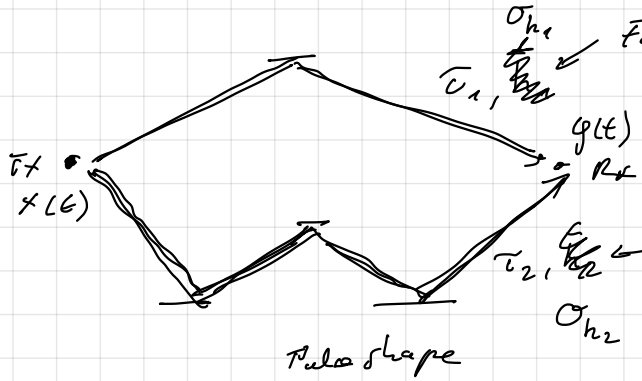
23/11/2020

Multipath Channel Reminder



$\tau_\ell - \tau_p \gtrsim T \quad \forall \ell \neq p$

with T = symbol duration

$$g(t) = \sum_{\ell=1}^{N} x(t-\tau_\ell)\, h_\ell$$

$h_{eff} = h * g_t * g_r^*$ MF

or $g(t) = h(t) * x(t)$

$$h(t) = \sum h_\ell\, \delta(t-\tau_\ell)$$

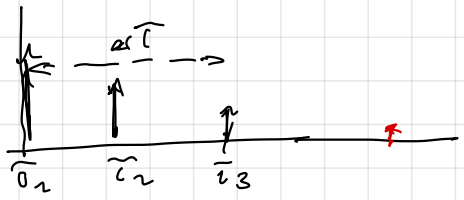
$h(t) \Rightarrow h[k]$ channel impulse response

Impact of Multipath channel

sampled @ nT, $\tau_0 = 0$ ← perfectly sync. to the first multipath comp.

$$y(nT) = h(0)\,x[n] + \underbrace{\sum x[n-k]\, h_{eff}[kT]}_{ISI}$$

Power-Delay-Profile: $\bar{h}_\ell = \sigma_{h_\ell}$

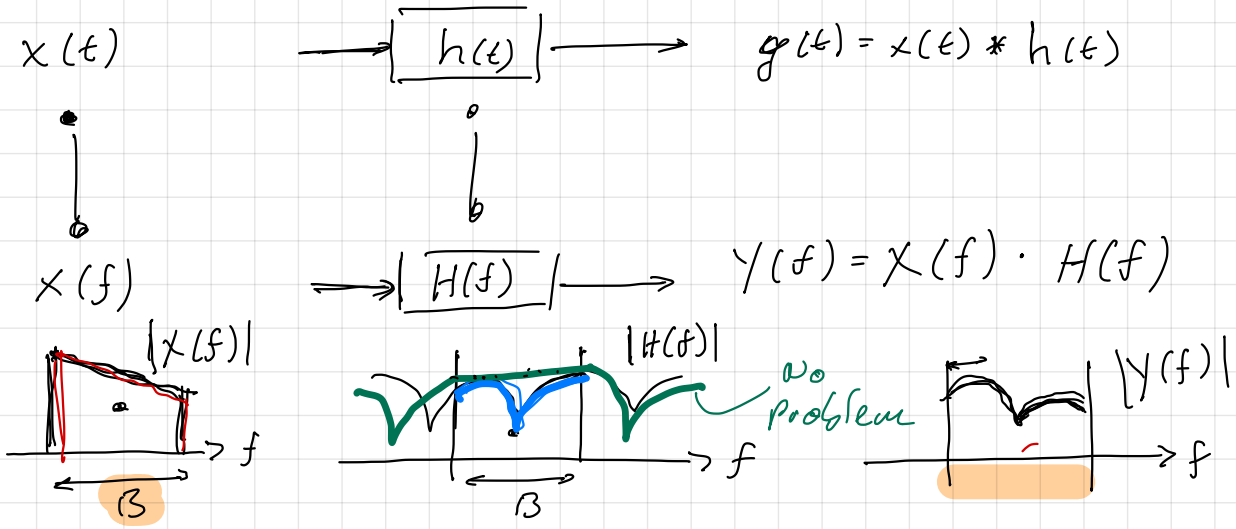


Measure for length of the channel that relates to ISI

RMS Delayspread: weighted length of the channel

$$\sigma_\tau = \sqrt{\frac{\sum (\tau_i - \bar{\tau})^2 |h_i|^2}{\sum |h_i|^2}} \qquad \bar{\tau} = \sum \tau_i$$

Frequency domain view of channel



Different parts of the spectrum are attenuated/rotated differently: Distorting the spectrum in a non-uniform way => ISI

=> Characterize the spectrum of the channel in the frequency domain

How much is the channel changing over frequency in the BW of the signal

How similar are $H(f)$ and $H(f+\Delta f)$?

$$\phi_h(\Delta f) = \mathcal{E}\{H^*(f)\, H(f+\Delta f)\}$$

Note: $\phi_h(\Delta f) = 0 \rightarrow H(f)$ and $H(f+\Delta f)$ are uncorrelated

Define: Coherence BW of the channel is the range of frequencies over which the channel has almost the same gain and phase (B_c = coherence bandwidth)

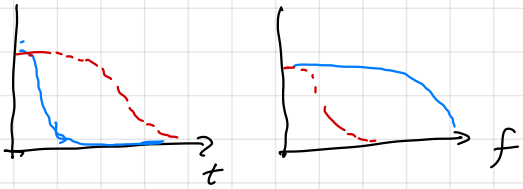
How to get the coherence BW?

