# Addendum
# Closed-box and vented-box formulations

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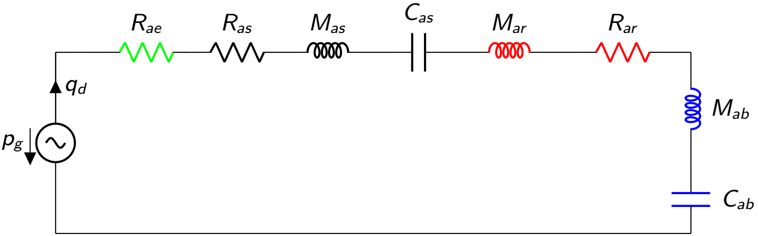
# Closed-box: definitions

Analogue acoustical circuit of a loudspeaker + closed cabinet (Volume $V_b$) system:



# Closed-box: definitions

The components of the analogue scheme are:

| | |
|---|---|
| **Electrical parts** | |
| $p_g = \dfrac{B\ell U_g}{S_d R_e}$ | source pressure analogue to the source electrical voltage $U_g$ |
| $R_{ae} = \dfrac{(B\ell)^2}{S_d^2 R_e}$ | acoustic resistances analogue to $R_e$, DC resistance of the coil |
| | (we neglect here the output resistance of voltage source $U_s$,<br>as well as the electrical inductance of the coil $L_e$) |
| **Mechanical parts** | |
| $R_{as} = \dfrac{R_{ms}}{S_d^2}$ | acoustic resistance corresponding to the mechanical resistance of the loudspeaker |
| $M_{as} = \dfrac{M_{ms}}{S_d^2}$ | acoustic mass corresponding to the mechanical mass of the loudspeaker |
| $C_{as} = S_d^2 C_{ms}$ | acoustic compliance corresponding to the mechanical compliance of the loudspeaker |
| **Acoustic parts** | |
| $M_{ar} = \dfrac{8}{3\pi} \dfrac{\rho a}{S_d}$ | radiation mass of a piston of radius $a = \sqrt{S_d/\pi}$ mounted on a screen |
| $R_{ar} \approx \dfrac{\rho c}{S_d} \dfrac{(ka)^2}{4}$ | radiation resistance of a "pulsating" source of radius $a$ (solid angle $4\pi$) |
| $M_{ab} \approx M_{ar} \approx \dfrac{8}{3\pi} \dfrac{\rho a}{S_d}$ | radiation mass inside the cabinet |
| | (deformation of particle velocity in the cabinet) |
| $C_{ab} = S_d^2 C_{ms}$ | acoustic compliance equivalent to the cabinet volume $V_b$ |

# Closed-box: response in flow velocity

The closed-box loudspeaker is assimilated to a small semi-monopolar (in $2\pi$ steradians) source of radius $a$ (radiation resistance $R_{ar} = 2\pi\rho\frac{f^2}{c}$) and flow velocity $q_d$.
The radiated power is then: $P_a = R_{ar}|q_d|^2$
The power can be derived by expressing $q_d$ as a function of voltage $U_g$.

Since: $q_d = \frac{p_g}{Z_{ac}}$, where $Z_{ac} = R_{ac} + j\omega M_{ac} + \frac{1}{j\omega C_{ac}}$ with:
$R_{ac} = R_{ae} + R_{as} + R_{ar}$: total losses of the closed-box loudspeaker
$M_{ac} = M_{as} + M_{ar} + M_{ab}$ : total acoustic mass
$C_{ac} = \frac{C_{as}C_{ab}}{C_{as} + C_{ab}}$: total acoustic compliance
Then $q_d = \left(\frac{B\ell U_g}{S_d R_e}\right) \frac{j\omega C_{ac}}{(j\omega)^2 M_{ac}C_{ac} + j\omega R_{ac}C_{ac} + 1}$

# Closed-box: response in flow velocity

$$q_d = \left(\frac{B\ell U_g}{S_d R_e}\right) \frac{j\omega C_{ac}}{(j\omega)^2 M_{ac} C_{ac} + j\omega R_{ac} C_{ac} + 1}$$

