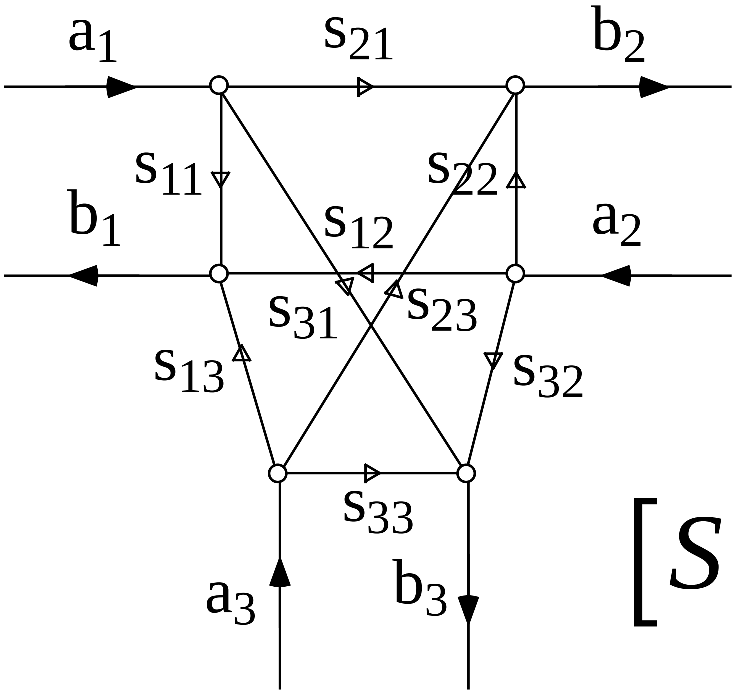
Three port components

Main application : power division and combination

Three port components



$$[S]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

Reciprocal

$$[S] = \begin{bmatrix} s_{11} & s_{21} & s_{31} \\ s_{21} & s_{22} & s_{32} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

Matched

$$[S]=\begin{bmatrix} 0 & s_{12} & s_{13} \\ s_{21} & 0 & s_{23} \\ s_{31} & s_{32} & 0 \end{bmatrix}$$

Lossless

$$\sum_{i=1}^{3} s_{ij}^{*} s_{ik} = \delta_{ik} \qquad j,k = 1,2,3$$

Lossless, matched and reciprocal

$$[S]=\begin{bmatrix} 0 & s_{21} & s_{31} \\ s_{21} & 0 & s_{32} \\ s_{31} & s_{32} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & s_{21}^{*} & s_{31}^{*} \\ s_{21}^{*} & 0 & s_{32}^{*} \\ s_{31}^{*} & s_{32}^{*} & 0 \end{bmatrix}\begin{bmatrix} 0 & s_{21} & s_{31} \\ s_{21} & 0 & s_{32} \\ s_{31} & s_{32} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Lossless, matched and reciprocal

Thus

$$|s_{12}|^2 + |s_{13}|^2 = 1$$

$$|s_{12}|^2 + |s_{23}|^2 = 1$$

$$|s_{13}|^2 + |s_{23}|^2 = 1$$

$$s_{13}^* s_{23} = 0$$

$$s_{12}^* s_{23} = 0$$

$$s_{12}^* s_{13} = 0$$

suppose :

$$s_{13} \neq 0 \;\Rightarrow\; s_{23} = 0 \;\; and \;\; s_{12} = 0$$

incompatible with

$$|s_{12}|^2 + |s_{23}|^2 = 1$$

A three port cannot be lossless, matched and reciprocal at the same time

Lossless, matched at 2 ports and reciprocal

$$[S]=\begin{bmatrix} 0 & s_{21} & s_{31} \\ s_{21} & 0 & s_{32} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

$$\begin{bmatrix} 0 & s_{21}^{*} & s_{31}^{*} \\ s_{21}^{*} & 0 & s_{32}^{*} \\ s_{31}^{*} & s_{32}^{*} & s_{33}^{*} \end{bmatrix}\begin{bmatrix} 0 & s_{21} & s_{31} \\ s_{21} & 0 & s_{32} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Lossless, matched at 2 ports and reciprocal

