4.2b Electrostatic transducers

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November 11 2015

Goals and prerequisites

Goals

The objectives of this lecture are

- to present in a straightforward manner the **physical phenomena** responsible of the electrostatic transduction;
- to describe these phenomena through **coupling equations** linking the electrical quantities (U,i) and the mechanical quantities (F,v);
- to translate these equations into **analogue electrical schemes**.

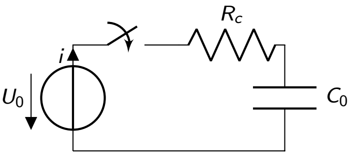
Prerequisites

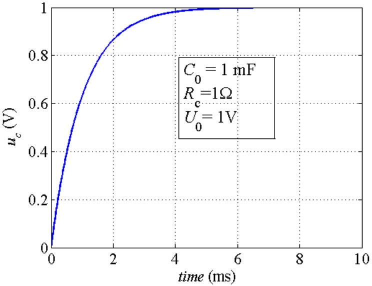
The following should be already known

- basic notions of electrostatic condensers,
- eletroctechnics,
- electrical-mechanical analogies.

Charge of a capacitor

Constant voltage U_0 applied to the capacitor.



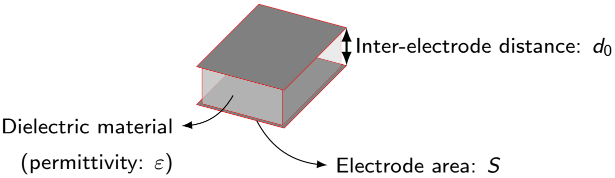


When switched on, electrons accumulate on the negative electrode: charge $-Q$.
On the positive electrode, lack of electrons represent an accumulated charge $+Q$.
The process slows down and eventually stops when voltage $u(t)$ at the capacitor ends equals that of the source: $u(\infty) = U_0$.

Capacitance of a capacitor

Capacitor: 2 fixed, planar electrodes area S, separated by a dielectric with thickness d_0.

Figure: Planar capacitor



Ratio between accumulated charge Q and voltage U_0 across the capacitor's eletrodes is the electrical capacitance C_0 of the capacitor:

$$C_0 = \frac{Q}{U_0} \text{ (unit: Farad).}$$

Capacitance depends on the geometry (electrodes area and thickness of the dielectric in between) and on permittivity ε. For a plane capacitor, C_0 is given by:

$$C_0 = \frac{\varepsilon S}{d_0}.$$

Electrostatic Force

The opposing electrodes being charged with opposite charges, an electrostatic force appears:

$$F_e = \frac{1}{2}\frac{1}{C_0 d_0}Q^2 = \frac{1}{2}\frac{1}{\varepsilon S}Q^2$$

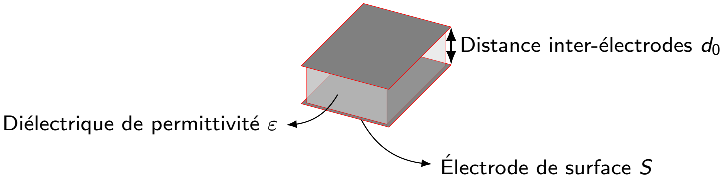


Figure: Planar capacitor

The electrostatic transducer

Electrostatic transduction relies on **variable capacitance**, e.g. with a mobile electrode. **Polarisation** ensures opposite charges on the electrodes.

Two-way transduction:

- a voltage across the electrodes appears, because of a change in the distance between the electrodes (sensor mode)
- the electrostatic force induces a movement of the mobile electrode (actuator mode)

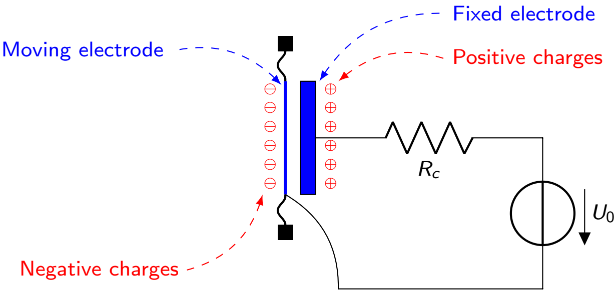


Figure: The electrostatic transducer

Electrostatic sensor (microphone)

Mobile electrode moves due to an incoming sound wave: inter-electrode distance varies.
The capacitance then also varies, inducing either the voltage across electrodes to vary (constant charge), or the accumulated charge to vary (constant polarization voltage). This varying voltage/charge is related to the **vibration** of the mobile electrode, itself related to the incoming **sound pressure**.