Defining: $\omega_c = \dfrac{1}{\sqrt{M_{ac} C_{ac}}}$ and $Q_{tc} = \dfrac{1}{\omega_s R_{ac} C_{ac}}$ the acoustic resonator's resonance frequency and quality factor, we show that:

$$C_{ac} = \frac{1}{\omega_c R_{ac} Q_{tc}}$$

$$M_{ac} C_{ac} = \frac{1}{\omega_c^2}$$

$$R_{ac} C_{ac} = \frac{1}{\omega_c Q_{tc}}$$

And then $q_d = \left(\dfrac{B\ell U_g}{S_d R_e}\right) \dfrac{1}{Q_{tc} R_{ac}} \dfrac{\left(\frac{j\omega}{\omega_c}\right)}{\left(\frac{j\omega}{\omega_c}\right)^2 + \frac{1}{Q_{tc}}\left(\frac{j\omega}{\omega_c}\right) + 1}$

Since $\dfrac{B\ell}{S_d R_e} = \dfrac{S_d R_{ae}}{B\ell}$, then we can define $q_c = \dfrac{B\ell U_g}{S_d R_e Q_{tc} R_{ac}} = \dfrac{S_d}{B\ell Q_{ec}} U_g$

and finally $q_d = q_c \dfrac{\left(\frac{j\omega}{\omega_c}\right)}{\left(\frac{j\omega}{\omega_c}\right)^2 + \frac{1}{Q_{tc}}\left(\frac{j\omega}{\omega_c}\right) + 1}$

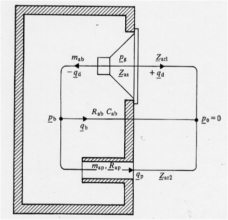
# Closed-box: response in power

$$P_a = R_{ar}|q_d|^2 = 2\pi\rho\frac{f^2}{c}q_c^2\left|\frac{\left(\frac{j\omega}{\omega_c}\right)}{\left(\frac{j\omega}{\omega_c}\right)^2+\frac{1}{Q_{tc}}\left(\frac{j\omega}{\omega_c}\right)+1}\right|^2$$

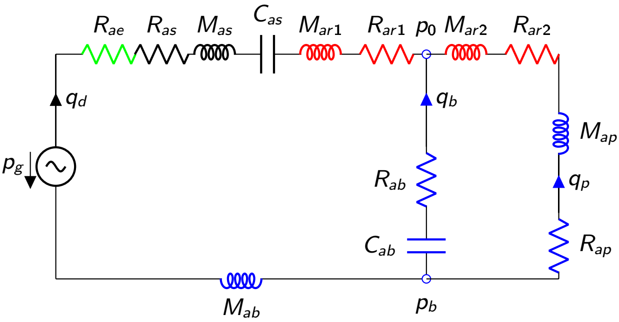
Then $P_a = P_{ac}\left|\frac{\left(\frac{j\omega}{\omega_c}\right)^2}{\left(\frac{j\omega}{\omega_c}\right)^2+\frac{1}{Q_{tc}}\left(\frac{j\omega}{\omega_c}\right)+1}\right|^2$

where $P_{ac} = \frac{2\pi\rho}{c}f_c^2 q_c^2$

# Vented-box loudspeaker: definitions




Analogue acoustic circuit of a vented-box + loudspeaker system (bass-reflex):



# Vented-box loudspeaker: definitions

In the analogue acoustic representation of the vented-box loudspeaker:

- the electrical voltage $U_g$ applied to the loudspeaker terminals is replaced by the equivalent pressure $p_g = \dfrac{B\ell}{S_d R_e} U_g$.
- the total acoustic impedance of the loudspeaker (excluding the Helmholtz resonator of the vented box) becomes:

