

I. Introduction to acoustics

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MA1 - Audio Engineering

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- 1 Introduction
- 2 Basic definitions and properties
- 3 Wave propagation
- 4 Scales in acoustics

Introduction

What is “acoustics” ?

- Science of sounds : branch of physics that addresses emission, propagation, and perception of sounds
- Quality of a space / room

Introduction

What is a *sound* ?

- variation of pressure
(rapid movement of particles back and forth)
- that can produce an auditive sensation

Introduction

What is a *noise* ?

(General language)

- acoustic phenomenon (sound) that is not wanted
- it is SUBJECTIVE

But it also denotes a a random signal (sound)

Sound is vibration

A **vibration** is:

- local variation of pressure (fluid & gases) / stress (solids)
- vibratory movement of the medium around an equilibrium position

Longitudinal vibration (fluids & solids) :

Transverse vibration (solids only) :

In airborne sound, we are mainly dealing with longitudinal vibrations of air particles.

Acoustic pressure

Acoustic vibrations = small fluctuations of pressure around a mean value.

Total pressure P_{tot}

Atmospheric pressure P_0 (constant)

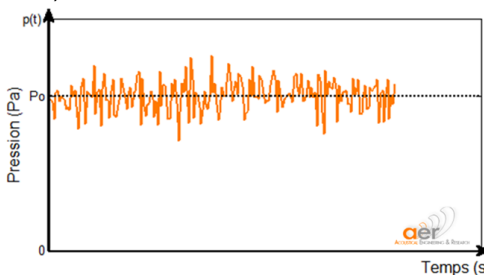
Acoustic pressure $p(t)$ (time dependent)

$$P_{tot} = P_0 + p(t)$$

Unit : *Pascal (Pa)*

Linear acoustic hypothesis :

$$p(t) \ll P_0$$



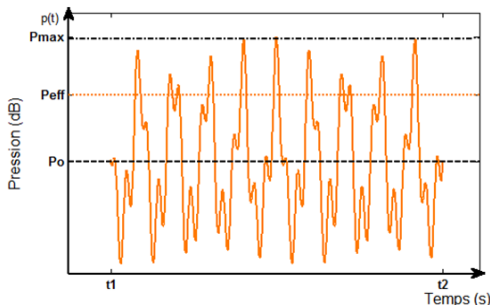
$$P_0 \approx 10^5 Pa, \text{ and } p \approx 2 Pa$$

Rms pressure

Quantity related to the acoustic pressure and to the *level* of sound
 Given acoustic pressure $p(t)$ between t_1 and t_2 :

$$p_{rms}^2 = \frac{1}{t_2 - t_1} \left(\int_{t_1}^{t_2} p(t)^2 dt \right)$$

$$= \langle p^2(t) \rangle$$



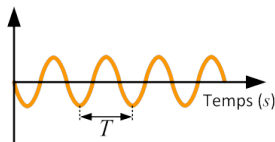
Period, frequency

Even though pure sinusoidal signals are rarely found in nature, sounds as signals are linear combinations of sinusoidal signals. Therefore sound signal description rely of Fourier analysis.

Period T : duration of one oscillation

Frequency f : number of oscillations per second

$$f = 1/T$$



Relation with *pitch*

Units : period in seconds (s) and frequency in Hertz ($\text{Hz} = 1/\text{s}$)

Angular frequency : ω (in rad/s) $\omega = 2\pi f$

Human ear sensitive to frequencies between 20Hz and 20kHz :

$< 20\text{Hz}$	20Hz to 200Hz^1	200Hz^1 to 2kHz^1	2 kHz^1 to 20kHz	$> 20\text{kHz}$
infrasounds	bass	medium	treble	ultrasounds

¹ Note: those frequency bounds should be considered indicative...

