$$

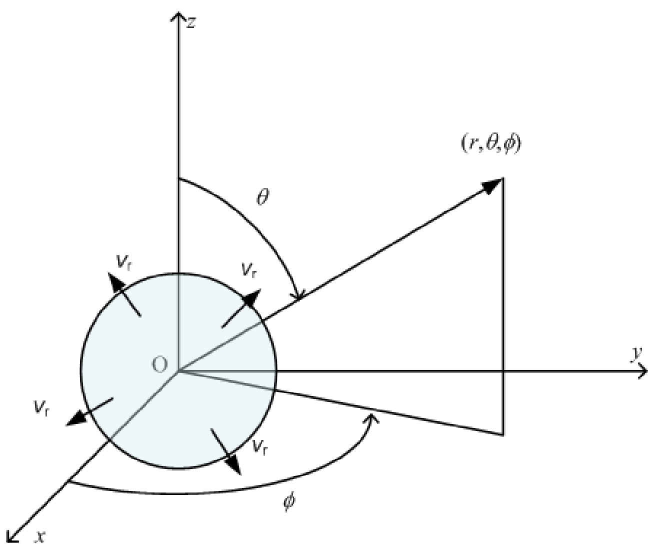
If vibrating sphere is wider than $M\lambda$, $\underline{p}(\theta)$ only depends on $\underline{v}_r(\theta)$

➔ Same as if each surface element on the radiating sphere radiates radially in this direction (θ)

EPFL

Examples of sources

Pulsating sphere



Vibrating sphere, with uniform radial movement:

$$v_r(a,\theta,\phi) = v_r(a) = \frac{q}{4\pi a^2}$$

Acoustic wave equation in 3D:

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Acoustic wave equation in spherical coordinates:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\frac{\partial^2 (rp)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2} = 0$$

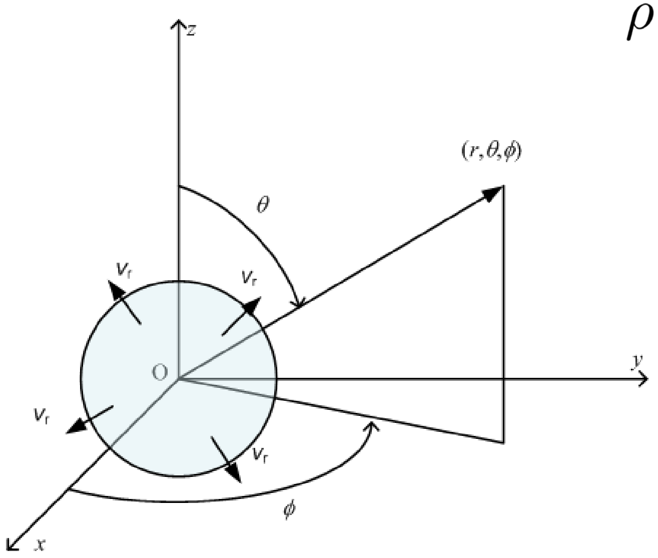
General solutions (p_+ unknown):

$$p(r,t) = p_+ \frac{e^{j(\omega t - kr)}}{r}$$

EPFL

Pulsating sphere

By applying Euler on the boundary of the Sphere:



$$\rho\frac{\partial v_r(a)}{\partial t} = \frac{\partial p}{\partial r}_{|r=a} \rightarrow p_+ = \rho\omega\frac{q}{4\pi}\frac{e^{+jka}}{ka-j}$$

Then the acoustic field reads

$$p(r) = \rho c k q \frac{e^{-jk(r-a)}}{4\pi r(ka-j)}$$

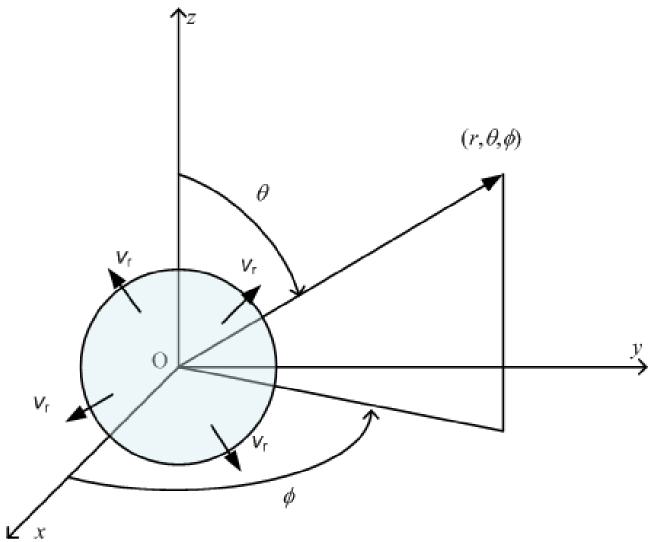
$$v(r) = \left[k - \frac{j}{r}\right] q \frac{e^{-jk(r-a)}}{4\pi r(ka-j)}$$

$$I(r) = \frac{\tilde{p}^2}{Z_c} = \frac{Z_c k^2 \tilde{q}^2}{16\pi^2 r^2((ka)^2+1)}$$

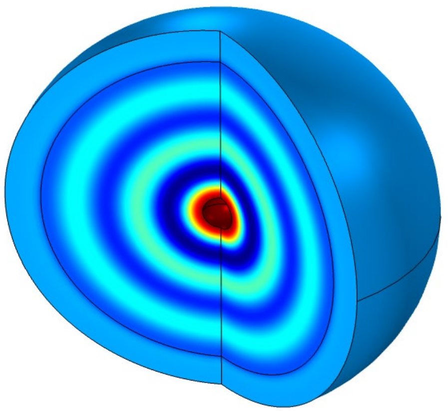
EPFL

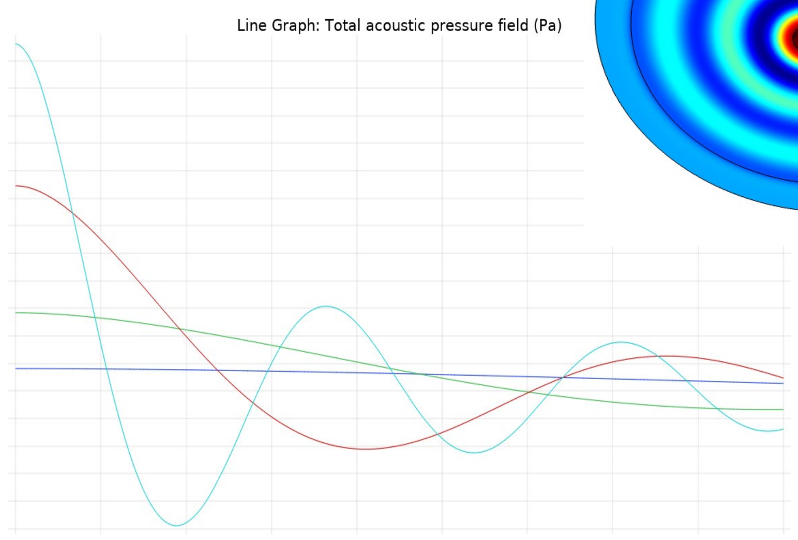
Pulsating sphere: properties

Pulsating sphere is omnidirectional (don't depend on θ and ϕ)



pressure distribution (Pa) radiated by a pulsating sphere of radius 10 cm at 100 Hz