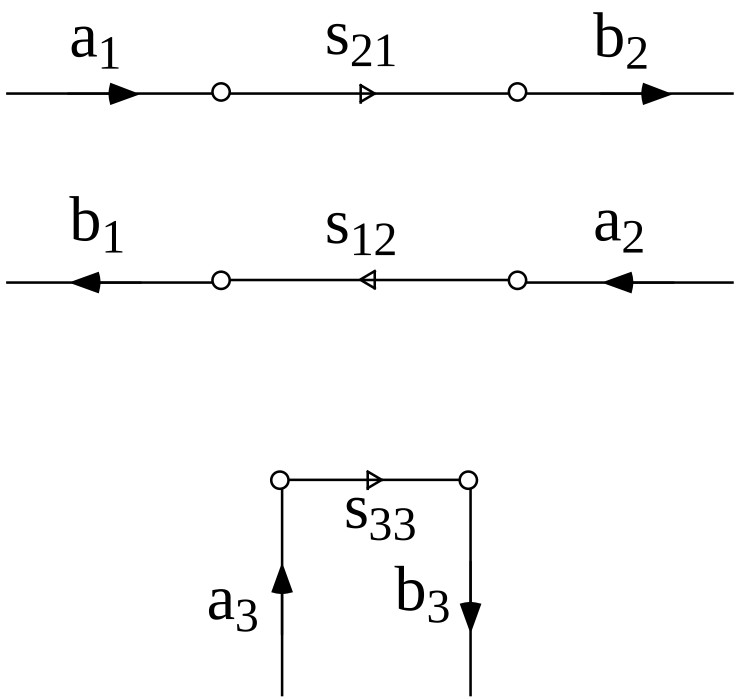
Thus

$$|s_{12}|^2 + |s_{13}|^2 = 1$$
$$|s_{12}|^2 + |s_{23}|^2 = 1$$
$$|s_{13}|^2 + |s_{23}|^2 + |s_{33}|^2 = 1$$
$$s_{13}^* s_{23} = 0$$
$$s_{12}^* s_{23} + s_{13}^* s_{33} = 0$$
$$s_{12}^* s_{13} + s_{23}^* s_{33} = 0$$

yields :

$$s_{13} = 0 \Rightarrow s_{12} = 1 \; and \; s_{23} = 0, \text{ thus } s_{33} = 1$$
$$s_{23} = 0 \Rightarrow s_{12} = 1 \; and \; s_{13} = 0, \text{ thus } s_{33} = 1$$

Lossless, reciporcal and matched at two ports



two decoupled components

Three port "nearly" matched at two ports

scattering matrix

$$[S]=\begin{bmatrix} \varepsilon & s_{21} & s_{31} \\ s_{21} & \varepsilon & s_{32} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \qquad \varepsilon \ll 1$$

Ref. planes at ports 1 and 2 selected to have ε real

$$\varepsilon^2 + |s_{12}|^2 + |s_{13}|^2 = 1$$

$$\Rightarrow \quad |s_{13}| = |s_{23}|$$

$$\varepsilon^2 + |s_{12}|^2 + |s_{23}|^2 = 1$$

Three port "nearly" matched at two ports

$$\varepsilon\left(s_{12}+s_{12}^{*}\right)+s_{13}s_{23}^{*}=0$$

and

$$\varepsilon s_{13}+s_{12}^{*}s_{23}+s_{13}^{*}s_{33}=0$$

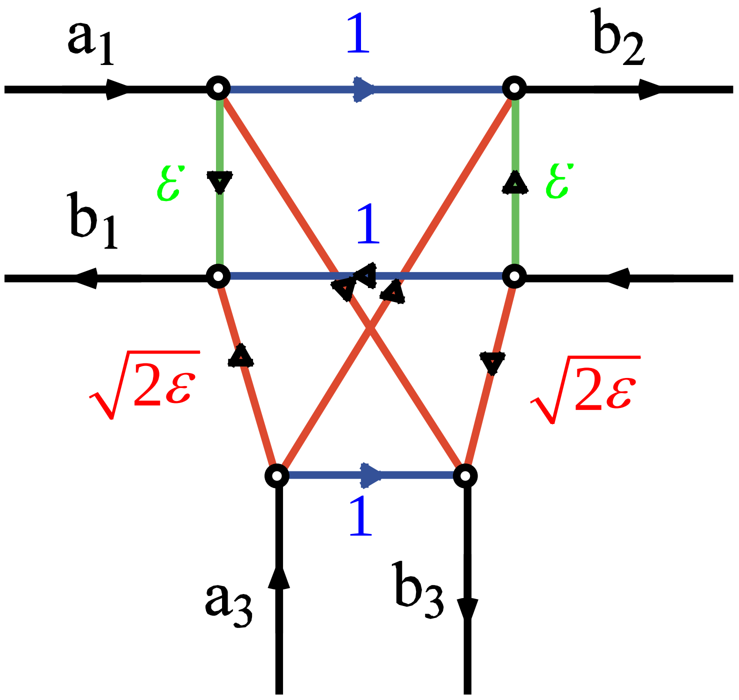
$$\left|s_{13}\right|=\sqrt{2\varepsilon\,\mathrm{Re}\left(s_{12}\right)}\quad\sim\sqrt{\varepsilon}$$

