LAB 2: MATCHING NETWORK

1. Introduction

The goal of this laboratory exercise is to understand the concept of microwave matching networks, familiarize yourself with the Smith chart, investigate various matching network architectures based on discrete reactive elements (capacitors, inductors) and transmission lines, and finally design one matching circuit for a specific application. After this exercise, you will be able to:

- Read the Smith chart
- Understand the operations inside the Smith chart
- Design matching networks for arbitrary load impedances
- Understand the limitations of each network architecture
- Design quarter-wavelength transformers
- Use a free open-source tool for microwave network analysis.

This exercise manual starts with a short theoretical background on microwave matching networks and some practical instructions on how to navigate the Smith chart in section 2. It is important to carefully read this section prior to the exercise, as this material will be used in all parts of the exercise. You are also welcome to briefly review any prior knowledge on transmission lines. The steps of the exercise itself are given in section 3.

2. Theoretical background

2.1. *Smith Chart*

The Smith chart is a very powerful tool in RF and microwave engineering. It allows for fast microwave calculations as it converts complex mathematical operations into simple geometrical transformations. The Smith chart is used to represent complex values of impedances, reflection coefficients, etc. Any complex impedance Z=R+jX or admittance Y=G+jB can be represented in one of the two variations of the Smith chart, shown in Fig. 2.1. The complex value is first normalized with a system impedance, which is typically the characteristic impedance of the transmission lines at the ports:

$$z = \frac{Z}{Z_0} = \frac{R}{Z_0} + \mathrm{j}\frac{X}{Z_0} = r + \mathrm{j}x, \qquad y = g + \mathrm{j}b$$

The admittance chart is obtained from the impedance chart by simply rotating it by 180°.

The real part is read from the horizontal axis and the imaginary part from the outer circle. Therefore, one impedance/admittance value corresponds to a single point in the Smith chart. The normalized impedance equal to 0 corresponds to the left-most point in the chart, whereas an infinite impedance corresponds to the right-most point. The center of the chart represents an impedance equal to the normalization impedance Z_0, or a normalized impedance equal to 1.

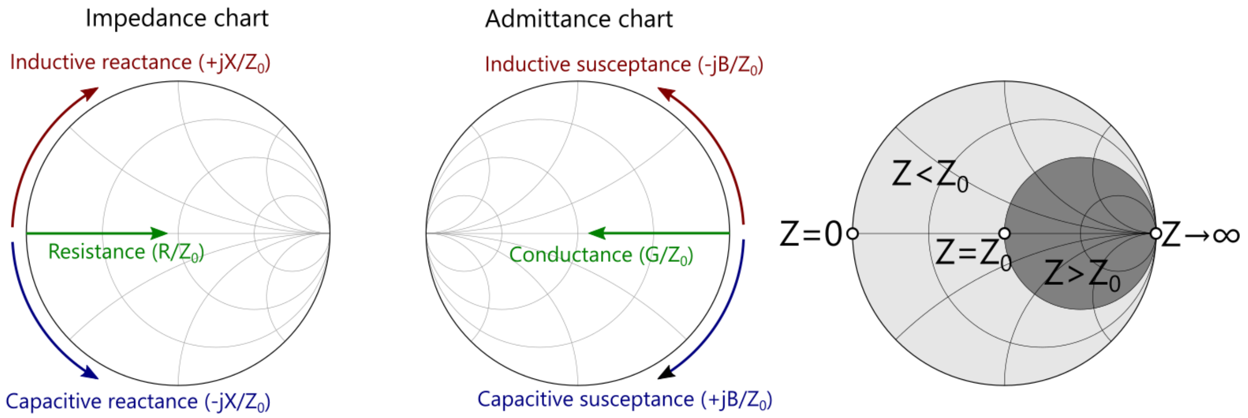
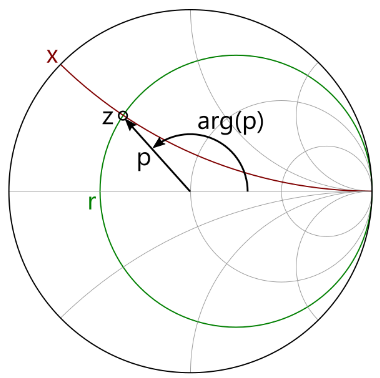


Fig. 2.1. Impedance and admittance Smith Charts.

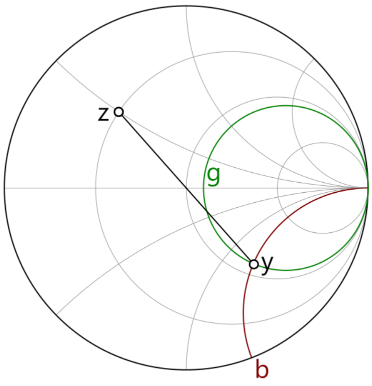
This chart has various interesting consequences, several of which are illustrated in the table below. The table will be very useful for this exercise and future reference.



The complex reflection coefficient corresponding to impedance *Z* is calculated using:

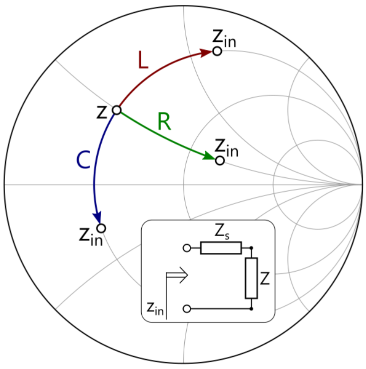
$$\rho = \frac{Z - Z_0}{Z + Z_0} = \frac{z - 1}{z + 1}$$

In the Smith chart, the distance between the chart center and the impedance is equal to the magnitude of the reflection coefficient, and the angle between this line and the positive part of the real axis represents its phase angle.