Frequency : properties

An *octave* is a 2-fold increase of frequency :

$$f \rightarrow 2f$$

A *decade* is a 10-fold increase in frequency :

$$f \rightarrow 10f$$

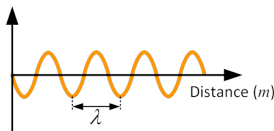
E.g. :

- the standard A is at 440Hz, the A an octave higher is 880Hz
- the audible frequency range covers 3 decades :
20Hz to 200Hz, 200Hz to 2kHz, 2kHz to 20kHz

Wave length : spatial period

The wave length λ is the characteristic length of periodicity in *space* of a sound :

$$\lambda = c/f$$



Length of propagation of a wave during one period

Units : meter (*m*)

Wave number : spatial frequency

Wave number k is the frequency in *space* of a vibration : $\propto$ number of oscillations per unit length.

spatial angular frequency of the vibration $k = \omega/c = 2\pi/\lambda$

Units : radian per meter (rad/m)

Pure sound

Sound with only one frequency

Pressure $p(t)$ is a pure sine :

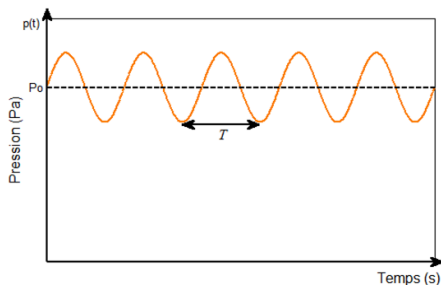
[Play Sound](#)

$$p(t) = P_{max} \sin(\omega t + \phi)$$

T : period (s)

$f = 1/T$: frequency (Hz)

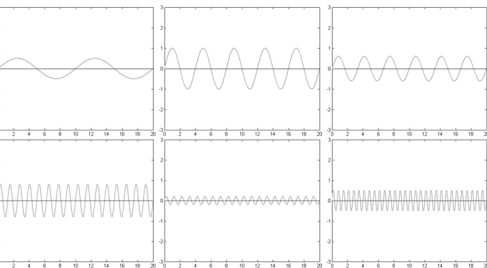
$\omega = 2\pi f$: angular freq. (rad/s)



Complex (harmonic) sound

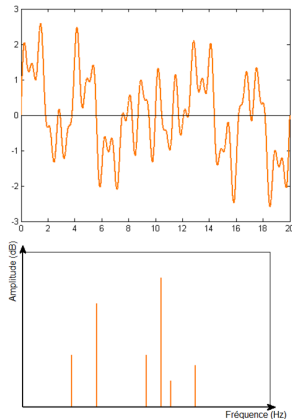
Sum of several pure sounds, the frequencies of which are in *harmonic* relation (integer multiple or rational)

$$p(t) = \sum_n P_n \sin(\omega_n t + \phi_n)$$



Play Sound

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Play Sound

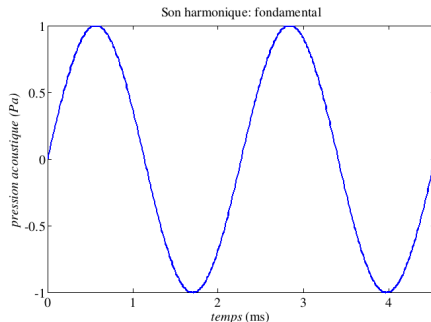
Complex (harmonic) sound

Sum of several pure sounds, the frequencies of which are in *harmonic* relation (**integer** multiple or rational)

$$p(t) = \sum_n^N P_n \sin(\omega_n t + \phi_n)$$

$$\omega_n = n\omega_0 = n \cdot 2\pi \cdot f$$

f_0 = fundamental frequency (Hz)



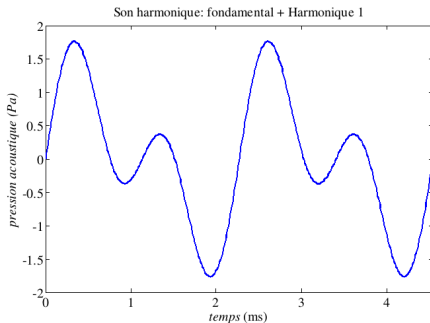
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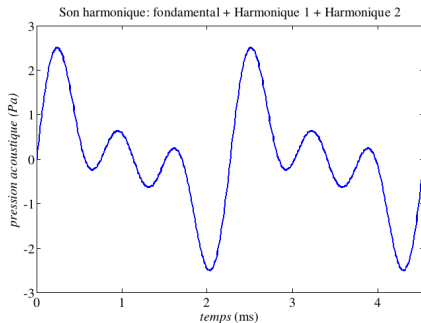
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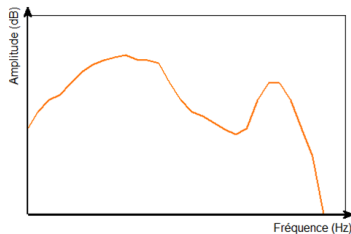
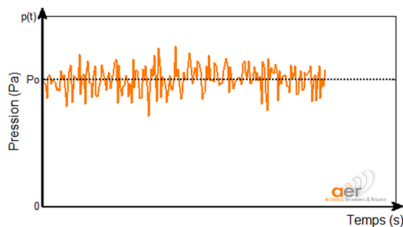