Thus

$$\left|s_{12}\right|=\sqrt{1-2\varepsilon\,\mathrm{Re}\left(s_{12}\right)-\varepsilon^{2}}$$

$$\cong 1-\varepsilon\,\mathrm{Re}\left(s_{12}\right)\approx 1$$

Three port "nearly" matched at two ports



Matched and Lossless three port circuits

$$|s_{21}|^2 + |s_{31}|^2 = 1$$

$$|s_{12}|^2 + |s_{32}|^2 = 1$$

$$|s_{23}|^2 + |s_{13}|^2 = 1$$

$$s_{12}^* s_{13} = 0$$

$$s_{21}^* s_{23} = 0$$

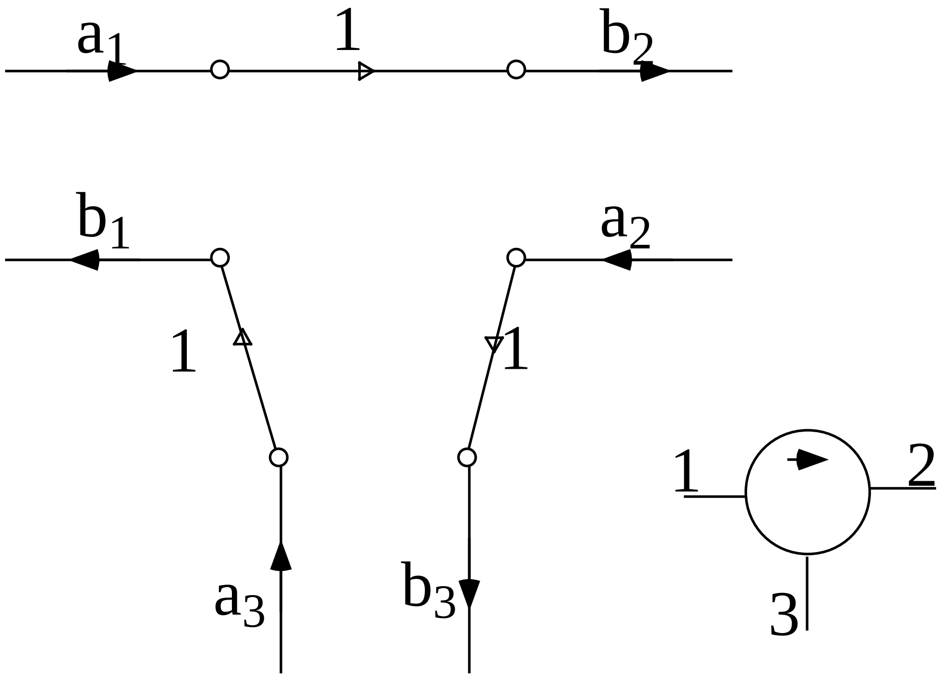
$$s_{31}^* s_{32} = 0$$

$$s_{13} \neq 0 \Rightarrow s_{12} = 0 \Rightarrow |s_{32}| = 1 \Rightarrow s_{31} = 0$$