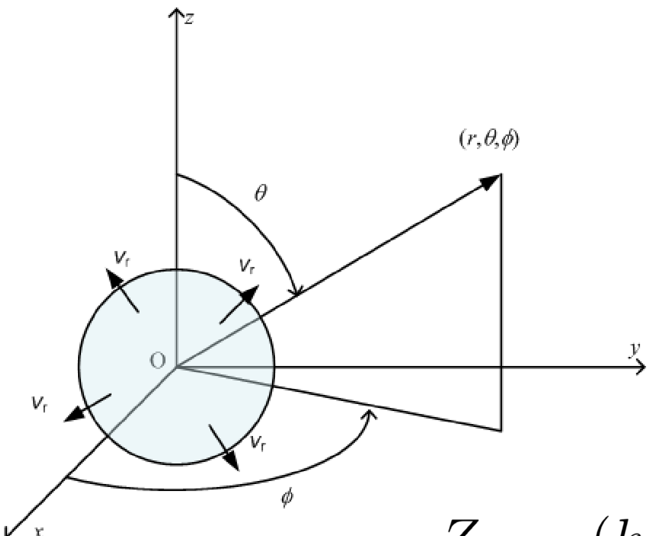


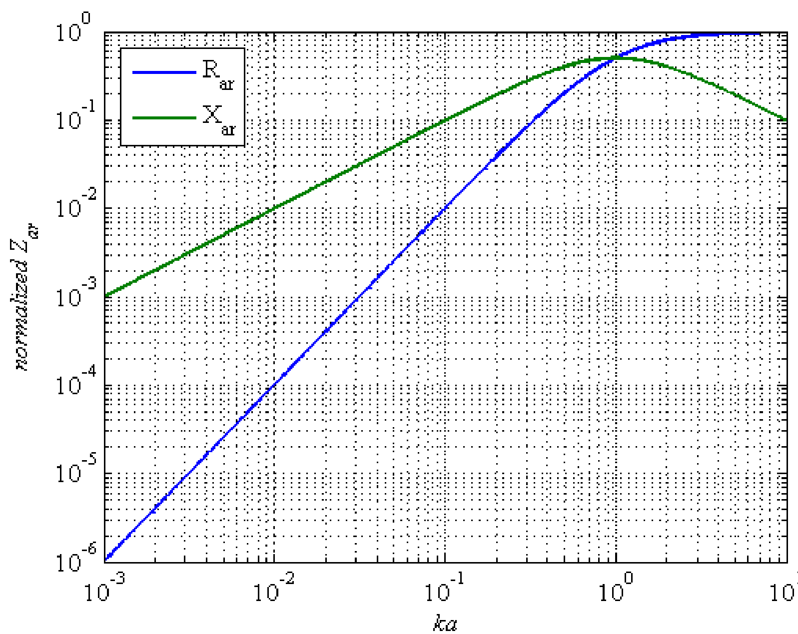
EPFL

Pulsating sphere: properties

Acoustic radiation impedance: accounts for the reaction of the medium to the vibration of the sphere

$$Z_{ar} = \frac{p(a)}{q} = \frac{Z_c}{4\pi a^2}\frac{(ka)^2 + j(ka)}{(ka)^2+1}$$





$$\Re(Z_{ar}) = R_{ar} = \frac{Z_c}{4\pi a^2}\frac{(ka)^2}{(ka)^2+1}$$

$$\Im(Z_{ar}) = X_{ar} = \frac{Z_c}{4\pi a^2}\frac{(ka)}{(ka)^2+1}$$

EPFL

Relationship between P_{as} and Z_{ar} (Z_{mr})

$$\begin{cases} R_{ar} = r_r.Z_c / S = Z_c \dfrac{k^2}{4\pi\left[(ka)^2+1\right]} \\ P_{aS} = \tilde{q}^2.Z_c.\dfrac{k^2}{4\pi\left[(ka)^2+1\right]} \end{cases}$$

$$\boxed{P_{aS} = R_{ar}.\tilde{q}^2 = R_{mr}.\tilde{v}_r^2}$$

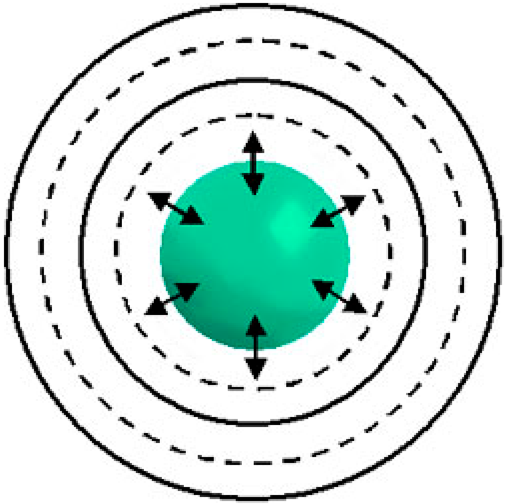
Formal analogy with $P_e = R.I^2$

- Justify the importance of $\underline{Z}_{ar}$ et $\underline{Z}_{mr}$
- Generalized to other sound sources

The monopole

Pulsating sphere with ka<<1

$$\begin{cases} \underline{p}(r) = +jZ_c.k.\underline{q}.\dfrac{\exp[-jkr]}{4\pi r} \\ \underline{v}(r) = \left[\dfrac{1}{r} + jk\right].\underline{q}.\dfrac{\exp[-jkr]}{4\pi r} \end{cases}$$



EPFL

The doublet

- constituted by 2 identical pulsating spheres at distance d
- synchronous flow velocities (harmonic), in opposite phases
- principle of superposition:

  the acoustic pressure $\underline{p}$ of the doublet is the sum of the acoustic pressures of the 2 pulsating spheres

  the doublet presents an axial symmetry: $\underline{p}$ does not depend on ϕ

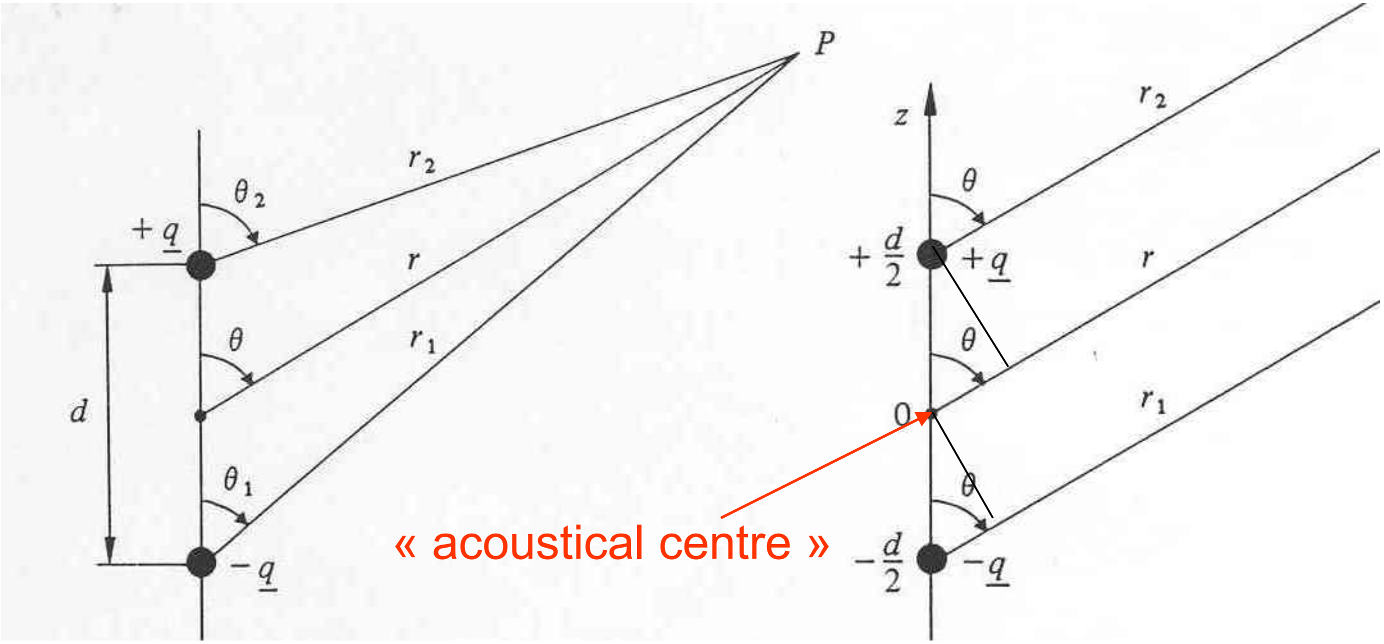
Field of the doublet

- r_1 and r_2, distances of the 2 spheres from the observation point
- in the median plane, p is null (phase opposition)
- outside the plane, p is not null

