Auxiliary functions in OFDM

1) Synchronization
2) Freq. offset estimation] Preamble based
3) PN an SR offset tracking] Pilot based

~~A typical OFDM frame:~~

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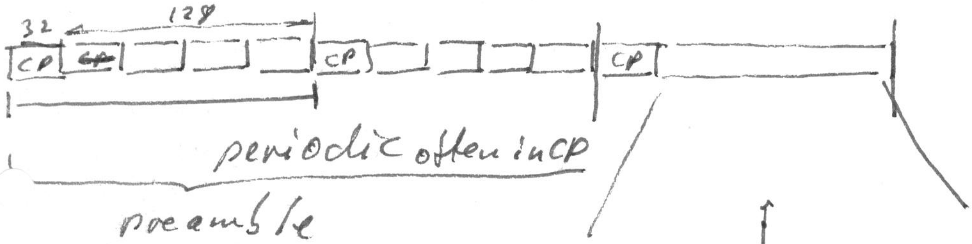
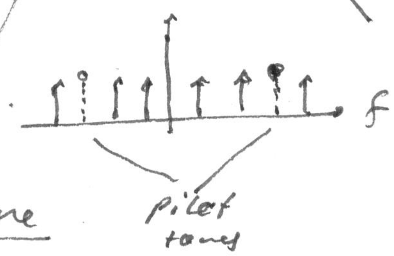




How to construct
- N/K size IFFT
- N-point IFFT using every Kth tone

Synchronization:

Idea 1: Matched filter

$$y[t] = p[t-\Delta T] \qquad Q: \text{preamble length}$$

$$\Delta\hat{T} = \arg\max_{\tau} \left| \sum_{t=0}^{Q-1} y[t+\tau]\, p^*[t] \right|^2$$

sampling time

add freq. offset: $y[t] = p[t-\Delta T]\, e^{j2\pi\Delta f t \cdot T_s}$

$$\Delta\hat{T} = \arg\max_{\tau} \left| \sum_{t=0}^{Q-1} p[t-\Delta T+\tau]\, p^*[t]\, e^{j2\pi\Delta f t T_s} \right|^2$$

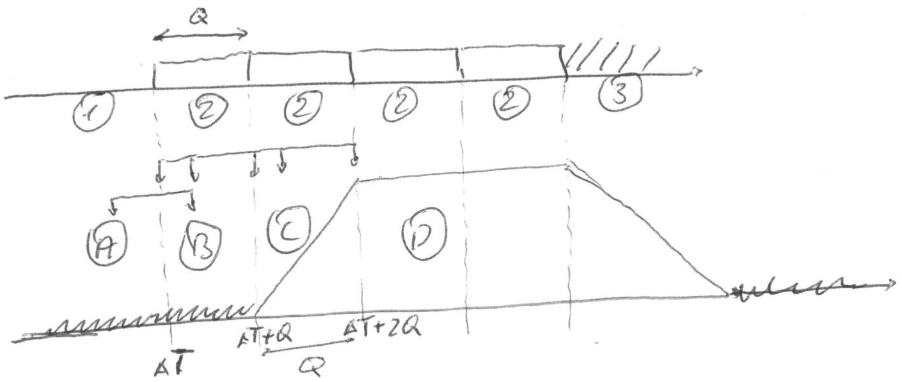
Example: $2\pi\Delta f T_s \cdot Q = 2\pi \rightarrow \Delta f = \frac{1}{QT_s} \Rightarrow \sum \ldots \Big|_{\tau=\Delta T} = 0$!

$\Rightarrow$ Sync fails due to freq. offset
MF is not matched to true receive preamble

Idea 2: Use received signal as MF
(detect similarities)

2017

$$\phi[t] = \left| \sum_{\tau=0}^{N/K-1} y[t-\tau]\, y^*[t-N/K-\tau] \right|^2 \qquad Q = N/K$$



1 $y[t] = n[t]$

$p[t] = 0$ for $t < 0$

2 $y[t] = e^{j2\pi\Delta f t T_s}\, p[(t-\Delta T)_{mod\, Q}] + n[t]$

3 $y[t] = x[t-\Delta T - C_p]$

$\phi[t]$:

A $t < \Delta T$ $\quad \sum_{\tau=0}^{Q-1} n[t-\tau]\, n^*[t-Q-\tau]$

B $\Delta T \leq t < \Delta T + Q$

$$\sum_{\tau=0}^{t-\Delta T} p[t-\Delta T-\tau]\, n^*[t-Q+\tau]\, e^{j2\pi\Delta f T_s t}$$

$$+ \sum_{\tau=t-\Delta T+1}^{Q-1} n[t-\tau]\, n^*[t-Q-\tau]$$

C $\Delta T + Q \leq t < \Delta T + 2Q$

$$\sum_{\tau=0}^{t-Q-\Delta T} p[t-\Delta T-\tau]\, p^*[t-\Delta T-\tau]\, e^{j2\pi\Delta f T_s t}\, e^{-j2\pi\Delta f T_s (t-Q)}$$

$$+ \sum_{\tau=t-Q-\Delta T+1}^{Q-1} p[t-\Delta T-\tau]\, e^{j2\pi\Delta f T_s t}\, n^*[t-Q-\tau]$$

D $t \geq \Delta T + 2Q$

$$\sum_{\tau=0}^{Q-1} p[t-\Delta T-\tau]\, p^*[t-\Delta T-\tau]\, e^{j2\pi\Delta f T_s t}\, e^{-j2\pi\Delta f T_s (t-Q)}$$

Sync: Detect transition between C and D

→ Align sync with AKF flattening

Schmidl-Cox approx sync:

$$m[t] = \sum_{\tau=0}^{Q-1} y[t-\tau]\, y^*[t-Q-\tau] \Big/ \sum_{\tau=0}^{Q-1} |y[t-\tau]|^2$$

Why approximate estimation?

→ Concurrently measure freq. offset

→ Compensate freq. offset

⇒ exact sync with MF

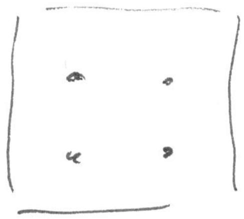
Freq. offset estimation during region D

$$\phi[t] = \sum_{\tau=0}^{Q-1} |p[t-\Delta T-\tau]|^2 e^{j2\pi\Delta f T_s Q}$$

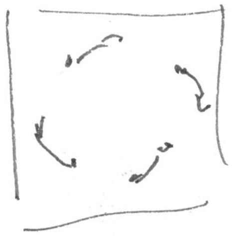
$$\rightarrow \arg(\phi[t]) = 2\pi \Delta f \cdot T_s \cdot Q \rightarrow \Delta f = \frac{\arg(\phi[t])}{2\pi T_s Q}$$

plotting:

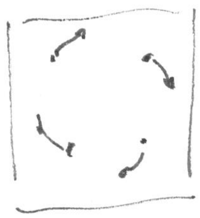
1) Tones of 1 OFDM symbol



2) ~~Tones~~ same tone across symbols with freq. offset



3) Tones in one symbol with time shift



Tracking:

Phase noise: $$y[t] \approx e^{j\phi(t)} \sum_K S_K[t-K(N+N_g)]$$

slow

$$\approx \sum_K e^{j\phi(K(N+N_g))} S_K[t-K[N+N_g]]$$

Freq. domain: $$DFT\{e^{j\phi(K(N+N_g))} S_K\} = e^{j\phi(K(N+N_g))} S_K[f]$$

Consider pilot tones: $P[f_P]$

$$S_K[f_P] = P[f_P]\, H[f_P]$$

$$\Rightarrow \phi(K(N+N_g)) = \arg\{ Y_K[f_P] \cdot P^*[f_P]\, H^*[f_P] \}$$

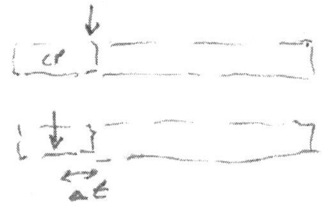
from ch. est.

Sampling offset

- Different sampling clocks at Tx/Rx
- Sampling instant drifts

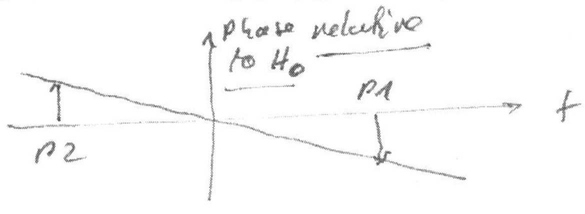
At time zero:

At time $t>0$



$$DFT\{y_K[t]\} = DFT\{S_K[t-\Delta t]\} = S_K[f] \cdot H[f]\, e^{-j\frac{2\pi}{N}\Delta t\, f}$$

$\rightarrow$ Rotation across tones



$$-\frac{2\pi}{N}\Delta t\, f_P = \arg\{ Y_K[f_P]\, P^*[f_P]\, H^*[f_P] \} \quad \Big| \cdot \frac{d}{df_P}$$