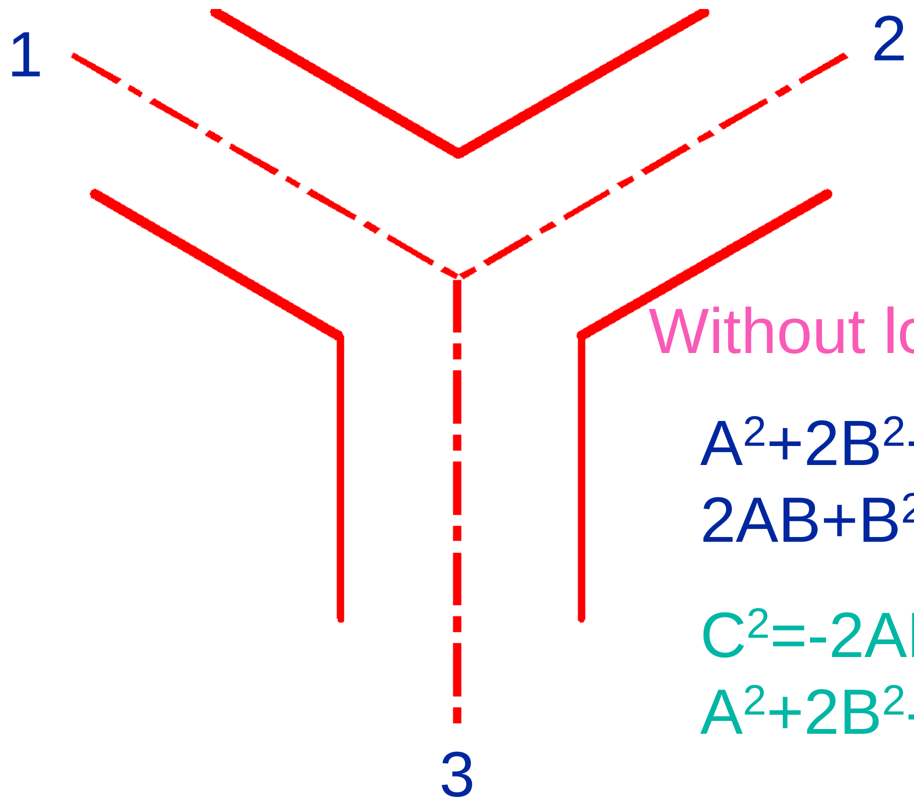
$$\Rightarrow |s_{21}| = 1 \Rightarrow s_{23} = 0 \Rightarrow |s_{13}| = 1$$

Circulator

$$[S]=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$



Y Junction



$s_{11}=s_{22}=s_{33}=A$

$s_{12}=s_{21}=s_{13}=s_{31}=s_{32}=s_{23}=$
$B+jC$

Without losses :

$A^2+2B^2+2C^2=1$ (ellipsoid)

