Fundamentals of Analog & Mixed Signal VLSI Design
Exercise 3 (2.10.2024)

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Problem 1 Cascode stage

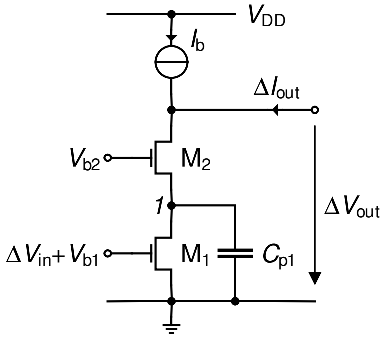

Figure 1: Cascode stage.

The schematic of the basic cascode stage is shown in Fig. 1.

1.1 Small-signal analysis

- Draw the small-signal equivalent schematic of the circuit assuming that the transistors are biased in saturation.
- Calculate the dc equivalent transconductance $G_{meq} \triangleq \Delta I_{out}/\Delta V_{in}|_{\Delta V_{out}=0}$ assuming that $G_{ms2} \gg G_{ds1}, G_{ds2}$.
- Calculate the output conductance $G_{out} = \Delta I_{out}/\Delta V_{out}$ neglecting capacitance C_{p1}. How does it compare to the output conductance G_{ds1} of M1?
- What is the effect of the parasitic capacitance C_{p1} at node 1 on the output conductance?

1.2 Noise analysis

- Calculate the output noise conductance G_{nout} and input-referred noise R_{nin} neglecting capacitance C_{p1}. Separate the input-referred noise resistance in its thermal and 1/f noise contributions $R_{nin}(f) = R_{nt} + R_{nf}(f)$.
- Calculate the cascode noise excess factor $\gamma_{cas} = G_{meq} \cdot R_{nt}$. How does it compare to the noise coming from M1 only?
- What is the effect of the parasitic capacitance C_{p1} at node 1 on the input-referred noise?

Problem 2 The Simple OTA

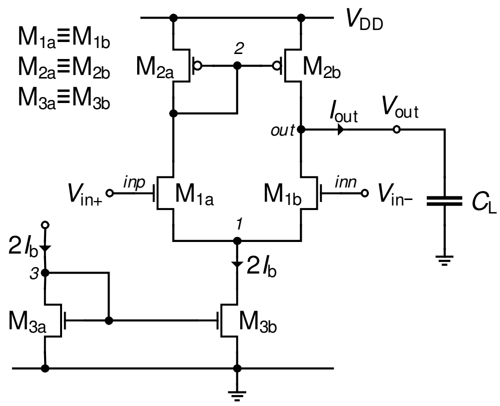

Figure 2: The simple OTA.

The schematic of the simple differential OTA is shown in Fig. 2.

2.1 Small-signal analysis

- Draw the small-signal equivalent schematic of the circuit assuming the transistors are biased in saturation.
- Calculate the differential dc transconductance $G_{md} \triangleq I_{out}/V_{id}$ where $V_{id} \triangleq V_{in+} - V_{in-}$ is the differential input voltage.
- Calculate the unity-gain frequency ω_u (or gain-bandwidth product).

2.2 Noise analysis

- Draw the small-signal equivalent schematic of the circuit assuming the transistors are biased in saturation including all the noise sources.
- Estimate the output noise conductance.
- Calculate the input-referred thermal noise PSD and the equivalent thermal noise resistance R_{nt}.
- Calculate the input-referred flicker noise PSD and the equivalent flicker noise resistance $R_{nf}(f)$.

Solutions to Exercise 3 (2.10.2024)

Problem 1 Cascode stage

1.1 Small-signal analysis

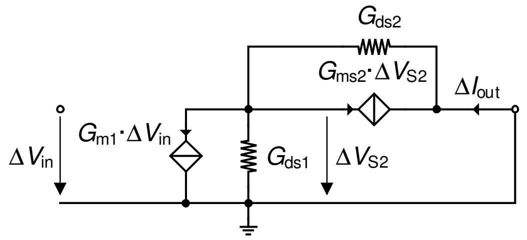

Figure 3: Small-signal schematic of the cascode stage of Fig. 1 for the calculation of the equivalent transconductance.