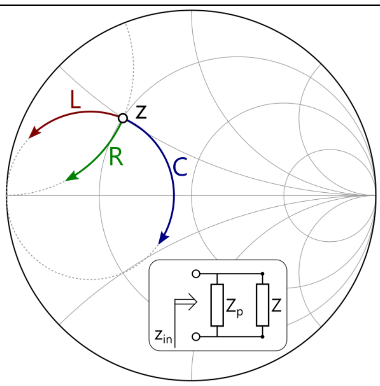
An admittance y is obtained from an impedance z by a simple mapping symmetrical to the center of the Smith chart, and reading of the values g and b. Note that the reading of the admittance real and imaginary components is to be done in the impedance coordinates.

Equivalently, the impedance point can be inserted in the same location in the admittance chart, and the values read from there.

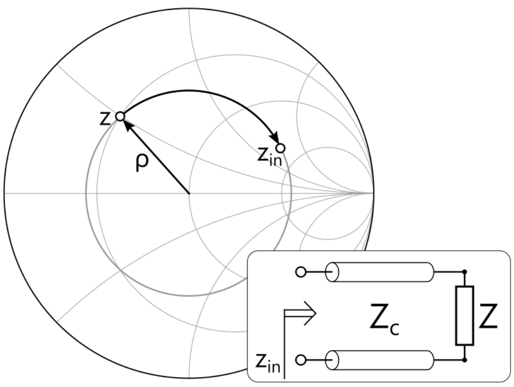


Adding a serial resistance (R) translates the input impedance along the constant-reactance circle.

Adding a serial reactance (L or C) translates the input impedance along the constant-resistance circle. **



Similar translation occurs for impedances added in parallel Z_p, except that the movement is done along circles of a fictional overlaid admittance Smith chart. **



Consider a lossless transmission line having a characteristic impedance Z_c terminated with an impedance Z. The complex reflection coefficient will have the same magnitude but different phase in all points along the line. These points are represented by a circle of radius ρ with a center at the center of the Smith chart. The impedance at the line input can be obtained by a translation of the load Z in a clockwise direction along this circle.

** Note that the arrow direction does not represent a growing L/C value in all cases.

2.2. *Matching Networks*

If a load with an impedance Z is connected to a generator having a complex impedance Z_g, the active power in the load will be maximal if the load is "matched" to the generator:

$$Z = Z_g^*$$

Achieving power matching is essential in microwave circuits to avoid reflections, which contribute to loss of precious microwave power and deterioration of the signal-to-noise ratio.

Microwave generator and load are typically connected with some sort of a transmission line, having a characteristic impedance Z_0, as seen in the figure below. In this exercise, we will consider that the generator input impedance and the characteristic impedance of the transmission line are both real and equal to 50 Ω. This is a valid assumption for a large number of modern microwave circuits.

To achieve matching of an arbitrary load impedance Z=R+jX to the line/generator impedance, a matching network is inserted somewhere along the line, typically close to the load. It typically consists of reactive elements and enables an input impedance of 50 Ω, removing any reflections.

2.2.1. Discrete Elements

The simplest matching network architecture consists of two discrete reactive elements – capacitors or inductors – one connected in series and one in parallel to the load. There are 8 possible combinations and depending on the elements used, some networks can be more suitable for a given load impedance (for example, some networks can require a smaller inductance value, or work in a larger frequency band). Furthermore, there exist the so-called "forbidden regions" in the Smith chart for each of the architectures. If a load impedance is located inside the forbidden region, the given matching network cannot be used to achieve a perfect match, and another one must be used. An illustration of these regions can be found at https://www.qsl.net/va3iul/Files/Impedance_Matching_Networks.pdf.

2.2.2. Transmission Lines

Sections of open-ended or short-circuited transmission lines can be used equally well for matching. A transmission line stub (open-ended or short-circuited) connected in parallel to the main line at a certain distance from the load can behave as a parallel inductance/capacitance, depending on the length of the added stub.

2.2.3. *Quarter-Wavelength Transformer*

If a load impedance is purely real, Z=R, a transmission line with a length of one-quarter wavelength at the working frequency and a characteristic impedance Z_t can be used for matching. Moving along a transmission line for one-quarter of a wavelength changes the sign of the reflection coefficient:

$$\rho(\lambda_g/4) = -\rho(0)$$

Therefore:

$$z(\lambda_g/4) = \frac{1+\rho(\lambda_g/4)}{1-\rho(\lambda_g/4)} = \frac{1-\rho(0)}{1+\rho(0)} = \frac{1}{z(0)},$$

$$\frac{Z(\lambda_g/4)}{Z_t} = \frac{Z_t}{Z(0)}$$

$$\frac{Z_{in}}{Z_t} = \frac{Z_t}{R}$$

It directly follows that the required characteristic impedance of a quarter-wavelength transmission line can be calculated as:

$$Z_t = \sqrt{Z_{in}R}$$

2.3. *Antenna Matching*

During the design process of an antenna the term “impedance matching” commonly appears. That happens because the antenna must be matched to the generator to operates in the desired frequency band with the maximum efficiency. Optimal efficiency results in maximum range, minimum power consumption and reduced heating. Matching the input impedance of the antenna (Z_L) to the impedance of the generator (Z_g), that usually is 50 Ω, is a requisite to ensure that the maximum power is transferred from the generator to the antenna with negligible reflections.