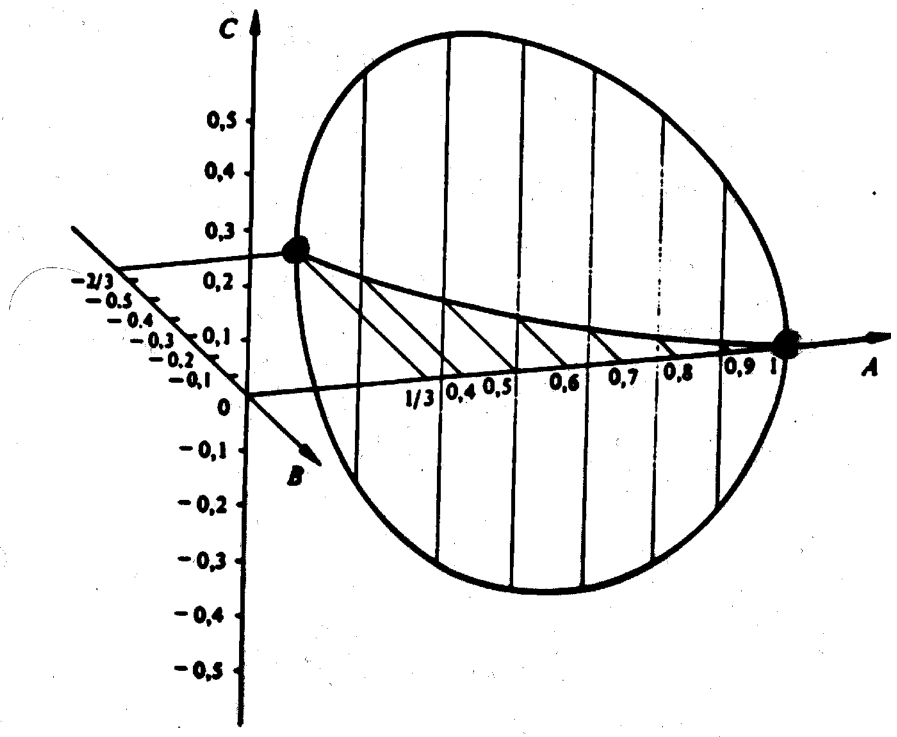
$2AB+B^2+C^2=0$

$C^2=-2AB-B^2 \quad => \quad AB<0$

$A^2+2B^2-4AB-2B^2=1$

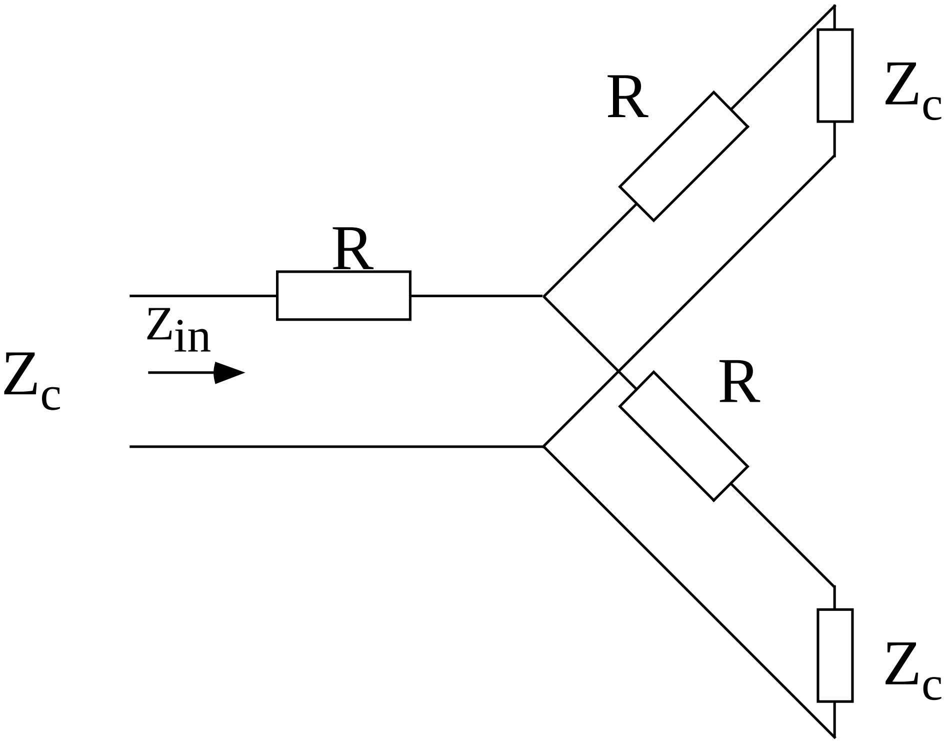
$B=1/4(A-1/A) \quad A=2B\pm(4B^2+1)^{1/2}$

Y-junction



Ellipsoid

Lossy three ports : resistive divider



Match

$$Z_{in} = Z_c$$

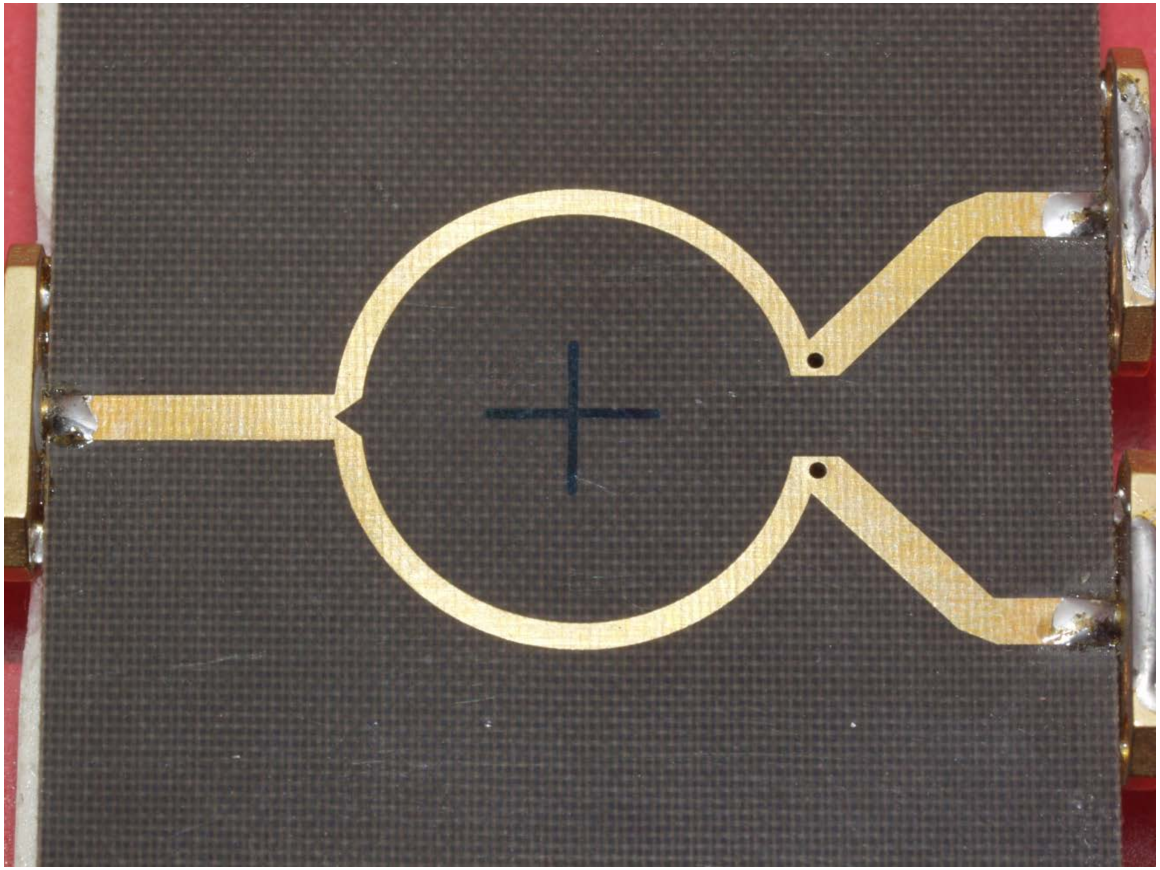
$$Z_{in} = \frac{3}{2}R + \frac{Z_c}{2}$$