  ϕ after the path-length difference between r_1 and r_2
- The median plane is a symmetry plane of radiation, but opposite phase

Let's denote: $\underline{p}=\underline{p}_1+\underline{p}_2$

Field of the doublet



Field of the doublet

- Hypothesis :
  - far field ($r >> d$, θ_1 et $\theta_2 \approx \theta$, r_1 and $r_2 \approx r$)

$$r_1 \cong r + \frac{1}{2} d \cos\theta \cong r$$

$$r_2 \cong r - \frac{1}{2} d \cos\theta \cong r$$

  - far enough from the source, p is not null after the propagation phase difference of p_1 and p_2
    ➔ Accounts r_1 and r_2 in the phase term of the monopole field, and r at the denominator (magnitude)

EPFL

Field of the doublet

$$\underline{p}_1 = Z_c k(-q)\frac{\exp(-jk(r_1 - a))}{4\pi r_1(ka-j)} \approx Z_c k(-q)\frac{\exp(-jk(r + 1/2d\cos\theta - a))}{4\pi r(ka-j)}$$

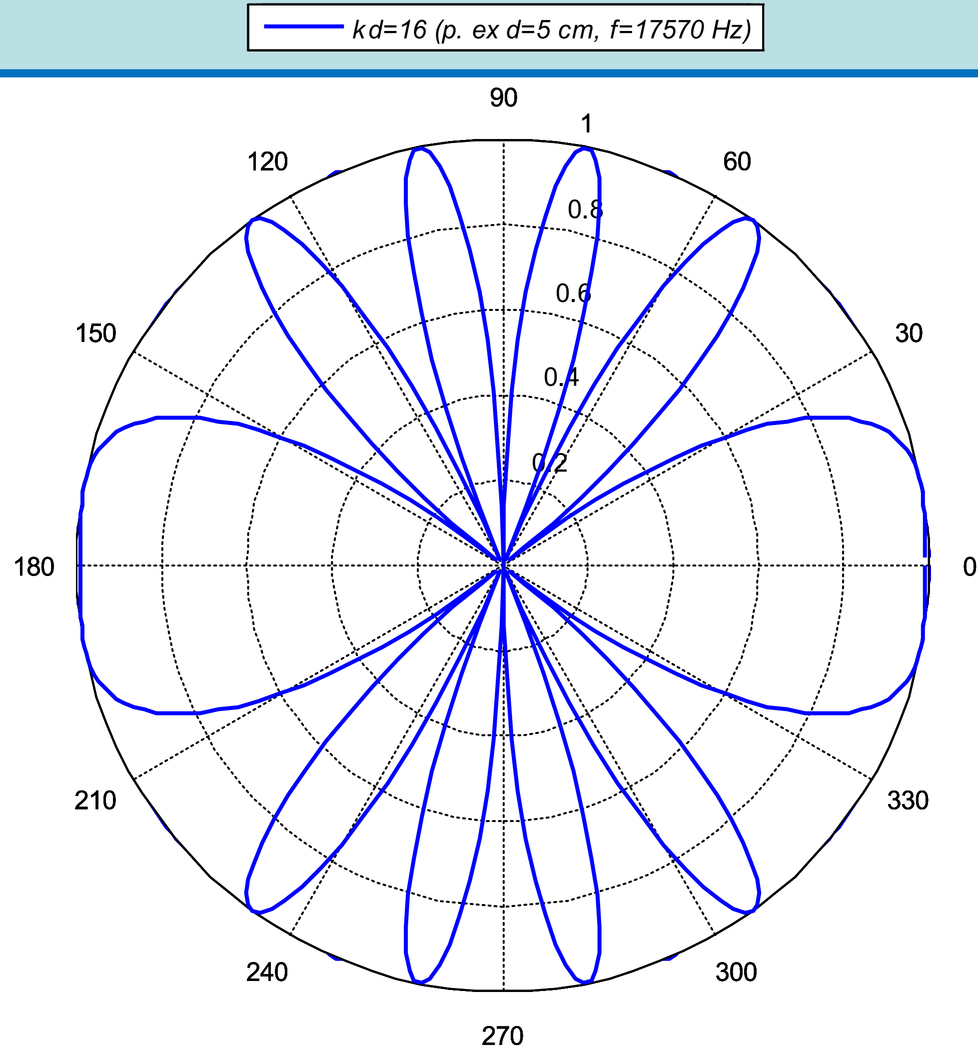
$$\underline{p}_2 \approx Z_c k(+q)\frac{\exp(-jk(r - 1/2d\cos\theta - a))}{4\pi r(ka-j)}$$

$$\underline{p} = \underbrace{\left[Z_c k \underline{q} / 4\pi r(ka - j)\right].\left\{\exp\left[-jk(r-a)\right]\right\}}_{\text{pulsating sphere located at the doublet centre}}.\underbrace{\left[-\exp(-jk\frac{1}{2}d\cos\theta) + \exp(+jk\frac{1}{2}d\cos\theta)\right]}_{2j\sin(\frac{1}{2}kd\cos\theta)}$$

$$\underline{p}(r,\theta) = j\ \underbrace{\underline{p}_{sp}(r)}_{\text{radial dependance}}\ .\underbrace{2\sin(\frac{1}{2}kd\cos\theta)}_{\text{directivity}}$$

EPFL

Field of the doublet



Properties of the doublet

- 2 omnidirectional puls. spheres ➔ directive source
- for $r >> d$, directivity due to $\Delta\Phi$ between the spheres
- directivity depends on ***kd***,

  ➔ depends on f and on the ***dimensions***
- if $(kr)^2 >> 1$, $\underline{v}(r,\theta)$ is almost radial

  at long distance, propagation is radial and approximated relationship $\underline{p}(r,\theta) \approx Z_c \underline{v}(r,\theta)$ is still valid

Far and near field

- **far field**: if r verifies the following relationships:

$$r >> d$$

$$kr >> 1 \ (r >> \lambda/2\pi)$$

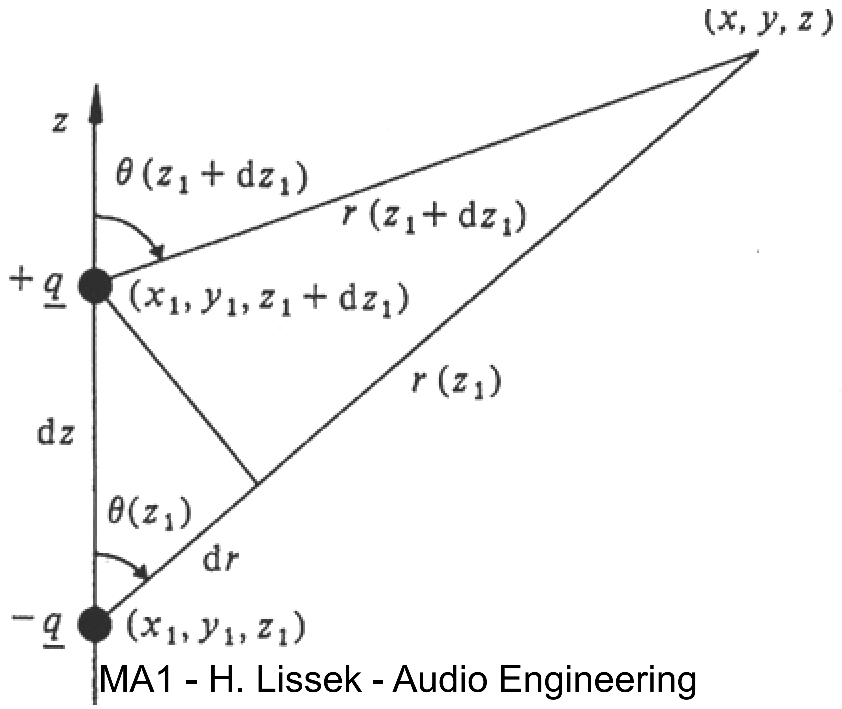
then $\underline{p} \approx Z_c . \underline{v}$ $\qquad$ $I \approx p^2/Z_c$