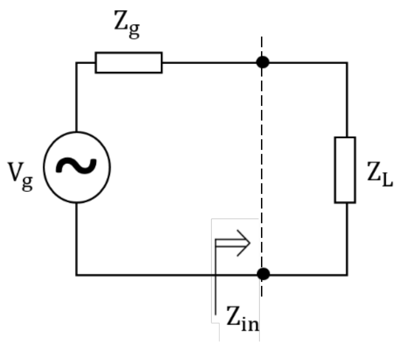
Antenna impedance is complex (Z_L=R+jX), consisting of both resistive and reactive parts. With the aim of getting a perfect match between the antenna and the generator (no reflections) the input impedance (Z_{in}) which is the impedance that is seen from the generator to the antenna/load, has to be the complex conjugate of the impedance of the generator (Z_g), i.e. $Z_{in}= Z_g^*$.

Standing Wave Ratio (SWR) is a measure that defines how well the antenna impedance is matched to the generator, a value smaller than 1.5 is desirable. SWR can be expressed as the reflection coefficient (ρ), which refers to the power reflected from the antenna. The reflection coefficient is a function of antenna impedance and the characteristic impedance. A perfect match is obtained when $Z_{in} = Z_g$, which gives $\rho = 0$ and SWR=1.

$$SWR = \frac{1 + |\rho|}{1 - |\rho|} \qquad \qquad \rho = \frac{Z_{in} - Z_g}{Z_{in} + Z_g}$$

There are two different ways of matching an antenna:

- **Changing antenna parameters:** in this case we modify different parameters of the antenna like its dimensions, the feeding network, etc. to adjust that the impedance of the antenna to be the complex conjugate of the impedance of the generator (Z_L=Z_g^*).

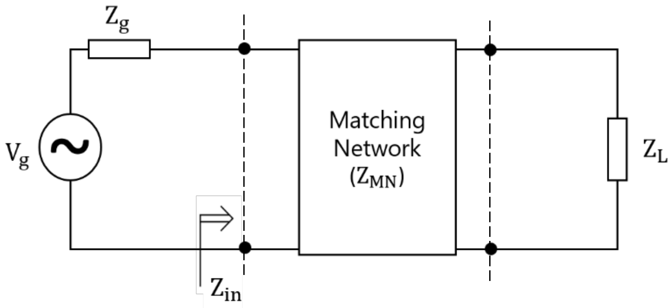


$$\left.\begin{array}{r} Z_L = Z_{in} = R_{in} - jX_{in} \\ Z_g = R_g + jX_g \end{array}\right\} Z_{in} = Z_g^*$$

$$Z_{in} = Z_g^* \begin{cases} R_g = R_{in} \\ X_g = -X_{in} \end{cases}$$

Fig. 2.2. Equivalent circuit conjugate matching changing the parameters of the antenna.

- **Using matching networks:** in this situation we insert a matching network between the antenna and the generator. The combination of the impedance of the matching network and the impedance of the antenna allow to get an input impedance that is the complex conjugate of the impedance of the generator (Z_{in}= Z_g^*). We can distinguish different types of matching networks:
  - **Discrete elements:** consist of the addition of a series L or C component, shunt L or C component or the combination of series and shunt lumped elements (usually one L and one C component one connected in series and one in parallel to the load).
  - **Transmission lines:** the addition of sections of open-ended or short-circuited transmission lines can be used equally well for matching.
  - **Quarter-Wavelength Transformer:** if the impedance is purely real a transmission line with a length of one-quarter wavelength at the working frequency and a characteristic impedance Z_t can be used for matching.



$$\left.\begin{array}{l} Z_{in} = R_{in} - jX_{in} \\ Z_g = R_g + jX_g \end{array}\right\} Z_{in} = Z_g^*$$

$$Z_{in} = Z_g^* \left\{\begin{array}{l} R_g = R_{in} \\ X_g = -X_{in} \end{array}\right.$$

Fig. 2.3. Equivalent circuit conjugate matching using matching network.

2.3.1. Aperture coupled microstrip antenna

One of the most extended microstrip antenna feeding technique is the **aperture coupling.** The antenna is composed by two parts: the microstrip patch, that could be placed over foam or over other different substrate, and the feeding network. The last contains the microstrip feedline, that will be connected with the generator, the feed substrate and the ground plane with the coupling aperture. A sketch of this antenna is shown in Figure.2.4. This antenna works as follows: the power is inserted in the microstrip feedline arriving to the coupling aperture that limits the amount of power that arrives to the patch depending on its dimensions. The power pass through the coupling aperture to the patch that radiates the energy.

