

Follow up on battery (equivalent circuit) models

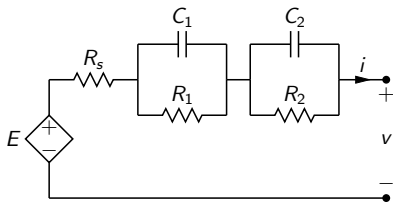
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Slides compiled on: November 6, 2024

Content of these slides

- Recap of last class main messages
- Solutions to last week exercises 5-7
- Some additional elements on equivalent circuit models (ECMs)

ECM of battery cell: two-time-constant model



Voltage dynamics: physical interpretation

Self-assessment of understanding of last week - Question 5

Prove by way of equations that a capacitor, in steady-state conditions, can be replaced by an open circuit. (not seen in this class; recall elements from linear circuit theory courses).

Two ways of doing this:

Impedance Z of a capacitor is

$$Z = \frac{1}{j\omega C}.$$

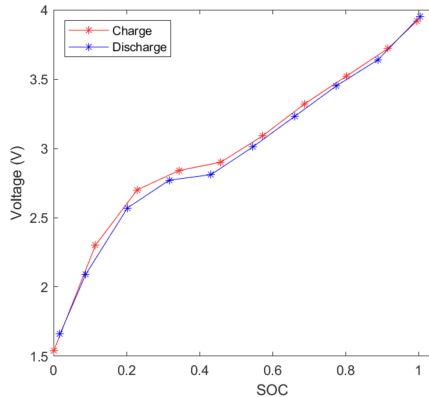
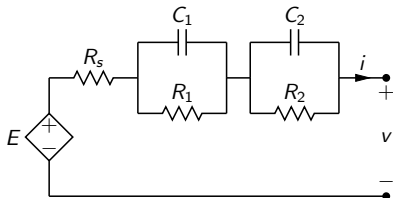
Steady-state conditions occurs when $\omega \rightarrow 0$ (ie, variations are extremely slow); this results in an impedance of $Z \rightarrow \infty$, which is an open-circuit.

The voltage-to-current (v_c -to- i_c) relation in a capacitor is:

$$i_c = C \frac{dv_c}{dt}$$

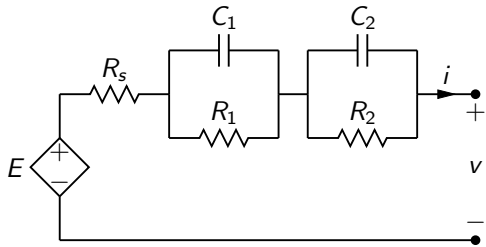
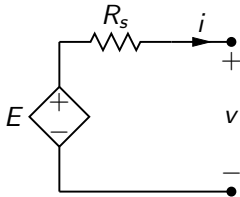
Steady state occurs, by definition, when time derivative terms are 0, thus $i_c = 0$, open circuit.

Cell internal voltage vs. state of charge (SOC)



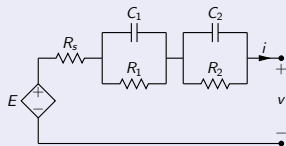
Self-assessment of understanding of last week - Question 6

In the internal-resistance model, the cell internal voltage E can be estimated easily by measuring the battery terminal voltage v in open-circuit conditions (so called OCV): why? Can you say the same about the TTC model? With the TTC model, which conditions do you need to make sure that v reflects E ?



Self-assessment of understanding of last week - Question 7

Prove that the TTC model



has the following state-space formulation:

$$\dot{x} = \mathcal{A}x + \mathcal{B}u$$

$$v = \mathcal{C}x + \mathcal{D}u$$

where state vector, input, and state-space matrices are as follows

$$x = [v_{C_1} \quad v_{C_2}]^T, u = [i \quad 1]^T$$

$$\mathcal{A} = \begin{bmatrix} -1/(R_1 C_1) & 0 \\ 0 & -1/(R_2 C_2) \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1/C_1 & 0 \\ 1/C_2 & 0 \end{bmatrix},$$

$$\mathcal{C} = [-1 \quad -1], \mathcal{D} = [-R_s \quad E].$$

Let's take the first RC branch. The current into the capacity is i_{C_1} , the voltage across the capacitor is v_{C_1} , the voltage on the resistance is also v_{C_1} (KVL), and the current into the resistance is i_{R_1} . The current in the capacity is by $i_{C_1} = i - i_{R_1}$ (KCL), $i_{C_1} = i - \frac{v_{C_1}}{R}$. Current-to-voltage in C_1 is:

$$\frac{dv_{C_1}}{dt} = \frac{1}{C_1} i_{C_1} = -\frac{1}{C_1 R_1} v_{C_1} + \frac{1}{C_1} i.$$

For the second branch, it is the same but with subscript 2 instead of 1. These two equations can be used to write the state dynamics of the state space.

Writing KVL to the whole circuit yields:

$$v = -v_{C_1} + -v_{C_2} - R_s i + E,$$

that one can use to derive the output equation of the state space.

All this fuss about battery voltage... why do we even bother?

Power (what we are interested in extracting from the battery) is voltage times current.

Battery limits are in voltage and current, not in power. Voltage models give us a way to map power into voltages and current and vice-versa.



Display of an electric car showing an estimation of the maximum (charging and discharging) power available.

If voltage is so important, why omitting it in the scheduling problem?

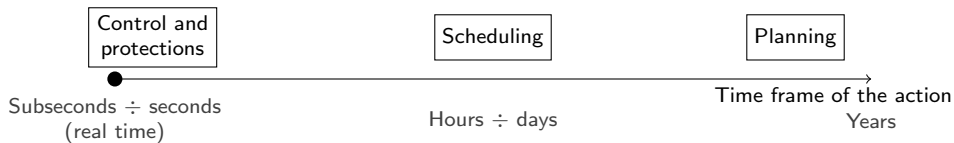
Great question!

Week 3's scheduler formulation

$$\begin{aligned} & \underset{B_0, B_1, \dots, B_{T-1}}{\text{minimize}} && \left\{ \Delta \sum_{t=0}^{T-1} p(t) \cdot (B(t) + D(t)) \right\} \\ & \text{subject to} && -\bar{P} \leq B(t) \leq \bar{P}, && t = 0, \dots, T-1 \\ & && 0 \leq SOE(0) + \Delta \sum_{\tau=0}^t B(\tau) \leq \bar{E}, && t = 0, \dots, T-1 \end{aligned}$$

Tacit assumption: battery terminal voltage is constant $\rightarrow$ we can omit voltage constraints.

Approximation deemed acceptable in scheduling, but not in real-time control.



Final notes on ECMs

- ECMs are typically estimated from measurements: one performs an experiment and then uses some numerical methods (eg, least squares estimation, LSE) to find the value of the model parameters that minimizes the estimation error.
- Why 2 states and not 3? Or 100? From grey-box modeling and system identification theory, the appropriate number of states is the lowest one that results in a time series of the model estimation error that is randomly and independently distributed (i.e., white noise).
- The value of the parameters of the TTC model depends on many variables: cell temperature, current itself, and aging.
- Compared to a physical model, the value of the parameters of the TTC model might not be (most of the time, it is not) directly linkable to a specific physical quantity or phenomenon, and that's a drawback.

Check the exercises on the slides uploaded on Moodle