- **near field**: $r<d$ <u>or</u> $kr<1$

EPFL

Dipole

- small doublet constituted of 2 monopoles at distance $d << \lambda$
- then $kd << 1$ and $ka << 1$ (a, radius of monopoles)



Field of the dipole

- $\underline{p}_d$ computed by applying the superposition principle :

$$\underline{p}_d = jZ_c k \underline{q}\left[-\frac{1}{r_1}\exp(-jkr_1)+\frac{1}{r_2}\exp(-jkr_2)\right]/4\pi$$

- We consider small distances (d=dz), then the difference in brackets becomes a differential :

$$-\partial_{z1}\left[\frac{1}{r}\exp(-jkr)\right]dz = -\partial_r\left[\frac{1}{r}\exp(-jkr)\right]\partial_{z1}r dz = \partial_r\left[\frac{1}{r}\exp(-jkr)\right]\partial_z r dz$$

$$= -j\frac{1}{r}\left(k-\frac{j}{r}\right)\exp(-jkr)\underbrace{\partial_z r}_{=\frac{dr}{dz}=\cos\theta} dz$$

EPFL

Field of the dipole

$$p_d = \underbrace{jZ_c kq\frac{\exp(-jkr)}{4\pi r}}_{\text{monopole field}}\left[kd\left(1+\frac{1}{jkr}\right)\frac{\cos\theta}{j}\right]$$

- In far field:

$$\underline{p}_d \cong \underline{p}_m.\left[-jkd\cos\theta\right]$$

$$= Z_c k^2 \underline{q}d\cos\theta\exp(-jkr)/4\pi r$$

+12 dB/oct bidirectionnal spherical
-6 dB/doubling of distance

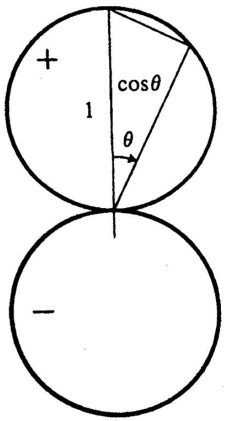
- In near field with $(kr)^2 \ll 1$:

$$\underline{p}_d \cong \underline{p}_m.\left[-d\cos\theta\right]\frac{1}{r} \propto \frac{1}{r^2}$$

+6 dB/oct bidirectionnal -12 dB/
doubling of
distance

Properties

- directivity in cosθ ➔ cos(θ+π)=-cosθ
- independant on r and k, then on f
- opposition of phase regarding the centre or the median plane of the dipole (same as doublet)
- far field analogue to doublet (kd<<1)
- close field, velocity $\underline{v}(\underline{v}_r,\underline{v}_\theta)$:



$$v_r = j\partial_r \underline{p}_d / kZ_c = \underline{p}_d(r,\theta)\left[1+\frac{2}{jkr}+\frac{2}{(jkr)^2}\right]/Z_c \propto \cos\theta$$

$$v_\theta = j\partial_\theta \underline{p}_d / kZ_c r = -jk\underline{q}d\left[1+\frac{1}{jkr}\right]\exp(.jkr)\sin\theta/(4\pi r^2) \propto \sin\theta$$

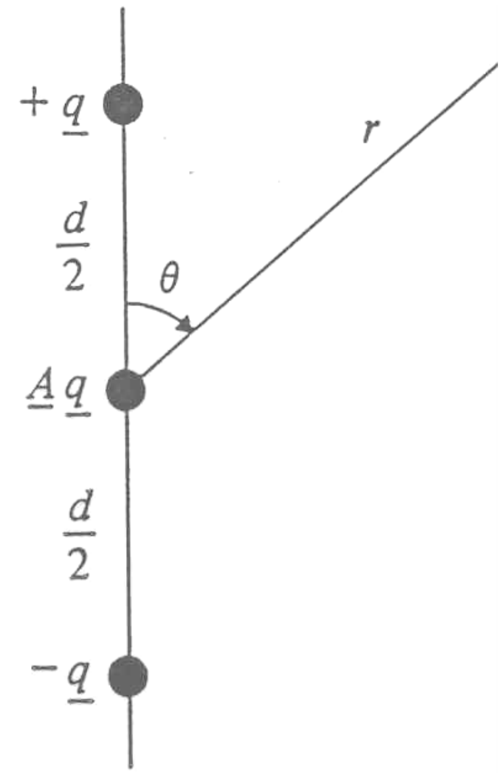
Cardioid source

- cardiod= 1 monopole + 1 dipole
- general expression

  $\underline{p}(r,\theta)=\underline{A}.\underline{p}_m(r)+\underline{p}_d(r,\theta)$
- In far field

  $\underline{p}(r,\theta)=\underline{p}_m(r)(\underline{A}-\mathrm{j}kd\cos\theta)$
- If we take $\underline{A}=-\mathrm{j}kd$

  $\underline{p}(r,\theta)=-\mathrm{j}kd\underline{p}_m(r)(1+\cos\theta)$

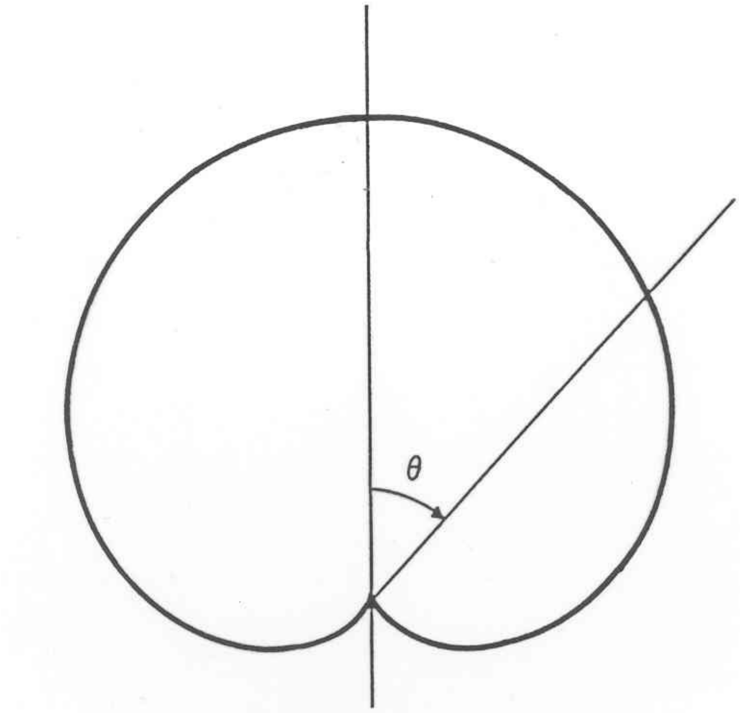


$1+\cos\theta$: unidirectional source (cardioid diagram)