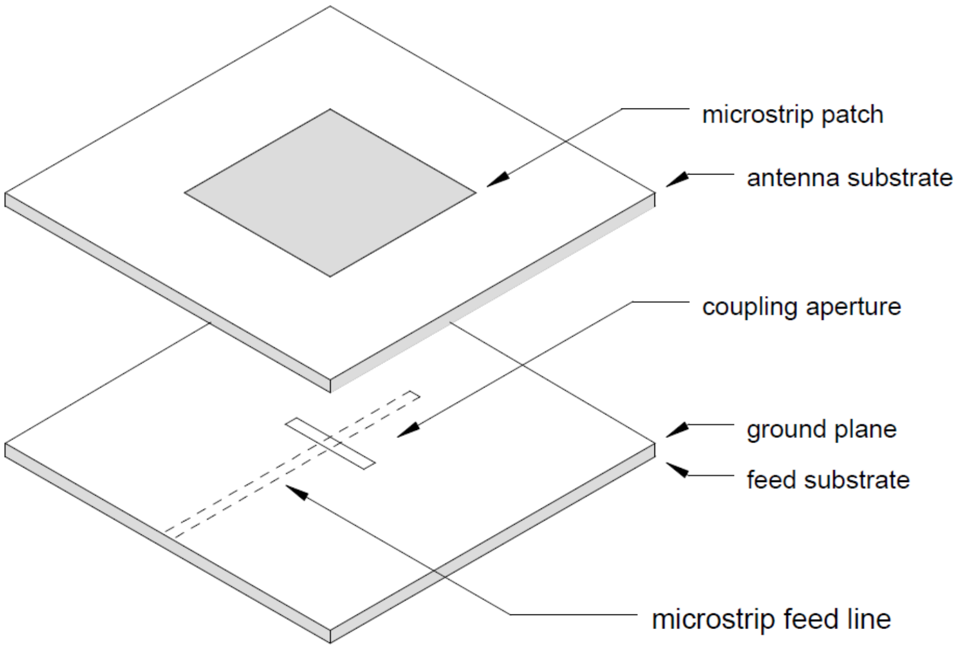


Fig. 2.4. Sketch of aperture coupled microstrip antenna.

In this lab we will match the aperture microstrip antenna. We will do it in two different ways: changing the antenna parameters and using a matching network. The objective of this lab is to compare these two matching techniques. At the same time, the change of the parameters of the antenna will allow us to study the coupling phenomenon. The parameters that we will modify to study the matching will be the length of the slot and the thickness of the foam, i.e. the distance between the feeding network and the patch.

As mentioned, the Smith Chart provides a very useful visualization of the input impedance of a microwave device. The evolution of the input impedance with frequency in the Smith Chart allows us to differentiate three scenarios, that are illustrated in Figure 2.5:

- **Overcoupling:** When the circle surrounds the center of the chart, too much power is coupled to the patch through the aperture and we can say that the resonant device is overcoupled.
- **Critical coupling:** When the circle crosses the center of the chart.
- **Undercoupling:** In this case the curve does not have its center of the chart in it, and that means that less power is coupled to the antenna.

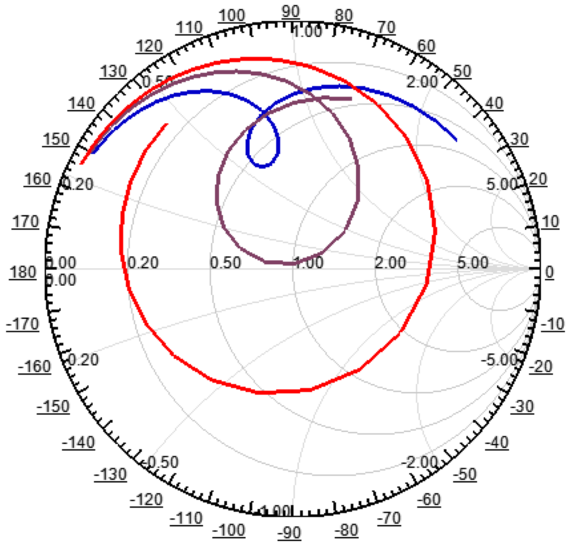


Fig. 2.5. Input impedance of a resonator represented on the Smith chart for undercopling (blue), critical coupling (purple), and overcoupling (red)

3. Matching Network Simulations

For this exercise, you will use a free software for circuit analysis – Quite Universal Circuit Simulator (Qucs). It is similar to popular software tools, such as ADS, Microwave Office (MWO), and others. Learning to use Qucs allows you to quickly become proficient in similar tools if necessary, while having a free-license tool for personal use in the future. The software can be downloaded [here](). You are welcome to download the software before the exercise and familiarize yourself with the environment. The window layout is shown below, and main sections and buttons are highlighted.

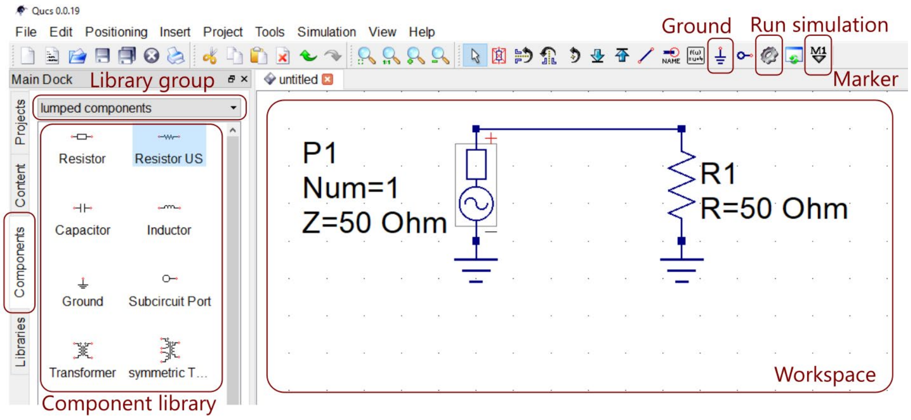


Fig. 2.5. QUCS interface.

After running the software, it is useful to enable the automatic wiring option:

File -> Application Settings -> Start wiring when clicking open node: Check.

Useful instructions:

- Mouse scroll moves the workspace up-down. Shift+scroll moves the screen sideways. Ctrl+scroll zooms in/out.
- To select a component, select the *Components* tab, select an appropriate library group from the drop list, left click on a component, and click in the workspace to place it. Right click rotates the component before placement, Esc to finish.
- Double click on a component to edit its properties.
- Ctrl+G places a ground node.
- Pressing '0' resets the view to fit the window.
- F2 runs the simulation.

3.1. *Impedance and Reflection Coefficient*

The first part of the exercise will be to create a basic circuit, run a simulation and read the results. The circuit to be created is shown below.