Random sound (random noise)

Random variations of acoustic pressure

- rich spectral content
- random vibrations

Complex sound with no time structure :



Play Sound

Complex notation of sound signals

In harmonic regime (which applies for most sounds), the complex notation ("phasor") is used:

$$\underline{p}(t) = P_{max} e^{j(\omega t + \Phi)}$$

To simplify the notation, we only use the complex amplitude $\underline{p} = P_{max} e^{j\Phi}$.
With this notation, the following mathematical operations become:

	Time domain	Phasor notation
Derivation	$\frac{\partial \underline{p}}{\partial t}$	$j\omega \times \underline{p}$
Integration	$\int \underline{p} dt$	$\frac{\underline{p}}{j\omega}$

Speed of sound c

Speed of sound waves

Depends on physical properties of the medium :

- density
- temperature
- ambient pressure ...

Does *not* depend on :

- amplitude
- frequency

Usually $c \approx 340\text{m/s}$
(in dry ambient air)

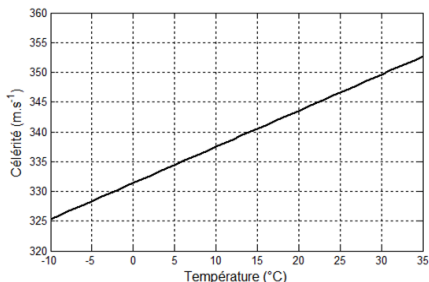
Speed of sound c

Units : meter per second (m/s)

Approximate dependence on temperature :

$$c_a(T) = 331.6 + 0.607T$$

(T in $^{\circ}C$)



WARNING :
 speed of sound
 $\neq$
 velocity of fluid particles !!

Acoustic velocity

Acoustic velocity v : at any given position $\vec{r}$ and time t , it represents the local perturbation of fluid particles velocity.

$$\vec{v}(\vec{r}, t) = \vec{V}_t(\vec{r}, t) - \vec{v}_m(\vec{r}, t)$$

In a medium at rest :

$$v_m = \langle V_t(\vec{r}, t) \rangle = 0$$

Acoustic *volume* velocity

Acoustic volume velocity (or volume flow) : integration of the acoustic velocity over a surface

$$\vec{q} = \int_S \vec{v} d\vec{S}$$

In pipe (1D, directing vector $\vec{e}_x$), S is the cross sectional area.

$$\vec{q} = q\vec{e}_x = Sv\vec{e}_x$$

In free field or room (3D), for a volume source, S is the contour surface of the source.

Acoustic power W

Quantity of energy through a given surface per unit time

$$W = \int_S \langle p.v \rangle dS = \langle p.q \rangle$$

Does not depend on the distance from the source

Units : Watt (W)

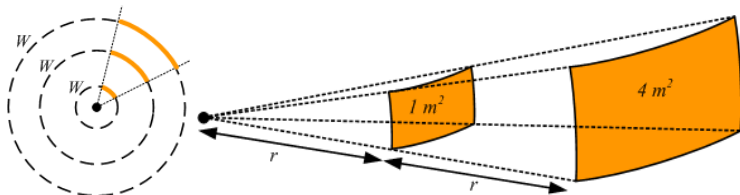
The power is linked to:

- acoustic velocity of the source (quadratic)
- radiation conditions (pipe, free field...)

Acoustic intensity I

Quantity of acoustic power *per unit surface*

Depends on the distance !!



Units : Watt per square meter (W/m^2)

$$I = \langle p \cdot v \rangle = \langle p^2 \rangle / (\rho c) = \rho c \langle v^2 \rangle$$

Relation between W and I

Intensity = acoustic power per unit surface

Power = integration of intensity over the surface :

$$W = SI$$

Spherical waves :

$$W = 4\pi r^2 I(r)$$

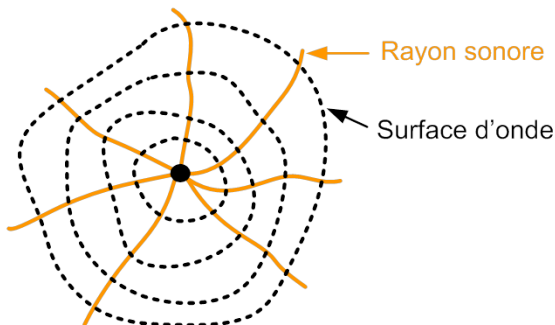
Intensity decreases with distance
(in $1/r^2$ for spherical waves)

Wave front and acoustic ray

Wave front definition : surface of equal phase

Property : perpendicular to the direction of propagation

Acoustic ray : line starting from the source, following the propagation direction. Always perpendicular to wave fronts.