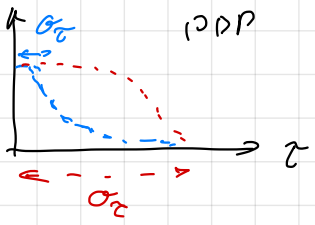
- Use that $\mathcal{E}\{H^*(f)\, H(f+\Delta f)\} = \mathcal{F}\{\underbrace{\mathcal{E}\{|h(t)|^2\}}_{\text{PDP}}\}$

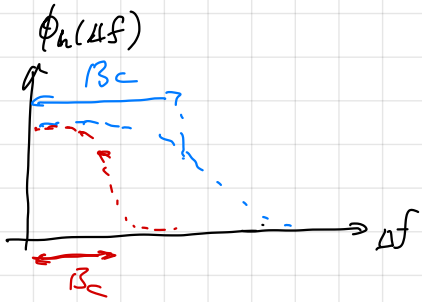
FD-AKF of the channel is the Fourier transform of the PDP

- Coherence BW = BW of $\phi_h(\Delta f)$
- Remember $BW \sim 1/T$ T: duration of the signal
- Duration: σ_τ = Delay Spread

$$B_c \sim \frac{1}{\sigma_\tau}$$

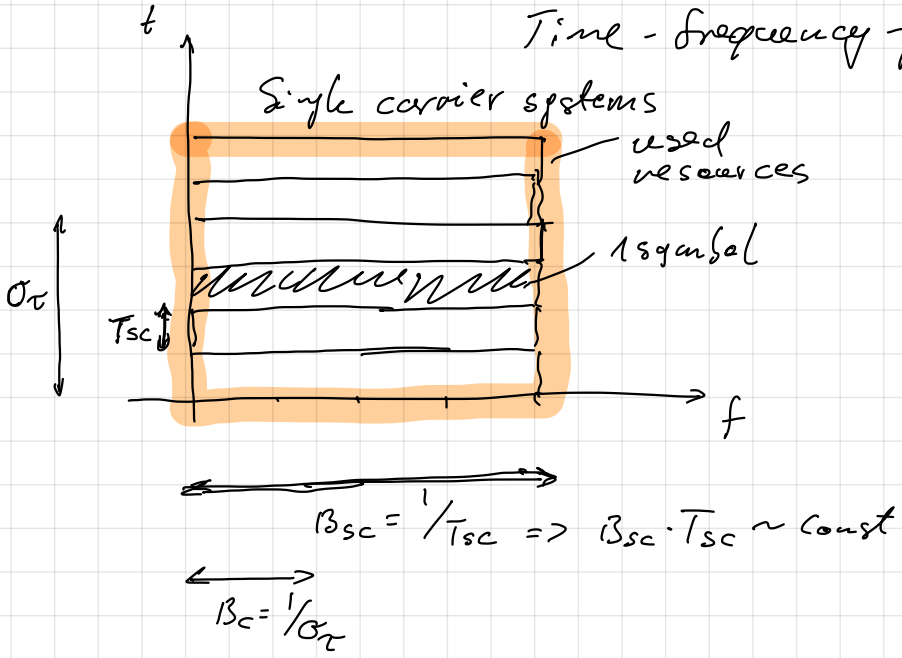


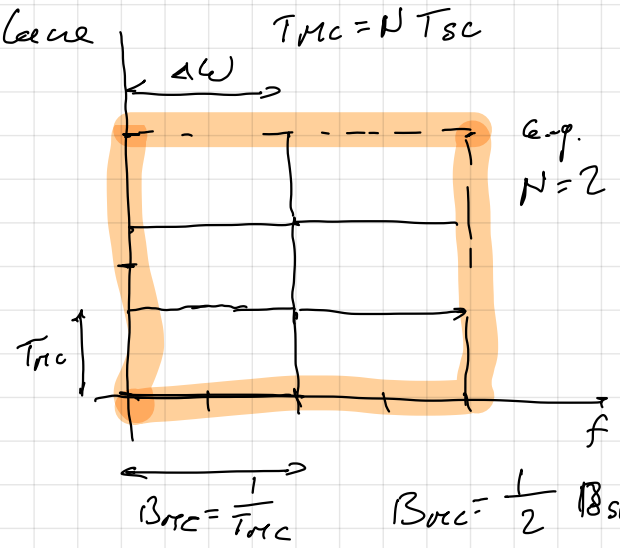




Multi-Carrier Modulation

Generate ISI-free Wideband signals



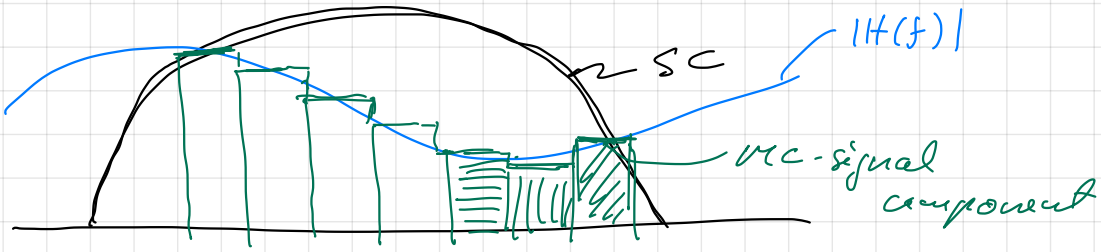


redistribute my Symbols in the time-freq.-plane

Impact on ISI

$T_{SC} < \sigma_\tau \rightarrow T_{MC} = N T_{SC} > \sigma_\tau$