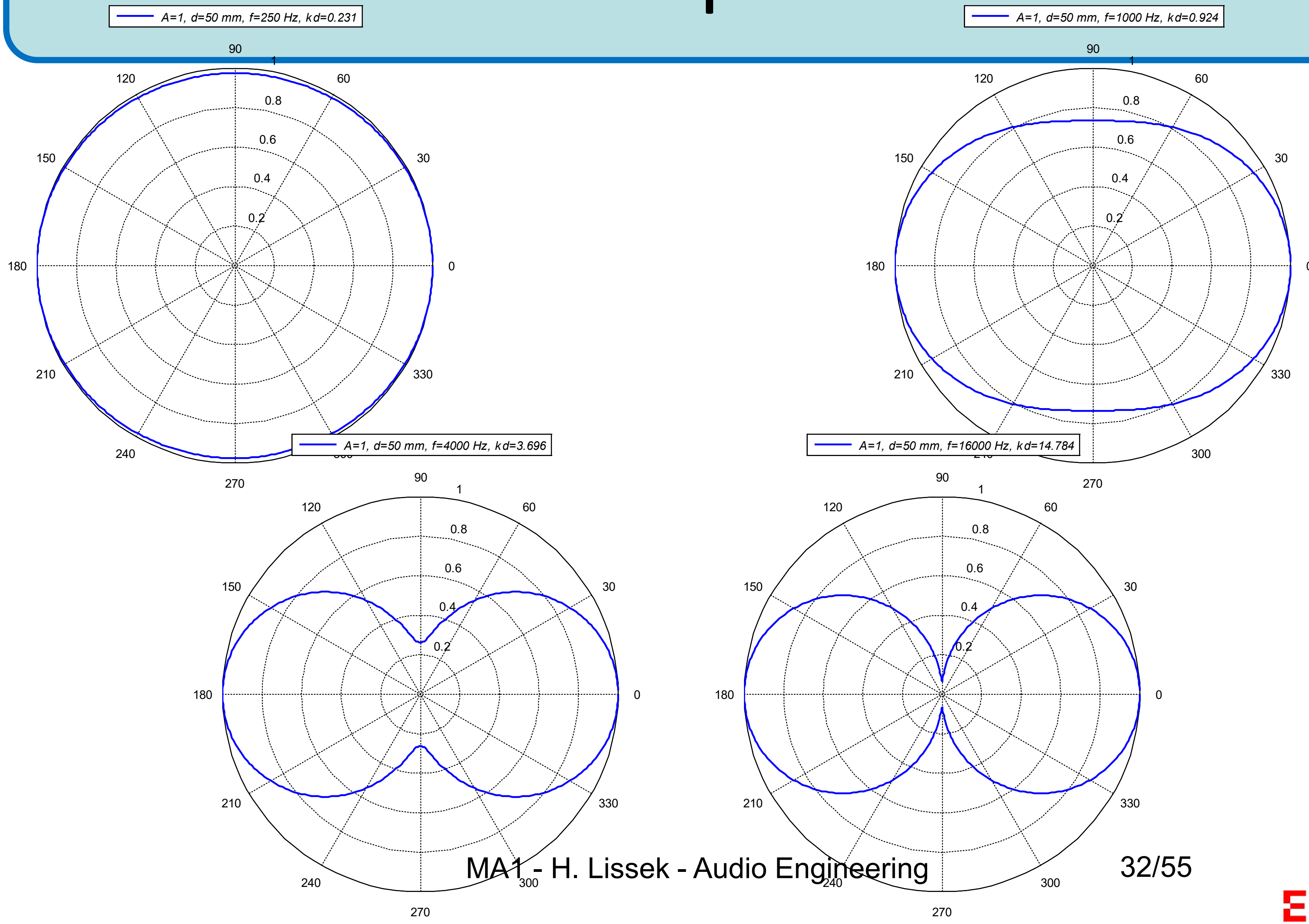
Cardioid source

- if $kd<A$ omnidirectional source
  (the monopole is more influent)
- if $kd>A$ bidirective source
  (the dipole is more influent)

➔ importance of kd



Examples



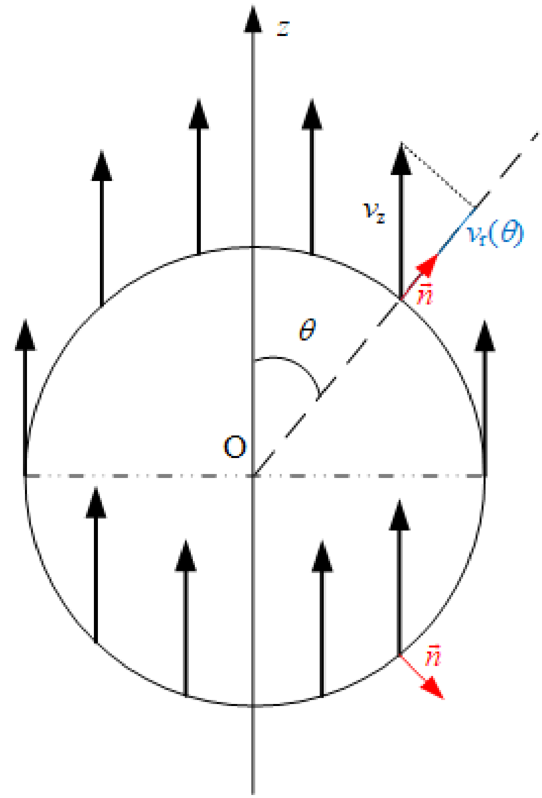
EPFL

Oscillating sphere

- rigid sphere (without strain) animated with small oscillation
- Total volume flow $\underline{q} \equiv 0$
- radial velocity transmitted to fluid

$$\underline{v}_r(\theta)=\underline{v}_z.\cos\theta$$

$\underline{v}_z$ axial velocity

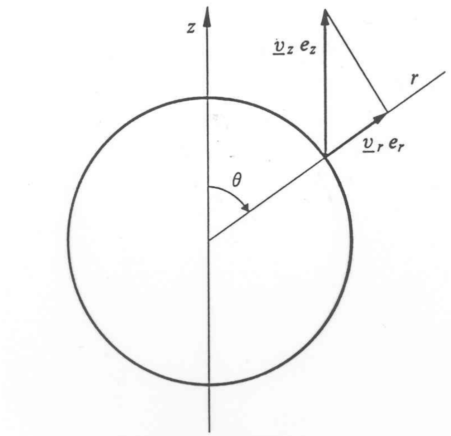


Oscillating sphere

- exact analytical computation after same method as vibrating sphere :

$$\underline{v}_r(\theta) = \underline{v}_z \cos\theta$$

$$\underline{v}_m = (m + \frac{1}{2}) \int_{-1}^{+1} \underline{v}_r(\theta) \mathcal{P}_m(\eta).d\eta$$



- m=0: $$\underline{v}_0 = \frac{1}{2} \underline{v}_z \int_{\cos\theta=-1}^{\cos\theta=+1} \cos\theta d(\cos\theta) = 0$$
- m=1: $$\underline{v}_1 = \frac{3}{2} \underline{v}_z \int_{\cos\theta=-1}^{\cos\theta=+1} \cos^2\theta d(\cos\theta) = \underline{v}_z$$

Orthogonality of $\mathcal{P}_m$

- m>1: $$\underline{v}_m = (m + \frac{1}{2}) \underline{v}_z \int_{\cos\theta=-1}^{\cos\theta=+1} \cos\theta.\mathcal{P}_m(\cos\theta).d(\cos\theta) = 0$$