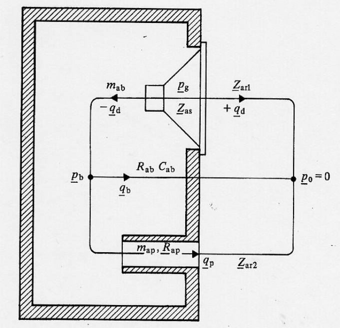
$$Z'_{as} = (R_{ae} + R_{as} + R_{ar}) + j\omega (M_{as} + M_{ab} + M_{ar}) + \frac{1}{j\omega C_{as}}$$
$$\approx R_{ae} + R'_{as} + j\omega M'_{as} + \frac{1}{j\omega C_{as}}$$

- the equivalent acoustic impedance of the Helmholtz resonator constituted with volume $V_b$ (acoustic compliance $C_{ab} = \frac{V_b}{\rho c^2}$, we neglect the resistance $R_{ab}$) and the port of length $l_p$ and cross-section $s_p$ (acoustic mass $M'_{ap} = M_{ap} + M_{ar2} = \rho \frac{a_p + 2\Delta L}{s_p}$, with $\Delta L$ the termination correction accounting for the acoustic radiation at each sides of the port $\Delta L \approx \frac{8a}{3\pi}$) reads:

$$Z_{ab} = \frac{j\omega M'_{ap}}{(j\omega)^2 M'_{ap} C_{ab} + 1} = \frac{j\omega M'_{ap}}{\left(\frac{j\omega}{\omega_p}\right)^2 + 1}, \text{ with } \omega_p = \frac{1}{\sqrt{M'_{ap} C_{ab}}}$$

# Vented-box loudspeaker: acoustic radiation

The radiated sound power is due to the combination of both the loudspeaker diaphragm and the port radiations. Then, we can consider the flow velocity $q_s + q_p = -q_b$ to process the radiated power, and consider the loudspeaker as a semi-monopole $R_{ar} = \frac{2\pi\rho}{c} f^2$.




The radiated power then writes:
$P_a = R_{ar}|q_b|^2$,
and can be derived by analyzing the analogue acoustic circuit of the system.

# Vented-box loudspeaker: acoustic radiation

We can derive $q_b$ in the analogue acoustic circuit as $q_b = j\omega C_{ab} p_b$, where $p_b$ is the pressure inside the cabinet of volume $V_b$.
Pressure $p_b$ can be simply deduced by analyzing the "pressure divider":

$p_b = \dfrac{Z_{ab}}{Z_{ab} + Z'_{as}} p_g$, then:

$$q_b = \frac{B\ell U_g}{S_d R_e} \frac{j\omega C_{ab} Z_{ab}}{Z_{ab} + Z'_{as}} = \frac{B\ell U_g}{S_d R_e} \frac{(j\omega)^2 C_{ab} M'_{ap}}{\left[\left(\frac{j\omega}{\omega_p}\right)^2 + 1\right]\left[j\omega M'_{as} + R'_{as} + \frac{1}{j\omega C_{as}}\right] + j\omega M'_{ap}}$$

$$= \frac{B\ell U_g \omega_s}{S_d R_e M'_{as} \omega_s} \frac{(j\omega)^3 C_{ab} M'_{ap} C_{as} M'_{as}}{(j\omega)^4 M'_{as} C_{as} M'_{ap} C_{ab} + (j\omega)^3 M'_{ap} C_{ab} (R_{ae} + R_{as}) C_{as} + (j\omega)^2 [M'_{ap} C_{ab} + M'_{as} C_{as} + M'_{ap} C_{as}] + (j\omega) C_{as} (R_{ae} + R'_{as}) + 1}$$