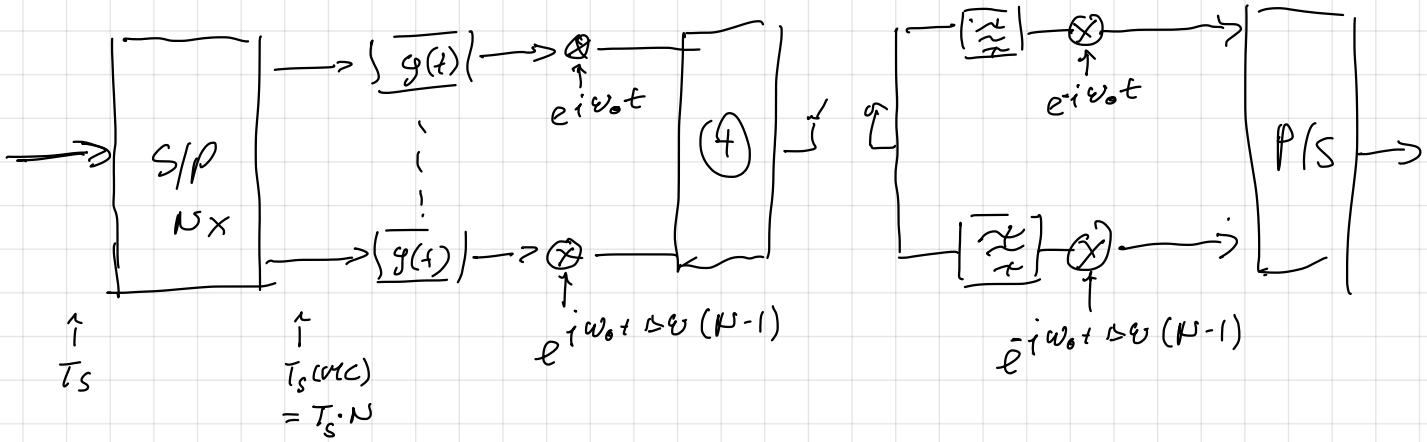
$B_{SC} > \underbrace{1/\sigma_\tau}_{B_C} \rightarrow B_{MC} = \frac{B_{SC}}{N} < B_C$



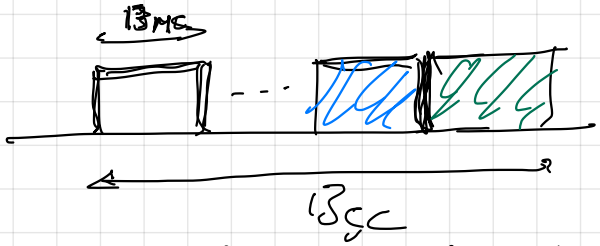
Wideband signal if the signal-BW is >> than the coherence BW of the channel → ISI

Narrowband signal if the opposite → NO ISI

Simple Multi-Carrier System



Let's look at the BW $\quad B_{MC} = \frac{B_{SC}}{N} = \frac{1}{N T_{SC}}$

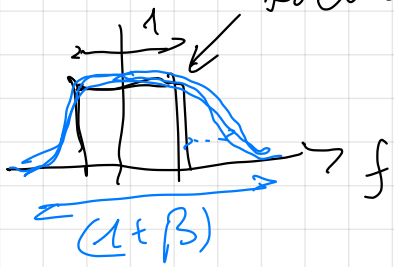


In theory requires infinitely good filters

In practice! No ideal filters

$$B_{MC} = (1+\beta)\frac{1}{N T_{SC}}$$

β: excess BW



Total BW in the non-ideal case

$$N \cdot B_{MC} = (1+\beta)\frac{1}{T_{SC}}$$

Efficiency $\frac{B_{SC}}{N B_{MC}} = \frac{1}{(1+\beta)} < 1$