EPFL

Field of the oscillating sphere

$$p(r,\theta) = \alpha_1 \mathcal{P}_1(\cos\theta) h_1^{(2)}(kr) = -\alpha_1 \cos\theta \left[1 + \frac{1}{jkr}\right] \frac{e^{-jkr}}{kr}$$

- α_1 depends on Z_c and ka, but neither on r nor θ
- comparison to $\underline{p}_d$ shows the same dependence on r and θ
- same for $\underline{v}(r,\theta)$ (computed after grad($\underline{p}$))

- the radial velocity due to a dipole and computed on r=a is equal to radial velocity of the oscillating sphere :

  $\underline{v}_r(\theta)=\underline{v}_z.\cos\theta$

EPFL

Properties

- $\underline{q} \equiv 0$
- $\underline{p}(r,\theta) \approx \underline{p}_d(\mathrm{r},\theta)$
- $\underline{v}(r,\theta) \approx \underline{v}_d(\mathrm{r},\theta)$
- $\underline{v}(r=a) = \underline{v}_z \cos\theta$

An oscillating sphere behaves as a dipole:

➔ dipole=model of the oscillating sphere

Radiation impedances

$$\underline{Z}_{mr} = \frac{\underline{F}_z}{\underline{v}_z}$$

$\underline{F}_z$, component along z of the medium reactions on the sphere

$$\underline{z}_r = \underline{Z}_{mr} / Z_c S = r_r + jx_r$$

$$\underline{F}_z = \int_S \underline{p}_d(r=a,\theta)dS = \int_S \underline{p}_d \cos\theta dS = 2\pi a^2 \int_0^\pi \underline{p}_d \cos\theta \sin\theta d\theta$$

$$= \frac{1}{3}\underline{Z}_c k^2 \underline{q} da \left[1 + \frac{1}{jka}\right].\exp(-jka)$$

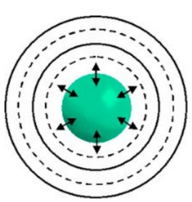
pulsating sphere (constant v_r)

$r_r=(ka)^2/[1+(ka)^2]$

$x_r=(ka)/[1+(ka)^2]$

$m_r=3m_s$

$P_a=R_{mr}v_r^{\,2}$



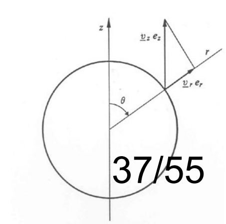
oscillating sphere (constant v_z)

$r_r=1/3(ka)^4/[4+(ka)^4]$

$x_r=1/3(ka).[2+(ka)^2]/[4+(ka)^4]$

$m_r=1/2m_s$

$P_a=R_{mr}v_z^{\,2}$



Field of the oscillating sphere

$$p(r,\theta) = 3Z_{mr}v_z \cos\theta \left[1+\frac{1}{jkr}\right]\frac{e^{-jkr}}{4\pi r}\cdot\frac{e^{+jka}}{a(1+\frac{1}{jka})}$$

EPFL

Radiation of the piston

Introduction

Loudspeakers: face = circular diaphragm

→ What directivity / radiation impedance?

Regarding dimensions, shape, and conditions of radiation, these properties vary

→ define good models

Hypothesis:

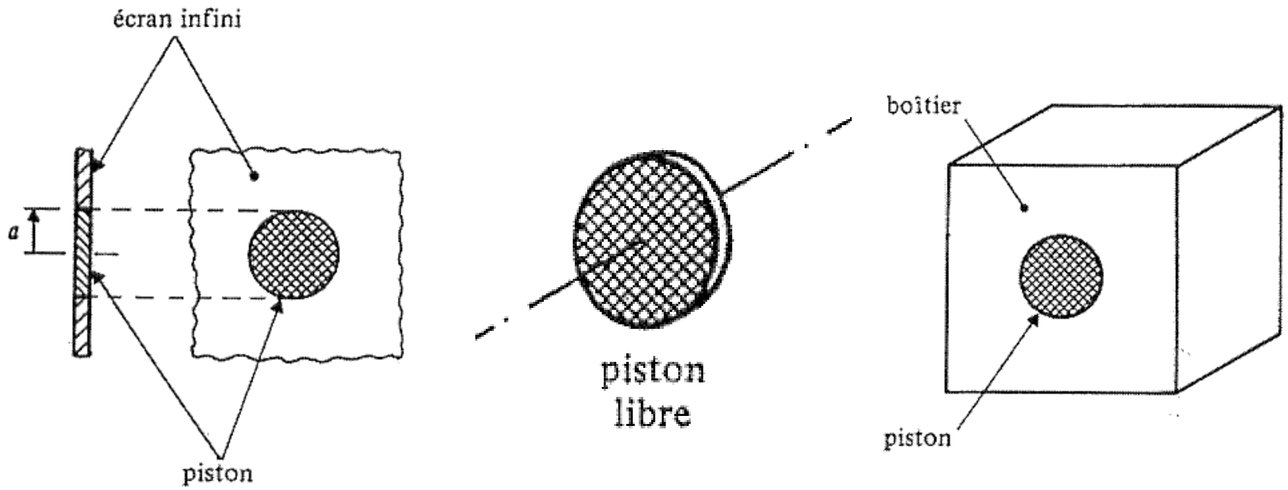
- Plane shapes, (circular, rectangular or squares)
- Moving with uniform velocity

Circular piston

Rigid disk, **planar**, **thin**, with oscillatory **translation** movement along axis of symetry

Different mounting conditions ➔ different radiations:

- Infinite screen
- Free piston
- Box

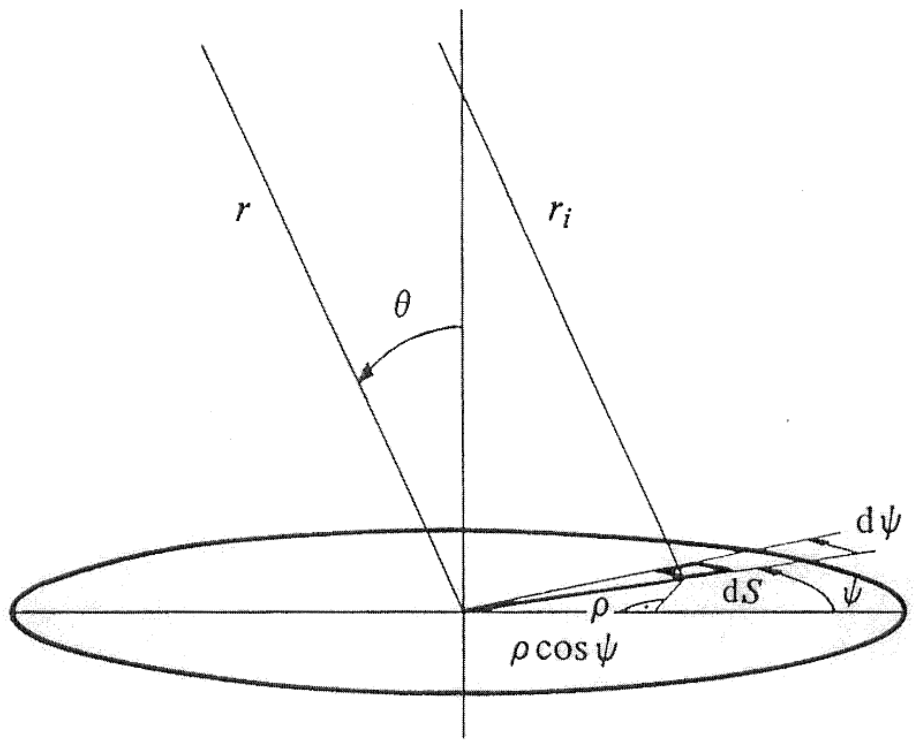


EPFL

Hypotheses: radiation of an element of surface

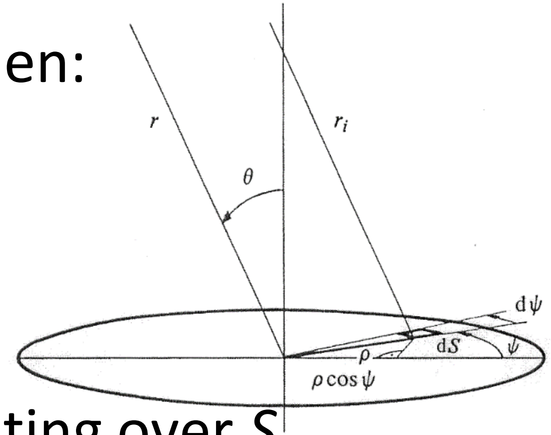
Let's consider the case of the infinite screen

d*S* is an element of the piston surface, assimilated to a semi-monopole with flow velocity $dq = v_0 dS$