$$R = \frac{1}{3}Z_c$$

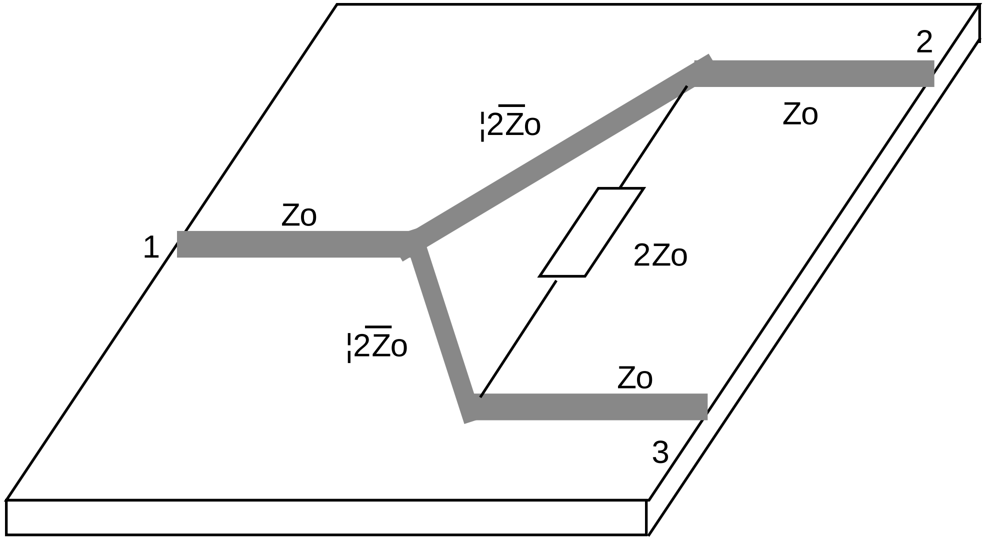
$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Wilkinson divider