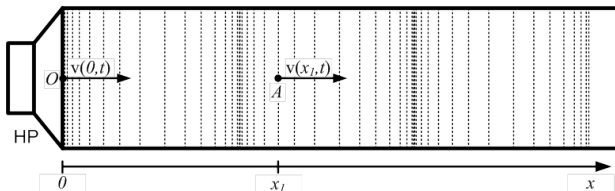


Plane waves : definition

Wave fronts = parallel, plane surfaces

Acoustic rays = parallel, straight lines

Wave fronts are perpendicular to the direction of propagation



The acoustic wave equation (on p):

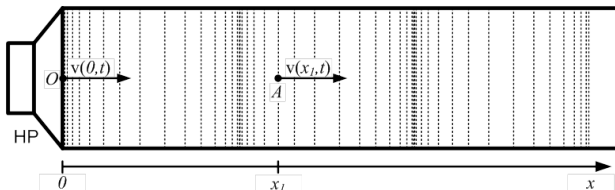
$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

Plane waves : definition

Wave fronts = parallel, plane surfaces

Acoustic rays = parallel, straight lines

Wave fronts are perpendicular to the direction of propagation



In 1D:

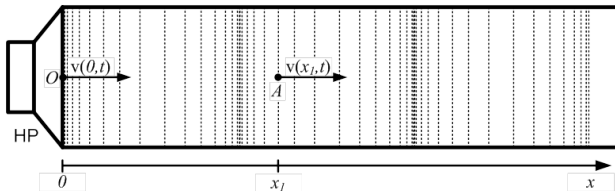
$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

Plane waves : solutions

Wave fronts = parallel, plane surfaces

Acoustic rays = parallel, straight lines

Wave fronts are perpendicular to the direction of propagation



$$p(x, t) = \left(p_+ e^{-jkx} + p_- e^{+jkx} \right) e^{j\omega t}$$

+ boundary conditions (if the medium is finite)

Plane waves : properties

Considering infinite tube (no reflected wave, only the $p_+ e^{-jkx}$ term)
 Pressure p and velocity v are proportional :

$$p(x, t) = Z_c v(x, t)$$

where $Z_c = \rho c$ is defined as the **characteristic impedance** of the medium.

Amplitude of p (resp. v) independent of distance

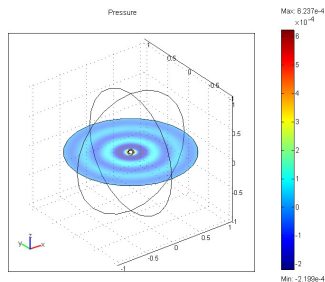
Intensity proportional to square of rms pressure :

$$I = \langle pv \rangle = \frac{p_{rms}^2}{\rho c}$$

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source

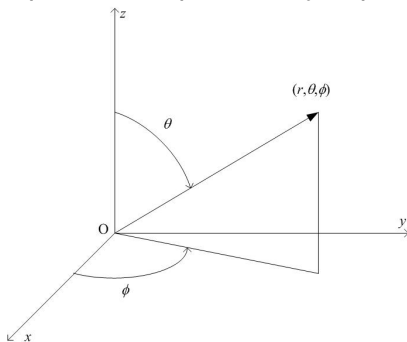


Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source



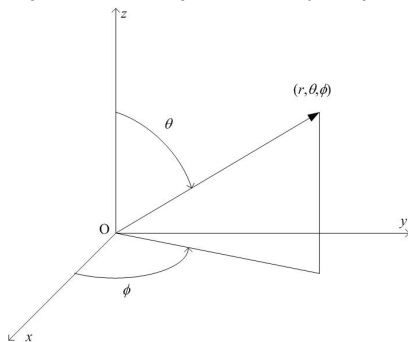
$$p(r, \phi, \theta, t) = p(r, t)$$

Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source



In spherical coordinates:

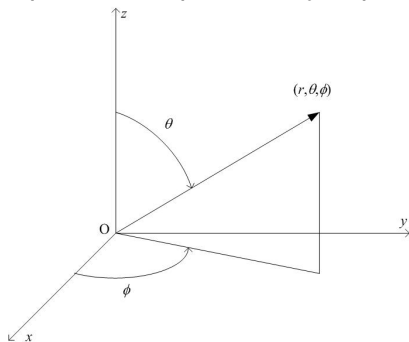
$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \cot \theta \frac{\partial}{\partial \theta} + \frac{\partial^2}{\partial \theta^2} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source



Wave equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} \frac{\partial p}{\partial r} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

Solutions (similar to plane waves):

$$p(r, t) = \frac{A}{r} e^{j(\omega t - kr)}$$

Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : properties

Pressure p decreases like $1/r$:

$$p(r, t) = \frac{A}{r} e^{j(\omega t - kr)}$$

Intensity prop. to $1/r^2$:

$$I(r) = \frac{\langle p^2(r, t) \rangle}{\rho c} = \frac{1}{\rho c} \frac{A^2}{r^2}$$

Power on concentric spheres is constant :

$$W = \frac{4\pi r^2 \langle p^2(r, t) \rangle}{\rho c} = \frac{4\pi A^2}{\rho c}$$

Sound wave superposition (I)

Uncorrelated sources

N sources $\rightarrow$ resulting pressure field is the sum of each pressure wave
(example : several cars in traffic, several instrument in a room...)

Property : the *interference* term is null

$$\begin{aligned} p_{rms}^2 &= \langle (\sum_n p_n(t))^2 \rangle \\ &= \sum_n \langle p_n^2(t) \rangle + 2 \sum_i \sum_{j>i} \langle p_i(t) p_j(t) \rangle \\ &= \sum_n \langle p_n^2(t) \rangle \end{aligned}$$

The energy of each source adds up :

$$I = \sum_{n=1}^N I_n \quad \text{and} \quad p_{rms}^2 = \sum_{n=1}^N p_{rms,n}^2$$

Sound wave superposition (II)

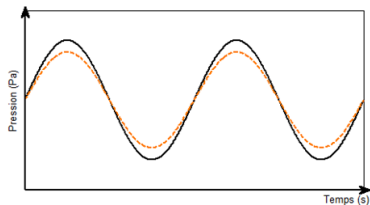
Correlated sources

Stable relation between sources (e.g. several speakers in a room, all playing the same signal)

Interferences are possible (application to active noise control)

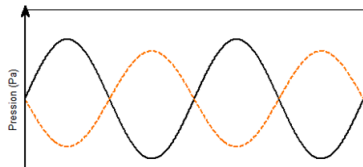
Case p_1 and p_2 in phase :

$$\begin{aligned} p(t) &= p_1(t) + p_2(t) = 2p_1(t) \\ p_{rms}^2 &= 4p_{rms,1}^2 \end{aligned}$$



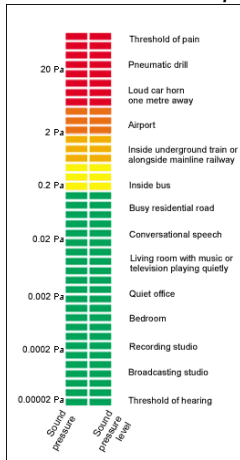
Case p_1 and p_2 out of phase :

$$\begin{aligned} p(t) &= p_1(t) + p_2(t) = 0 \\ p_{rms}^2 &= 0 \end{aligned}$$



Sound levels

From the weakest audible sound (hearing threshold p_0) to the pain threshold : factor 10^6 to 10^8 on p !!



⇒ we need a logarithmic scale !!

Sound levels

Acoustic *sound pressure level* (SPL) :

$$L_p = 10 \log_{10} \left(\frac{p_{rms}^2}{p_0^2} \right) = 20 \log_{10} \left(\frac{p_{rms}}{p_0} \right)$$

where the reference pressure p_0
is usually $20 \mu Pa$ (mean hearing threshold, *in the air*)

Acoustic *intensity level* :

$$L_I = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

where usually $I_0 = \frac{p_0^2}{\rho c} = 10^{-12} W/m^2$

Note: generally, we can write $L_p = L_I$

Acoustic *power level* :

$$L_W = 10 \log_{10} \left(\frac{W}{W_0} \right)$$

where usually $W_0 = I_0 \times 1 m^2 = 10^{-12} W$

Power and pressure levels

Units : $dB \rightarrow$ need to detail what kind of level (L_p , L_I , or L_W)

Reference level : refer to a reference value used in the log ($20\mu Pa$, $1W$, ...)