Hypotheses: radiation of an element of surface

The acoustic pressure due to this element is then:



$$dp = jZ_c k dq \frac{e^{-jkr_i}}{\mathbf{2}\pi r_i}$$

The total pressure due to the piston by integrating over *S*

far field: $$p(r,\theta) = \left\{ jZ_c k v_0 \frac{e^{-jkr}}{\mathbf{2}\pi r} \right\} \cdot \int_s e^{\overbrace{-jk(r_i - r)}^{-jk\rho\cos\psi\sin\theta}} dS$$

$$= \mathbf{2} p_m(r) \left\{ \frac{2J_1(k_\theta a)}{\underbrace{k_\theta}_{k\sin\theta}\ a} \right\} = \mathbf{2} p_m(r) D_0(\theta)$$

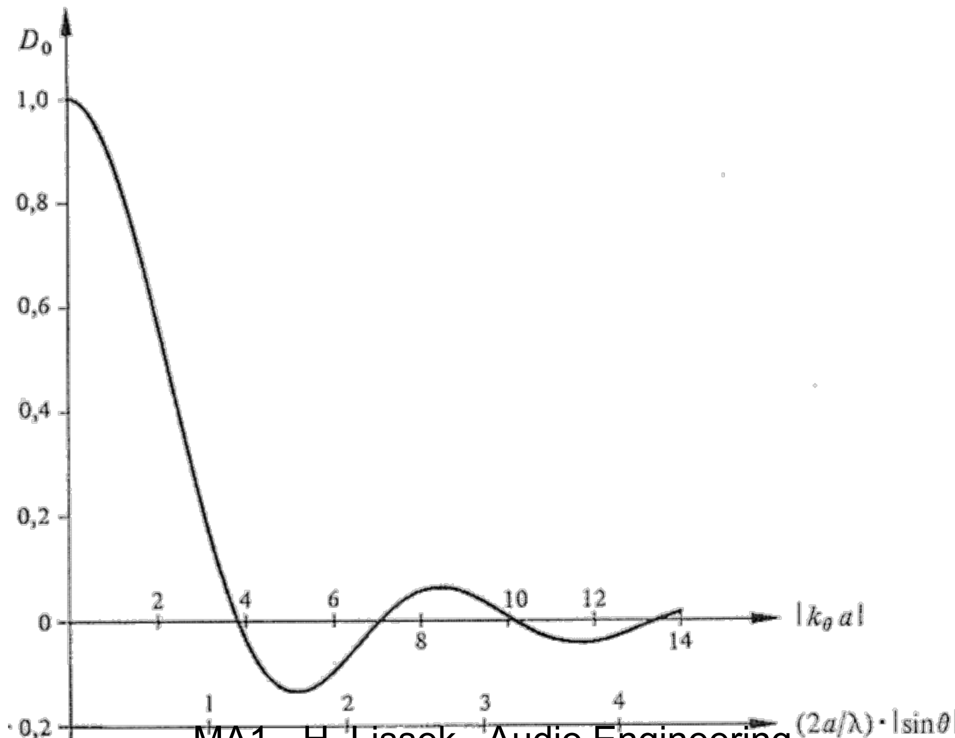
EPFL

Piston in infinite screen: directivity

The piston is not directive when *ka*<1

For *ka*>2, increased directivity

From 2*ka*=3,83, secondary lobes appear



Piston in infinite screen: properties

Directivity factor in the principal axis:

$$\Delta = \frac{4\pi}{\displaystyle\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} D_0^2(\theta)\sin\theta d\theta d\phi} = \frac{(ka)^2}{1-\dfrac{J_1(2ka)}{ka}}$$

$$ka < 1 \rightarrow \qquad L_\Delta = 10.\log_{10}\Delta = 3\text{dB}$$

$$ka = 1 \rightarrow \qquad L_\Delta = 10.\log_{10}\Delta = 3.7\text{dB}$$

$$ka > 2 \rightarrow \qquad L_\Delta = 20.\log_{10}(ka)$$

EPFL

Piston in infinite screen: near field

Complex expression

But in axis: $$\tilde{p}_p(r,0) = 2Z_c\tilde{v}_0\left|\sin\left[\underbrace{\frac{1}{2}k\left\{\sqrt{a^2+r^2}-r^2\right\}}_{\text{minima et maxima}}\right]\right|$$

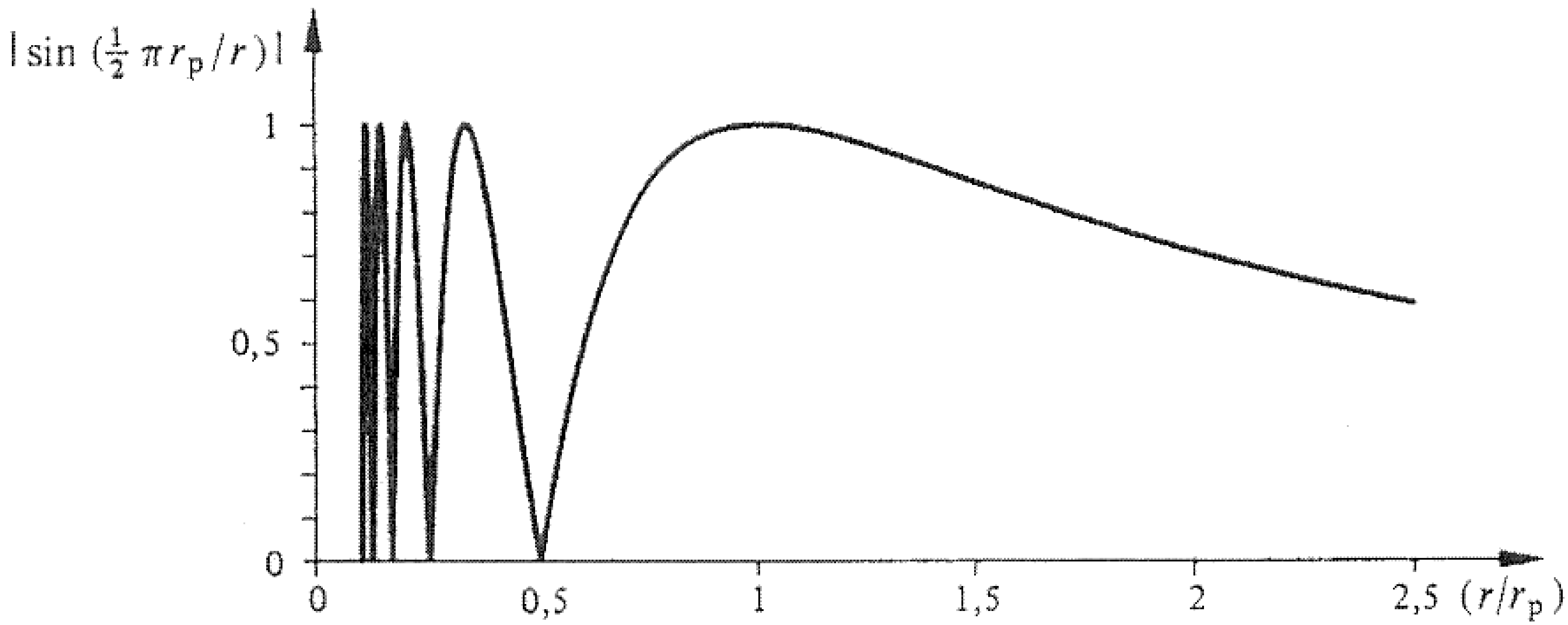
For *r*<<*a*: $$\tilde{p}_p = 2Z_c\tilde{v}_0\left|\sin\left(\frac{1}{2}ka\right)\right|$$

