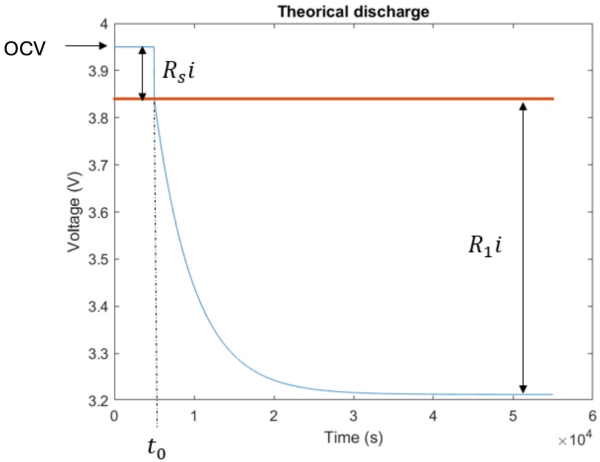
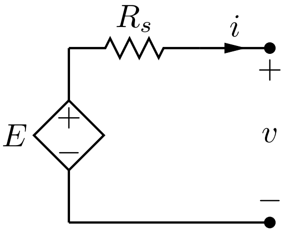
Voltage dynamics as a function of the charging/discharging current



Voltage response of a battery cell to a stepwise variation of the discharging current.

Equivalent Circuit Models (ECMs)

ECM of battery cell: internal resistance model



Symbols:

- i: cell current, supplied to an external load if positive, consumed (provided by a power supply) if negative. Measurable.
- E: cell internal voltage. It is a modelling abstraction and is not measurable (it can be estimated, though).
- v: cell terminal voltage. This is the voltage measurable at the cell terminals.
- R_s: cell resistance

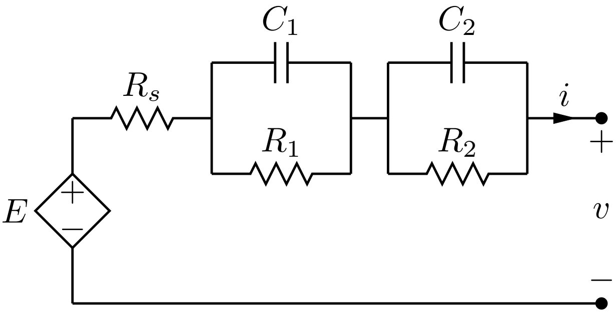
The cell internal voltage E depends on the battery SOC (as shown later), and that's why in the schematic is denoted as a variable voltage source.

A recall from electric circuit theory: the open-circuit voltage (OCV) denotes the value of the voltage v when $i = 0$.

ECM of battery cell: two-time-constant model

To capture voltage dynamics, one should add "dynamic" elements to the circuit above. In linear circuit theory, these are inductors and capacitors.

Two RC branches added in series to the series resistance are able to capture reasonably well voltage dynamics. This is known as"TTC model" (two-time-constant model).



Two-time-constant model: state-space model

The governing equations of the TTC model, in state-space form, are:

$$\dot{x} = \mathcal{A}x + \mathcal{B}u$$
$$v = \mathcal{C}x + \mathcal{D}u$$

where the state and input vectors are

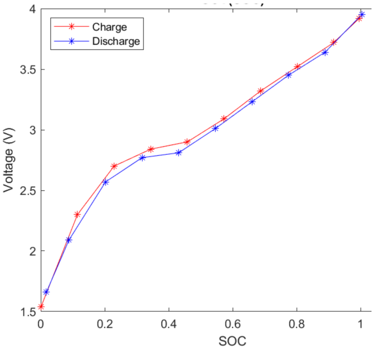
$$x = \begin{bmatrix} v_{C_1} & v_{C_2} \end{bmatrix}, u = \begin{bmatrix} i & 1 \end{bmatrix}^T$$

and the state-space matrices A, B, C, and D are:

$$\mathcal{A} = \begin{bmatrix} -1/(R_1 C_1) & 0 \\ 0 & -1/(R_2 C_2) \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1/C_1 & 0 \\ 1/C_2 & 0 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} R_s & E \end{bmatrix}.$$

Cell internal voltage vs. state of charge (SOC)

The cell internal voltage E varies with the cell state-of-charge, reflecting the different concentration of Li-ions in the electrodes following the charge and discharge of the battery.



The plot on the left shows the voltage-to-SOC dependency identified from measurements.

The two (red and blue) curves (one referring to the charging process, the other to the discharging process) are to verify voltage hysteresis phenomena (which does not appear because the two are very similar, ie their difference is a very small fraction of the raw measurement and has comparable order of magnitude to the voltage sensor accuracy).

In the laboratory next week, your task will be to identify this relation in a battery cell by way of measurements.

Self-assessment of understanding and exercises

1. Explain the difference between the SOC models seen in weeks 1 and 8 in terms of meaning, input, and parameters.
2. Losses (present in week 1's SOC model) are not present any longer in week 8's SOC model ($\eta \approx 1$). Explain where you expect them to appear again.
3. Explain the qualitative difference between a physical model of the cell voltage and an equivalent circuit model.
4. Explain the limitations of the internal-resistance model of the battery (slide 12) when applied to model the behaviour of the cell voltage of slide 10.
5. Demonstrate formally (ie, by way of equations) that a capacitor, in steady-state conditions, can be replaced by an open circuit. (not seen in this class; recall elements from linear circuit theory courses).
6. In the internal-resistance model, the cell internal voltage E can be estimated easily by measuring the battery terminal voltage v in open-circuit conditions (so called OCV): why? Can you say the same about the TTC model? With the TTC model, which conditions do you need to make sure that v reflects E?
7. (Extra:) Prove that the state-space formulation of the TTC model shown in slide 13 is as given in slide 14.