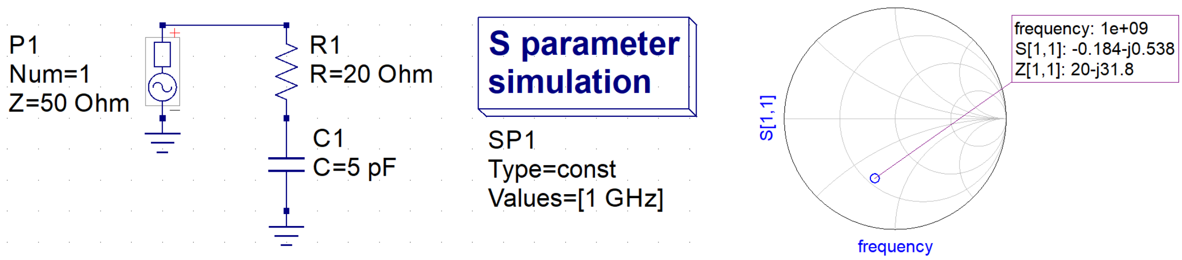


Fig. 2.6. QUCS interface.

1. Start Qucs, create a new project and save it as **Lab_2_Ex_1.sch**.
2. Select *Components* tab, *Sources* group, select and place a *Power Source* (Esc to finish).
3. *Lumped components* -> Resistor US -> place. Repeat for the capacitor.
4. Place the two grounds (Ctrl+G or select from the dock).
5. Click on the two open nodes to connect them.
6. *Components* -> *Simulations* -> *S-parameter simulation*.
7. Double click on the S parameter simulation to edit component properties: *Sweep parameter*: frequency, *Type*: constant, *Values*: 1 GHz, OK.
8. Set the resistor to 25 Ω, capacitor to 5 pF.
9. Save the project and run the simulation .

If the schematic is correctly designed, result display window is automatically displayed. You can navigate back to the schematic by selecting the '.sch' tab. We can now display the simulation result of the network. By default, the normalization impedance of the Smith chart is 50 Ω.

10. *Components* -> *Diagrams* -> *Smith Chart*.
11. Double click on the *S[1,1] Dataset* to add it to the *Graph*. Click on S[1,1] under *Graph*, select a color, *Style*: circles, *Thickness*: 2, OK.
12. Click on the Marker button and place a marker on the only point in the chart.

Is the result expected? In which part of the Smith chart is the impedance located: capacitive or inductive? Based on the marker readings, note and calculate the following:

Impedance	Reflection coef. ρ (S_{11}) (magnitude/phase)	Magnitude ρ (dB)	VSWR (σ)

If we fix a criterion for a good matching of the reflection coefficient magnitude below -10 dB, is this load correctly matched to 50 Ω? ______

Modify the values of the resistance and capacitance to see the resulting effects in the Smith chart.

Optional: Replace the capacitor with a 10 nH inductor and observe the results.

3.2. ***Discrete Matching Network***

For the capacitive load in the previous exercise, we will first design a matching network using discrete reactive elements. For this specific load, only a few network architectures are suitable to obtain a perfect matching. We will select the one with an inductor in series to the load, followed by a capacitor in parallel, as shown below.

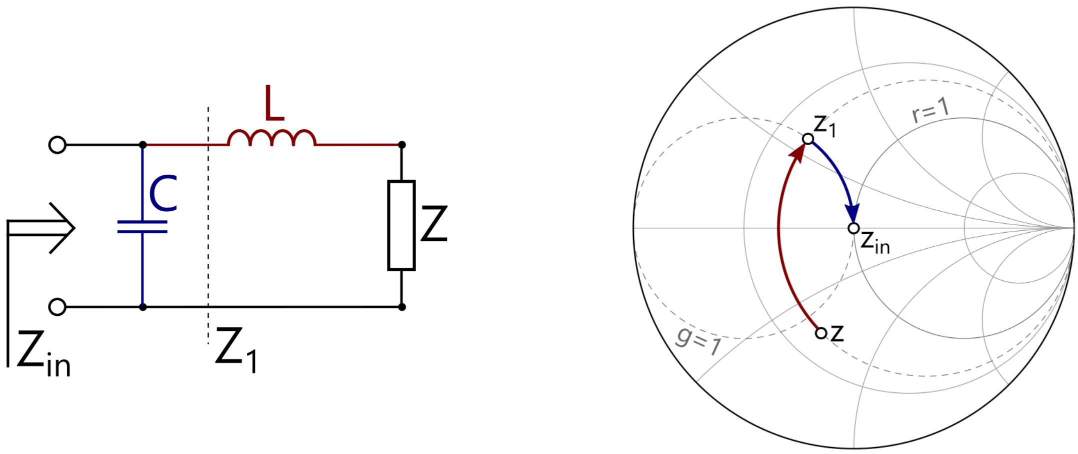


Fig. 2.7. Discrete matching network example in Smith Chart.

The load consists of a series 25 Ω resistor and a 5 pF capacitor, as in section 3.1. For a perfect matching, the impedance seen at the network input should be exactly 50 Ω.

If we suppose this network is matched, then the input impedance is located at the center of the Smith chart (normalized impedance $z_{in}=1$). The normalized input admittance $y_{in}=1/z_{in}=1$ is in the same location. This input admittance is equal to the sum of the capacitor admittance and the admittance seen from the cut Z_1 to the right. Since the capacitor admittance is purely imaginary, the admittances y_{in} and y_1 must have the same real part ($g=1$). As seen in Section 2.1, adding a capacitor in parallel moves the impedance downwards along fictional constant-conductance circles of the admittance Smith chart (in this case, the circle $g=1$). This means that the impedance z_1 must be located on the circle $g=1$, in the top half of the Smith chart.