Reminding $\omega_s = \dfrac{1}{\sqrt{M'_{as} C_{as}}}$ and $Q_{ts} = \dfrac{1}{\omega_s R'_{as} C_{as}}$, as well as $\omega_p = \dfrac{1}{\sqrt{M'_{ap} C_{ab}}}$ it comes:

$$q_b = \frac{B\ell U_g}{S_d R_e M'_{as} \omega_s} \frac{\left(\frac{j\omega}{\omega_s}\right)^3 \frac{\omega_s^2}{\omega_p^2}}{\left(\frac{j\omega}{\omega_s}\right)^4 \frac{\omega_s^2}{\omega_p^2} + \left(\frac{j\omega}{\omega_s}\right)^3 \frac{1}{Q_{ts}} \frac{\omega_s^2}{\omega_p^2} + \left(\frac{j\omega}{\omega_s}\right)^2 \left[\frac{\omega_s^2}{\omega_p^2} + 1 + \frac{C_{as}}{C_{ab}} \frac{\omega_s^2}{\omega_p^2}\right] + \left(\frac{j\omega}{\omega_s}\right) \frac{1}{Q_{ts}} + 1}.$$

# Vented-box loudspeaker: acoustic radiation

$$q_b = \frac{B\ell U_g}{S_d R_e M'_{as} \omega_s} \frac{\left(\frac{j\omega}{\omega_s}\right)^3 \frac{\omega_s^2}{\omega_p^2}}{\left(\frac{j\omega}{\omega_s}\right)^4 \frac{\omega_s^2}{\omega_p^2} + \left(\frac{j\omega}{\omega_s}\right)^3 \frac{1}{Q_{ts}} \frac{\omega_s^2}{\omega_p^2} + \left(\frac{j\omega}{\omega_s}\right)^2 \left[\frac{\omega_s^2}{\omega_p^2} + 1 + \frac{C_{as}}{C_{ab}} \frac{\omega_s^2}{\omega_p^2}\right] + \left(\frac{j\omega}{\omega_s}\right) \frac{1}{Q_{ts}} + 1}.$$

Introducing $\alpha = \frac{C_{as}}{C_{ab}}$ et $h = \frac{\omega_p}{\omega_s}$, it comes:

$$q_b = \frac{B\ell U_g}{S_d R_e M'_{as} \omega_s} \frac{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2}}{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2} + \left(\frac{j\omega}{\omega_s}\right)^3 \frac{h^{-2}}{Q_{ts}} + \left(\frac{j\omega}{\omega_s}\right)^2 \left[1 + h^{-2}(1+\alpha)\right] + \left(\frac{j\omega}{\omega_s}\right) \frac{1}{Q_{ts}} + 1}.$$

# Vented-box loudspeaker: response in power

$$P_a = \frac{2\pi\rho}{c} f^2 \left(\frac{B\ell U_g}{S_d R_e M'_{as}\omega_s}\right)^2 \left| \frac{\left(\frac{j\omega}{\omega_s}\right)^3 h^{-2}}{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2} + \left(\frac{j\omega}{\omega_s}\right)^3 \frac{h^{-2}}{Q_{ts}} + \left(\frac{j\omega}{\omega_s}\right)^2 \left[1 + h^{-2}(1+\alpha)\right] + \left(\frac{j\omega}{\omega_s}\right) \frac{1}{Q_{ts}} + 1} \right|^2$$

$$= \frac{2\pi\rho}{c} f_s^2 \left(\frac{B\ell U_g}{S_d R_e M'_{as}\omega_s}\right)^2 \left| \frac{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2}}{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2} + \left(\frac{j\omega}{\omega_s}\right)^3 \frac{h^{-2}}{Q_{ts}} + \left(\frac{j\omega}{\omega_s}\right)^2 \left[1 + h^{-2}(1+\alpha)\right] + \left(\frac{j\omega}{\omega_s}\right) \frac{1}{Q_{ts}} + 1} \right|^2$$

$$= P_{ao} \left| \frac{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2}}{\left(\frac{j\omega}{\omega_s}\right)^4 h^{-2} + \left(\frac{j\omega}{\omega_s}\right)^3 \frac{h^{-2}}{Q_{ts}} + \left(\frac{j\omega}{\omega_s}\right)^2 \left[1 + h^{-2}(1+\alpha)\right] + \left(\frac{j\omega}{\omega_s}\right) \frac{1}{Q_{ts}} + 1} \right|^2 .$$