The very near field is independant of *r*, but depends on *ka*

Conditions of far field: from $r_p=a^2/\lambda$:

$$\tilde{p}_p \approx \left|\sin\left(\frac{1}{2}\pi r_p / r\right)\right|$$

Piston in infinite screen: near field



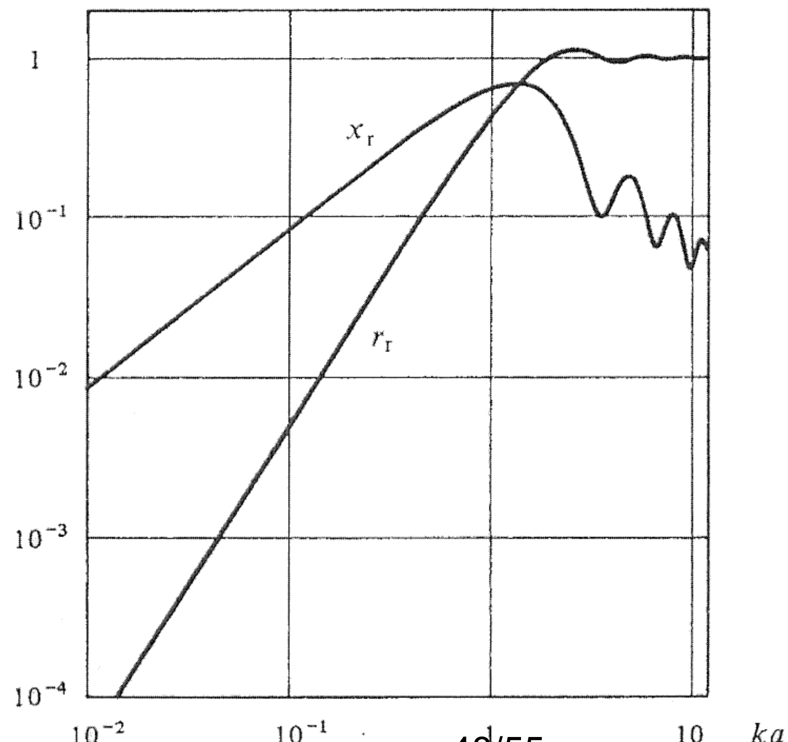
Piston in infinite screen: radiation impedance

Same definition as for pulsating sphere

➔normalized impedance of the piston in infinite screen

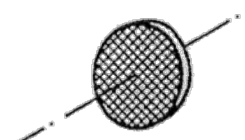
$$r_r = 1 - \left[\frac{J_1(2ka)}{ka}\right] \approx \frac{1}{2}(ka)^2.\left[1 - \frac{(ka)^2}{6}\right]$$

$$x_r = \frac{S_1(2ka)}{ka} \approx \left(\frac{8}{3\pi}\right).(ka)\left[1 - \frac{(2ka)^2}{15}\right]$$



EPFL

Free piston: far field



Free piston = oscillating source

General expression is complex

approximation of the acoustic pressure in far field, on one side of the piston

$$\underline{p}(r,\theta) = \left[jk \underbrace{\underline{F}}_{\substack{\text{total} \\ \text{radiation force} \\ \text{(2 faces)}}} \frac{\exp(-jkr)}{4\pi r} \right] \cdot \underbrace{\cos\theta}_{\text{oscillating sphere}} \cdot \underbrace{\left[\frac{2J_1(k_\theta a)}{k_\theta a}\right]}_{\text{piston on screen}}$$

Free piston: radiation impedance

$$r_r \approx \left(\frac{8}{27\pi^2}\right)(ka)^4$$

$$x_r \approx \left(\frac{4}{3\pi}\right)ka$$

For ka>>1, r_r=1 and x_r=0

For ka<<1, radiation mass = one face of the piston in infinite screen

For ka<1:

- better radiation on screen than free
- free radiation is bidirectional
- screen radiation omnidirectional

Piston in a box: models

Same as the case of the piston in infinite screen, but the rear face is not in contact with the medium:

➔ Only radiating on the front side

Pulsating source with flow velocity $\underline{q}=\pi a^2 \underline{v}_0$

+

diffraction due to the box (dimension max d)

Model: piston in small box

For kd<1/2, no diffraction, omnidirectional (ka<1),
=small pulsating source

(5.85): $$R_{mr} = \frac{Z_c S^2 k^2}{4\pi} \cong Z_c \pi a^2 \frac{(ka)^2}{4}$$

Half the one of the piston on screen

EPFL

Model: piston on a wide face

- Same as infinite screen

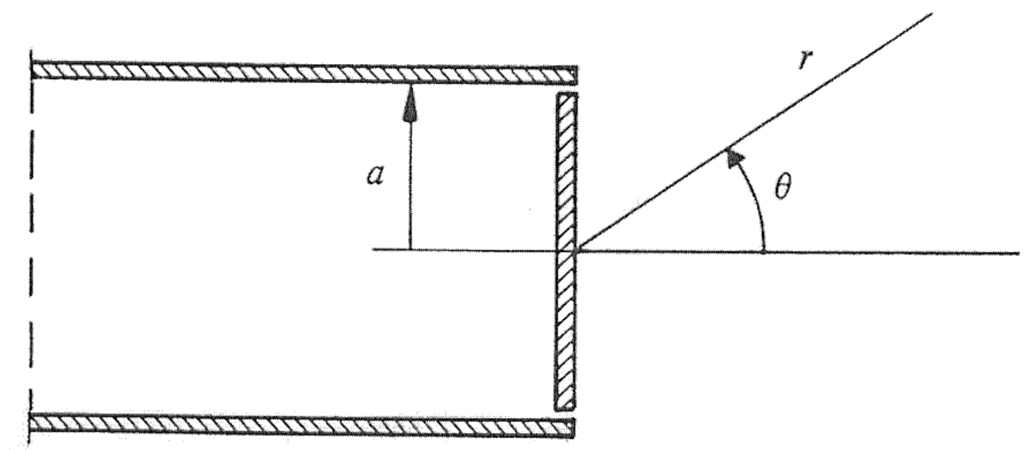
Model: piston at the termination of a duct

Piston=one face of a box → model of a the piston at the termination of a long duct

$ka<1/2$: $D(\theta)\approx 1$, $r_r=(ka)^2/4$ and $m_r=1{,}9a^3\rho$ kg

$ka>2$: $r_r=1$ and $m_r=0$ kg, and

$$D(\theta)\cong\left(1+\cos\theta\right)\frac{J_1(k_\theta a)}{\sqrt{1-(2ka)^{-1}}\sin\theta}$$



EPFL

Remarks

- In practice, it is impossible to determine analytically the radiation of the speaker face on a box with arbitrary shape.
- Certain simple situations can be treated, but experimentation (incl. numerical models) is generally preferred