Impedance Z_1 consists of a series inductor and the impedance Z. Since the inductor impedance is purely imaginary, the impedances z and z_1 must have the same real part. As seen in Section 2.1, adding an inductance in series moves the impedance upwards along constant-resistance circles (in this case, the circle $r=r_L$). This means that the load impedance is located on the circle $r=r_L$ below impedance z_1.

From this analysis, it directly follows that the location of the impedance z_1 is the crossing point of circles $g=1$ (admittance chart) and $r=r_L$!

To perform the exercise, open the schematic **Lab_2_Ex_2.sch** and follow the steps:

1. Connect the generator with the load and run the simulation. You should observe the same impedance as in Exercise 3.1. You will also notice an additional Smith (admittance) chart to assist you for the following steps.
2. Insert an inductor in series between the generator and the load, and connect the circuit.
3. Manually adjust the inductance and observe the effect. Change the inductance until the input impedance is moved to the circle $g=1$ in the admittance chart. Note the value in the table below.
4. Insert a capacitor in parallel between the generator and the inductor, and connect the circuit (don't forget the ground!).
5. Manually adjust the capacitance and observe the effect. Change the capacitance until the input impedance is translated to the Smith chart center (the network is matched to 50 Ω). Note the value in the table below.

After completing the exercise steps, select another arbitrary load impedance. Then, choose an appropriate matching circuit, design a circuit schematic in Qucs starting from the previous one and note down the results in the table. Repeat this process up to two times.

Load	$Z_L^{(1)} = 25\Omega - \frac{j}{\omega\ 5\text{pF}}$	$Z_L^{(2)} =$	$Z_L^{(3)} =$
First element	Series		
	Inductor		
	___ nH		
Second element	Parallel		
	Capacitor		
	___ pF		

3.3. *Transmission-Line Matching Network*

This part of the exercise will show how matching can be done using only transmission line sections. If the use of discrete components is avoided, the matching network can often be quite simplified in terms of manufacturing. A simple transmission-line matching network consists of a line section (stub) inserted in the main line, at a certain distance from the load. The stub can be short-circuited or open-ended; in this exercise, we will use an open-ended stub.

An open-ended stub essentially represents a reactance connected in parallel to the main line. Its purpose is to cancel the susceptance seen from the point 2 towards the load. There are several ways to explain how this matching network works, and we will explain it using the admittance chart.

Suppose that the network is matched at the input (Z_{in}=50 Ω). This means that the input admittance and $y2$ are both in the center of the Smith chart (as the reflection coefficient in the input line is 0). Since the input admittance of the stub is purely imaginary, that means that it cannot affect the real part of the admittance y_1. Therefore, the both the admittances y_1 and y_2 must lie on a constant-g circle, in this case g=1, as they have the same real parts (conductance).

Admittance y_1 is obtained from the load admittance y_L by travelling along the main transmission line in the clockwise direction. Along the line, the reflection coefficient ρ_L does not change its magnitude, only the phase. Therefore, both y_1 and y_L are on the constant-ρ circle, in this case $\rho=\rho_L$.

It directly follows that the location of the admittance y_1 is
the crossing point of circles g=1 and $\rho=\rho_L$!

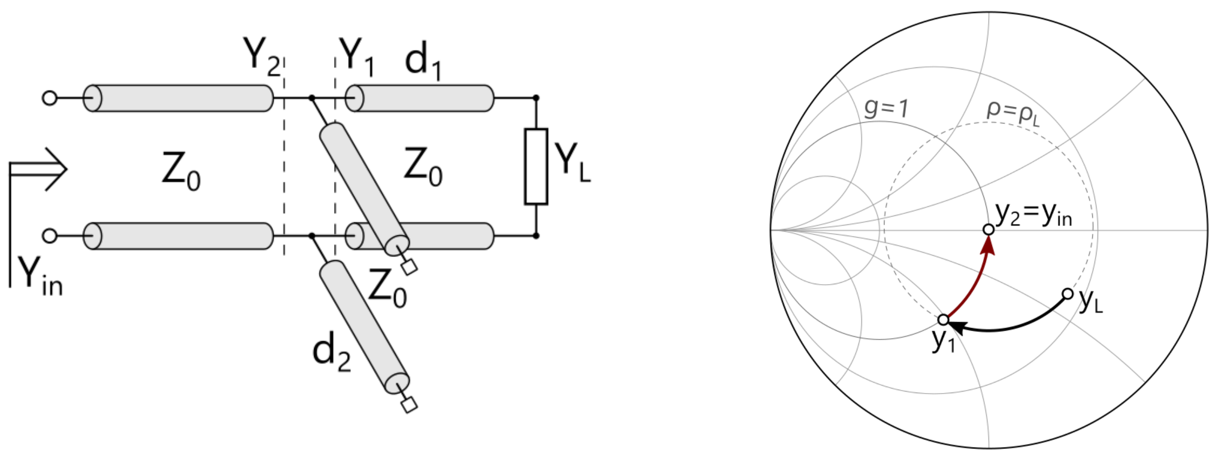


Fig. 2.8. Transmission line matching network example in Smith Chart.