Example in microstrip line

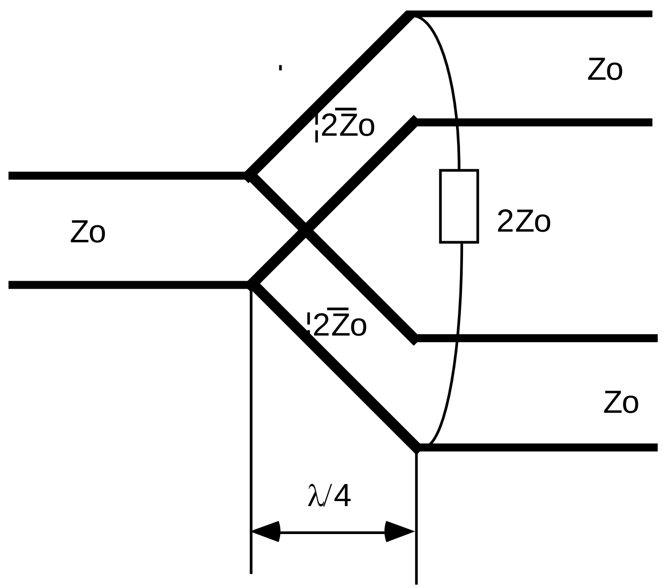


Wilkinson

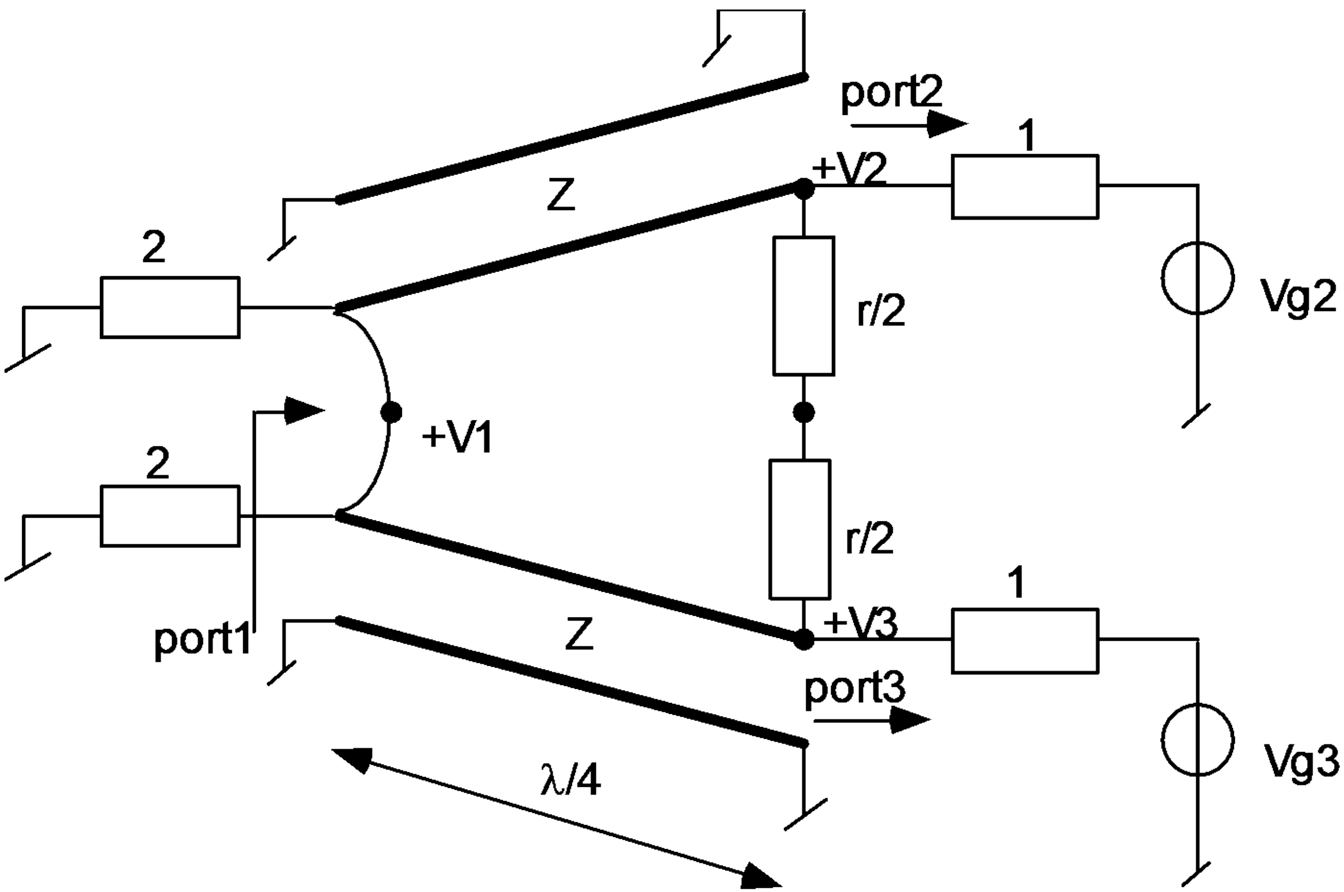


We want :
3 ports matched
port 2 and 3 isolated

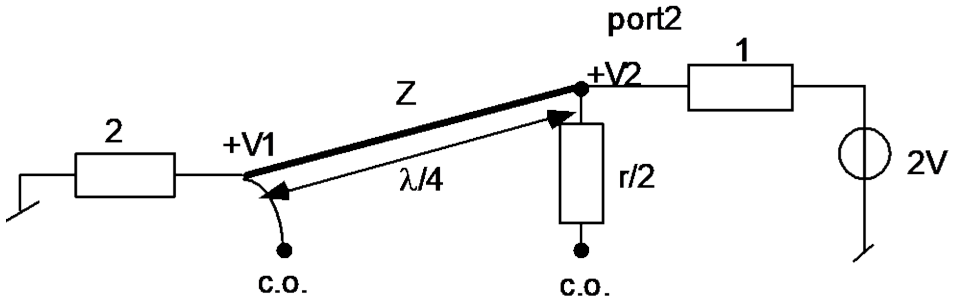
Equivalent network



Symmetry



Symmetric excitation



Looking into port 2 2:

$$Z_{in}^{e} = \frac{Z^2}{2}$$

We obtain :

$$Z = \sqrt{2}$$

Symmetric excitation

Computation of s21 :

$$V(x) = V^+\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$$

$$V(0) = V^+(1+\Gamma) = V_2 = V$$

$$V_1 = V(\lambda/4) = jV^+(1-\Gamma) = jV\frac{\Gamma-1}{\Gamma+1} \rightarrow \boxed{V_1 = jV\frac{-1}{\sqrt{2}}}$$

where Γ is the reflexion seen at port 1 looking towards resitor 2

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

Symmetric excitation

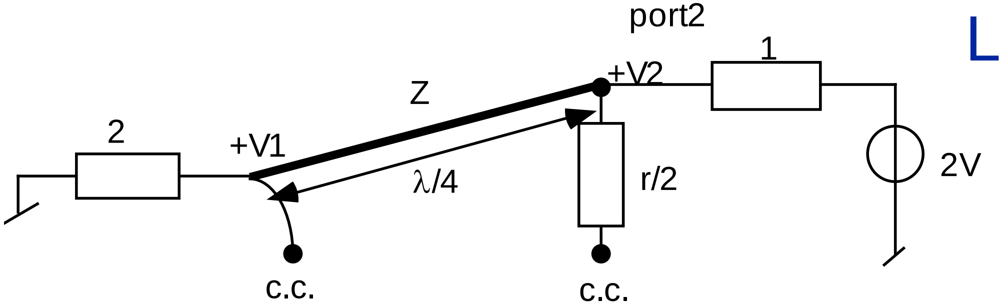
Thus

$$S_{12} = S_{21} = \frac{V_1}{V_2} = \frac{-j}{\sqrt{2}} = -j0.707$$

By symmetry

$$S_{13} = S_{31} = \frac{V_1}{V_2} = \frac{-j}{\sqrt{2}} = -j0.707$$

Asymmetric excitation



Looking into port 2:

$$Z_{in}^{e} = \frac{r}{2}$$

We get

$$r = 2$$

In summary

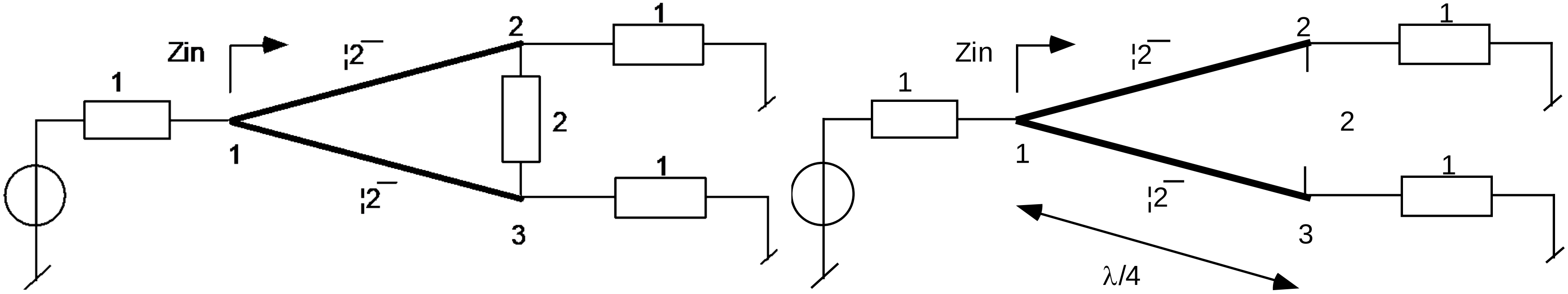
$s_{33} = s_{22} = 0$ (ports 2 and 3 are matched for both excitations)

$s_{12} = s_{21} = -j0.707$ (The component is reciprocal)

$s_{13} = s_{31} = -j0.707$ (The component is reciprocal)

$s_{23} = s_{32} = 0$ (Because of the short circuit or open circuit on the symmetry axis)

Match at port 1



$$Z_{in} = \frac{1}{2}\frac{\left(\sqrt{2}\right)^2}{1} = 1$$

Thus $\boxed{S_{11} = 0}$