Figure: 1" Brüel & Kjaer electrostatic microphone. Left: protection grid was removed, so that the moving electrode is apparent. Right: mobile electrode was removed, so that the fixed electrode is apparent.

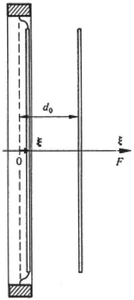
Electrostatic actuator (loudspeaker)

A varying voltage across the electrodes induces a variation of the accumulated charge. The electrostatic force then varies, and the mobile electrode moves.
This vibration produces a pressure wave: a sound is emitted, related to the **electrical voltage signal** across the electrodes.

Figure: Quad electrostatic loudspeaker

Electrostatic coupling: sensor

External load imposed on the mobile electrode.
Mobile electrode moves: strain $\xi(t)$.
Capacitance $C(t)$ varies as:



$$C(t) = \varepsilon \frac{S}{d_0 - \xi(t)} = \frac{\varepsilon S}{d_0} \frac{1}{1 - \dfrac{\xi(t)}{d_0}}$$

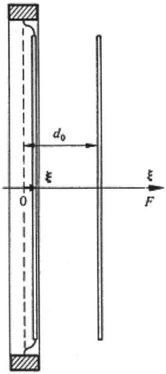
thus the voltage across the electrodes varies as:

$$u(t) = \frac{Q(t)}{C(t)} = \frac{Q(t)}{C_0} - \frac{1}{C_0 d_0} Q(t)\xi(t)$$

with $C_0 = \dfrac{\varepsilon S}{d_0}$

Electrostatic coupling: actuator

Varying voltage imposed across the electrodes.
The accumulated charge $Q_c(t)$ changes. The resulting electrostatic force varies as:



$$F_e(t) = \frac{1}{2}\frac{1}{\varepsilon S}Q^2(t)$$

Nonlinearity of the coupling

Equations describing the transduction are non linear:

$$\begin{aligned} u(t) &= \frac{Q(t)}{C_0} - \frac{1}{C_0 d_0} Q(t)\xi(t) \\ F_e(t) &= \frac{1}{2}\frac{1}{\varepsilon S} Q^2(t) \end{aligned}$$

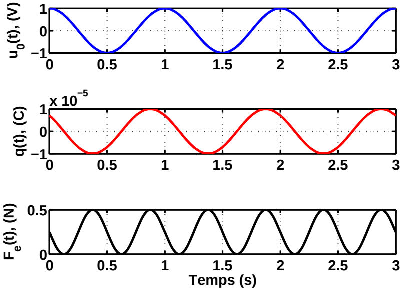


Figure: Nonlinearity of the electrostatic transduction: $F_e(t) \geq 0$ and $F_e(t)$ oscillates with frequency $2f$.

Linearisation of the actuator (1/2)

The capacitor is **polarised**, using a dc bias voltage U_p, on top of the ac component: $u(t) \leftrightarrow U_p + u(t)$

The inter-electrode distance is: $d(t) = d_0 - \xi(t)$.

The charge Q also exhibits dc and ac components: $Q(t) = Q_p + Q_a(t)$, as well as the capacitance:

$$C(t) = C_0 \frac{1}{1 - \dfrac{\xi(t)}{d_0}}$$

by linearization:

$$C(t) \approx C_0 \left(1 + \frac{\xi(t)}{d_0}\right)$$

Linearisation of the actuator (2/2)

Considering that $u(t) \ll U_p$ and $\xi(t) \ll d_0$, the electrostatic force reads:

$$
\begin{aligned}
F_e(t) &= \frac{\varepsilon S}{2} \frac{[U_p + u(t)]^2}{[d_0 - \xi(t)]^2} \\
&\simeq \frac{\varepsilon S}{2} \frac{[U_p + u(t)]^2}{d_0^2} \left[1 + \frac{\xi(t)}{d_0}\right]^2 \\
&\simeq \frac{C_0 U_p^2}{2 d_0} + \frac{C_0 U_p}{d_0} u(t) + \frac{C_0 U_p^2}{d_0^2} \xi(t)
\end{aligned}
$$

Red = dc component (polarisation). Blue = linearised ac component.
Dc force = change in rest position.
Keeping only the ac components:

$$
\boxed{F_e = \Gamma_s \, u + \frac{\Gamma_s^2}{j\omega C_0} \, v}
$$

with $\Gamma_s = C_0 U_p / d_0$ the electrostatic coupling coefficient.