To perform the exercise, open the schematic **Lab_2_Ex_3.sch** and follow the steps:

1. Connect the generator with the load and run the simulation. Observe the input impedance/admittance. You have two Smith charts to assist you, but focus on the admittance chart.
2. Insert a 50-Ω transmission line in series (*Components* -> *Transmission lines* -> *Transmission Line*) and connect the circuit.
3. Manually adjust the line length d_1 and observe the effect. Change the length d_1 until the input admittance is moved to the circle *g*=1. Note the value in the table below (Option #1).
4. Insert an open-ended 50-Ω stub (transmission line) in parallel and connect the circuit.
5. Manually adjust the stub length d_2 and observe the effect. Change the length d_2 until the input admittance is translated to the Smith chart center the network is matched to 50 Ω). Note the value in the table below.
6. Insert a 50-Ω transmission line in series, between the generator and the stub, and connect the circuit.
7. Manually change the line length and observe the effect. Explain the result.

	Option #1	Option #2
d_1		
d_2		

It is obvious that another intersection point exists between circles *g*=1 and $\rho=\rho_L$. This intersection point can also be used in the matching procedure. Repeat the design steps (3) – (5) and note the values in the table under Option #2.

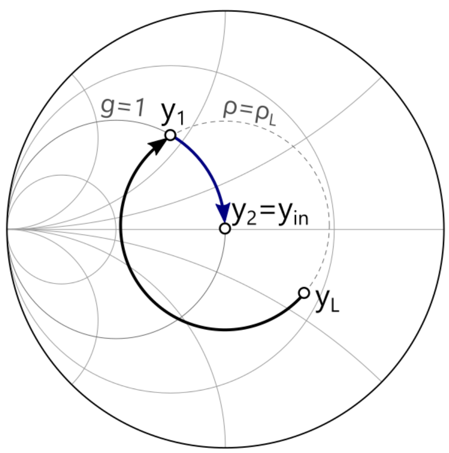


Fig. 2.9. Transmission line matching network example, second intersection point.

3.4. *Quarter-Wavelength Transformer*

In Section 2.2.3, it was shown how a transmission line, having a length of $\lambda_g/4$ and a characteristic impedance Z_t, can be used to match purely real load impedances. This exercise will show how arbitrary impedances can be matched using a section of a transmission line followed by a quarter-wavelength transformer.

Impedance z_1 can be obtained in the Smith chart by starting from the load impedance z_L and going along a constant-ρ circle (along a line). If the length d_1 is properly selected, the impedance z_1 can be made purely real, in the intersection between the constant-ρ circle and the horizontal (real) axis. Then, a quarter-wave transformer can be used to match this purely real impedance.

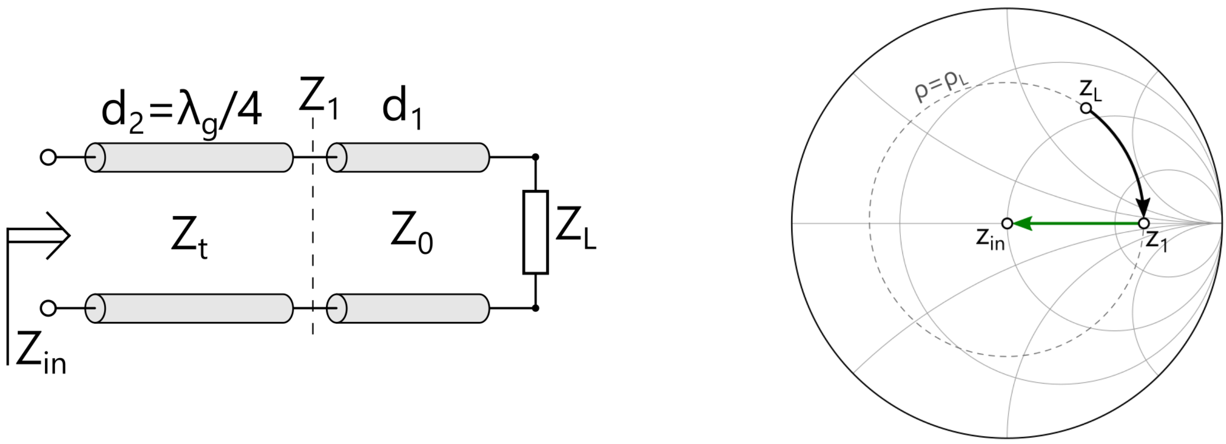


Fig. 2.10. Quarter-wavelength transformer example in Smith Chart.

To perform the exercise, open the schematic **Lab_2_Ex_4.sch** and follow the steps:

1. Connect the generator with the load and run the simulation. Observe the input impedance.
2. Insert a 50-Ω transmission line in series (*Components* -> *Transmission lines* -> *Transmission Line*) and connect the circuit.
3. Manually adjust the line length d_1 until the input impedance is moved to the intersection with the real (horizontal) axis. Note the value in the table below (Option #1).
4. Use a marker to determine the input impedance Z_1. Note the value in the table.
5. Calculate the required characteristic impedance Z_t to match the impedance Z_1 to 50 Ω.
6. Insert a quarter-wavelength-long transmission line in series, with the characteristic impedance equal to Z_t. Run the simulation and observe the result.

	Option #1	Option #2
d_1		
d_2		
Z_1		
Z_2		

It is obvious that another intersection point exists between the circle $\rho=\rho_L$ and the real axis. This intersection point can also be used in the matching procedure. Repeat the design steps (3) – (6) and note the values in the table under Option #2.

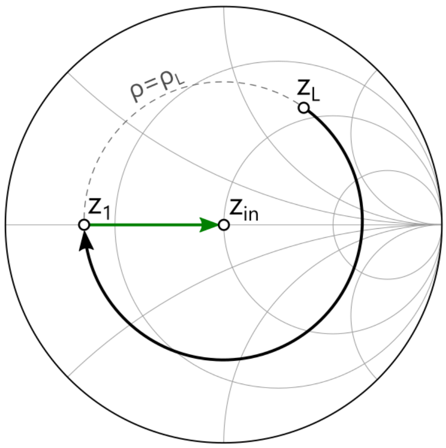


Fig. 2.11. Quarter-wavelength transformer example, second intersection point.