Relation between L_p (L_I) and L_W for an omnidirectional source ($I(r) = \frac{W}{\pi r^2}$):

$$L_p = L_I = L_W - 10 \log_{10}(4\pi r^2)$$

two correlated sources of equal amplitude and in phase :

$$2 \times p \rightarrow L_p + 6dB$$

two uncorrelated sources of equal intensity :

$$2 \times p_{rms}^2 \text{ (or } 2 \times I) \rightarrow L_p + 3dB$$

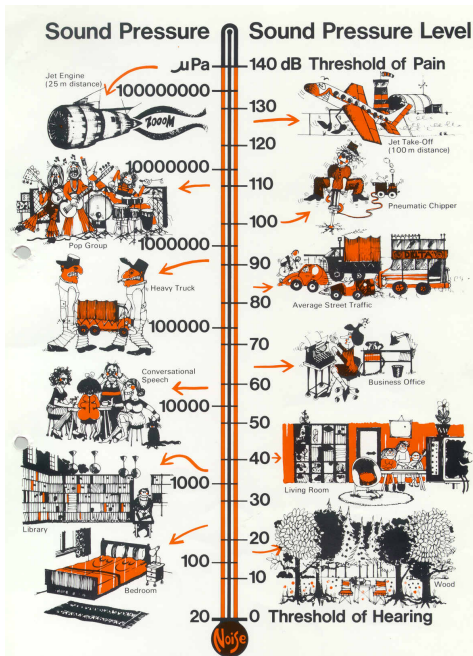
Logarithmic scale

Useful :

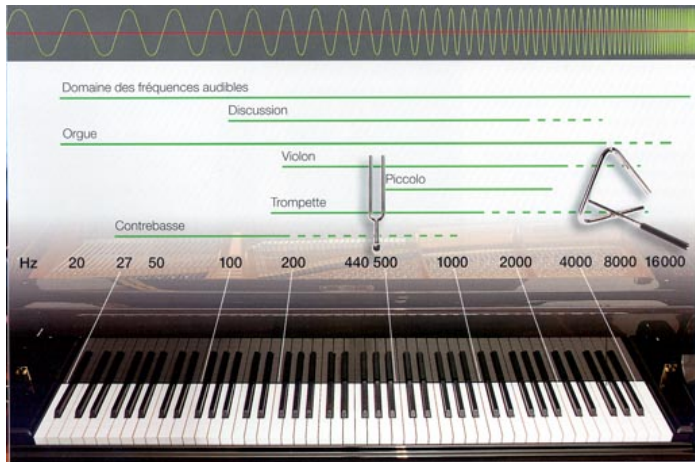
- hearing threshold $p_0 = 20\mu Pa \rightarrow 0dB$
- pain threshold $\sim 20Pa \rightarrow 120dB$

Relation to perception :

- human ear sensitive to the *log* of pressure
- doubling of sound pressure [Play Sound](#)
- just noticeable difference of level between 2 sounds [Play Sound](#)
- tripling of sound pressure [Play Sound](#)



Frequency scales



Frequency scales

Acoustical scales:

- **Octave** : $f_0 \rightarrow 2f_0$
- **Semi-octave** : $f_0 \rightarrow \sqrt{2}f_0$ - **Full octave** : $f_0 \rightarrow \sqrt{2}f_0 \rightarrow 2f_0$
- **Third-octave** : $f_0 \rightarrow \sqrt[3]{2}f_0$ - **Full-octave** : $f_0 \rightarrow \sqrt[3]{2}f_0 \rightarrow \sqrt[3]{4}f_0 \rightarrow 2f_0$
- **Full-octave** : $f_0 \rightarrow 10f_0$
- etc.

Musical scales:

- **Octave** : $(f_0 \rightarrow 2f_0)$
- **Major third** : $f_0 \rightarrow 5/4f_0$
- **Minor third** : $f_0 \rightarrow 6/5f_0$
- **Perfect fifth** : $f_0 \rightarrow 3/2f_0$
- etc.