Linearisation of the sensor

We recall the relationship between $C(t)$ and $\xi(t)$ (very small compared to d_0):

$$C(t) \approx C_0 \left(1 + \frac{\xi(t)}{d_0}\right)$$

The charge $Q(t)$ reads:

$$Q(t) = C_0 U_p + C_0 u(t) + \frac{C_0 U_p}{d_0}\xi(t) + \frac{C_0}{d_0}\xi(t)u(t)$$

Linearisation of the sensor

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Keeping first order terms only:

$$Q(t) \simeq \boxed{C_0 U_p} + \boxed{C_0 u(t) + \frac{C_0 U_p}{d_0}\xi(t)}$$

Differentiating w.r.t. time:

$$i(t) \simeq C_0 \frac{du(t)}{dt} + \frac{C_0 U_p}{d_0} v(t)$$

Or, with complex notation:

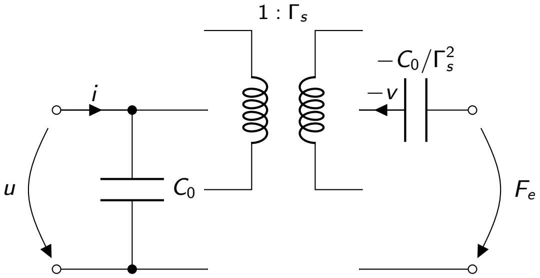
$$\boxed{i = j\omega C_0 u + \Gamma_s v}$$

Representing the electrostatic coupling

Global view of electrostatic transduction with two equations:

$$\begin{cases} F_e = \Gamma_s\, u + \dfrac{\Gamma_s^2}{j\omega C_0}\, v \\ i = j\omega C_0 u + \Gamma_s v \end{cases}$$

Or as a schematics, we get:



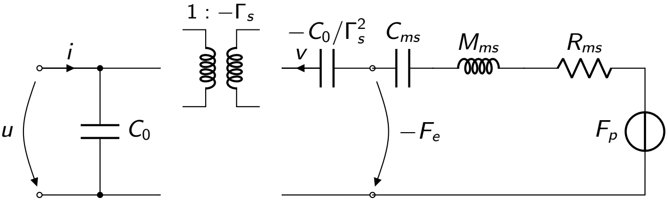
(actuator convention)

Example: electrostatic microphone (1/3)

The microphone membrane may be modelled with a single degree of freedom mechanical resonator (R_{ms}, M_{ms},C_{ms}) with area S_d, subject to a pressure force $F_p = S_d p$. This force is added to the electrostatic force, according to the sign convention.

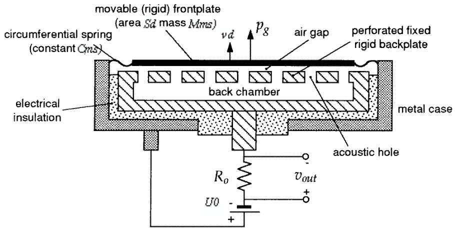
$$F_p + F_e = \left(R_{ms} + j\omega M_{ms} + \frac{1}{j\omega C_{ms}} \right) v$$

$$\begin{cases} -F_e = -\Gamma_s\, u - \dfrac{\Gamma_s^2}{j\omega C_0}\, v \\ i = j\omega C_0 u + \Gamma_s v \end{cases}$$

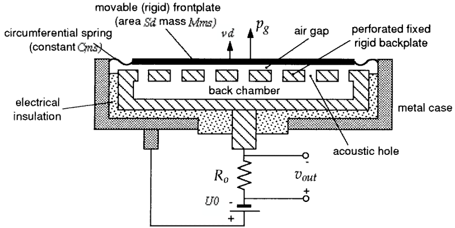


Example: electrostatic microphone (2/3)

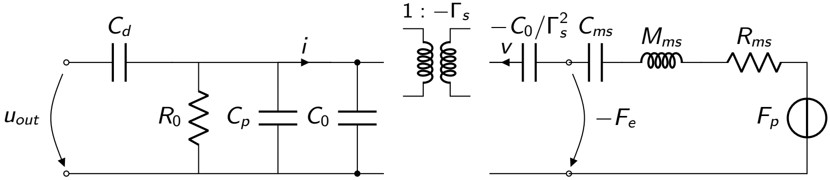
On the electric side, the microphone is connected to a conditioning device, or a preamplifier, comprising a permanent polarization U_p, decoupled from the sensed sound pressure signal.
A resistance R_0 and a decoupling capacitance C_d allows filtering the DC voltage U_p on the electric side, and a parasitic capacitance C_p is created between the microphone body (same potential than the mobile membrane) and the fixed back armature.



Example: electrostatic microphone (3/3)



If we neglect the DC voltage and the decoupling circuit, the full circuit of the electrostatic microphone can be drawn as:



bibliographie

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