4. Antenna Matching Hands-on

In this part of the lab we are going to analyse the difference between matching an antenna by adjusting its own parameters and adding a matching network. For this purpose, we use an aperture coupled microstrip antenna that you will find explained in the theoretical background of this document.

4.1. *Antenna matching using discrete elements*

Open the file **Lab_2_Ex_5.sch** and follow the steps:

1. Connect the generator with the box. This box contains a *.s1p* file with the S11 parameters of an unmatched aperture coupled microstrip antenna. Observe the input impedance in the Smith Chart.
2. Design a matching network using discrete elements to make the antenna operate at 5.3 GHz and

measure the bandwidth.

3. Now the effect of losses will be studied. Create a new file **Lab_2_Ex_5_Losses.sch** and copy the previous circuit**.** For adding losses to the matching network, you need to add a series resistance each one of your lumped components (inductances and/or capacitances). Change the value of this resistances between 0 and 2 Ohms. How does the addition of this resistances change the S11 parameter?

4.2. *Antenna matching changing antenna parameters*

Open the file HFSS simulation file and combine the next values of the slot length (*slot_l*) and the foam thickness (*foam_h*) to analyze the behavior of the antenna.

- Foam thickness: 1 mm, 2mm and 3 mm
- Slot length: 10 mm, 13mm, 15mm

Explain how the behavior of the antenna change with the different parameters and how the coupling of the antenna changes. Are you able to get that the antenna works at 5.3 GHz in one of the combinations? If the answer is yes, what is the difference of this configuration with the case when you use a matching network?

4.3. *VNA measurements*

In this part of the lab you need to measure the different combinations that you have simulated in HFSS with the VNA. The material that you need to use is:

- Pieces of foam with thickness of: 1 mm, 2mm and 3 mm
- Feeding networks with slot lengths of: 10 mm, 13mm, 15mm
- Patch size: 21 mm

Note: Remember to calibrate the VNA before measuring the antennas.

Paying attention to the obtained results in this part of the lab, which matching technique would you consider that is better: the technique of the matching network using discrete elements or the one changing the parameters of the antenna? Why?

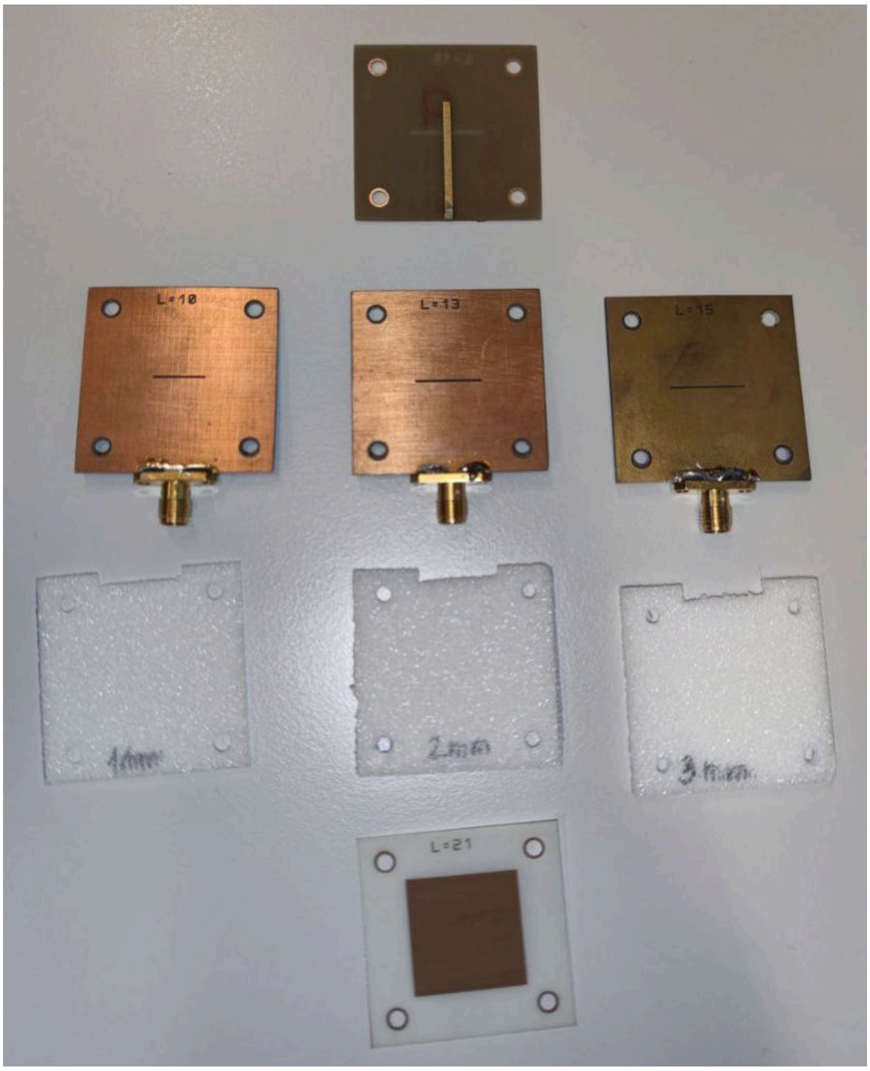


Fig. 2.12. Material for the hands-on of the matching network lab.