The small-signal schematic corresponding to Fig. 1 for the derivation of the equivalent transconductance is given in Fig. 3. Note that the output is short-circuited for calculating the short-circuit output current and the corresponding transconductance. The equivalent transconductance is given by

$$G_{meq} \triangleq \left.\frac{\Delta I_{out}}{\Delta V_{in}}\right|_{\Delta V_{out}=0} = \frac{G_{m1}(G_{ms2} + G_{ds2})}{G_{ms2} + G_{ds1} + G_{ds2}} \cong G_{m1}, \tag{1}$$

which shows that assuming $G_{ms2} \gg G_{ds1}, G_{ds2}$, the equivalent transconductance of the cascode stage is equal to the transconductance of the driver transistor M1. This result is expected since the cascode transistor is a common gate stage which has a unity current gain so that the current coming from the driver transistor M1 is directly directed to the output.

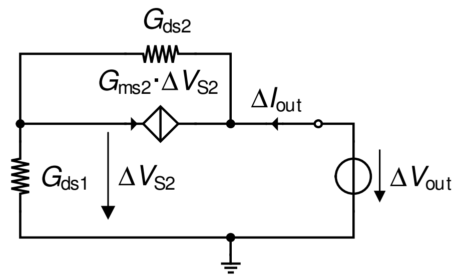

Figure 4: Small-signal schematic of the cascode stage of Fig. 1 for the calculation of the output conductance.

The small-signal schematic for the calculation of the output conductance is shown in Fig. 4. The output conductance is given by

$$G_{out} \triangleq \frac{\Delta I_{out}}{\Delta V_{out}} = \frac{G_{ds1}\, G_{ds2}}{G_{ms2} + G_{ds1} + G_{ds2}} \cong \frac{G_{ds1}}{G_{ms2}/G_{ds2}}, \tag{2}$$

which is equal to the output conductance of M1, G_{ds1}, divided by the voltage gain of the cascode G_{ms2}/G_{ds2}. This means that, at low-frequency, the output conductance of a single transistor can be reduced by adding a cascode stage at the cost of some voltage headroom to maintain M2 in saturation.

The impact of the parasitic capacitance at node 1 on the output conductance can be investigated by adding the capacitance as shown in Fig. 5. The output admittance then becomes

$$Y_{out} = G_{out} \cdot \frac{1 + s/\omega_z}{1 + s/\omega_p}, \tag{3}$$

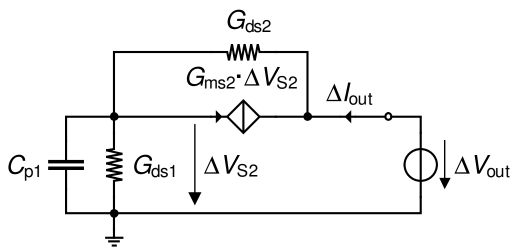

Figure 5: Small-signal schematic of the cascode stage of Fig. 1 for the calculation of the output conductance including the parasitic capacitance C_{p1}.

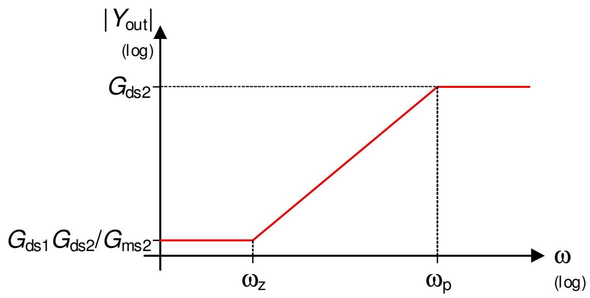

Figure 6: Effect of the parasitic capacitance C_{p1} on the output admittance Y_{out}.

where

$$G_{out} \cong \frac{G_{ds1}\,G_{ds2}}{G_{ms2}}, \tag{4a}$$

$$\omega_z \triangleq \frac{G_{ds1}}{C_{p1}}, \tag{4b}$$

$$\omega_p \triangleq \frac{G_{ms2}}{C_{p1}}. \tag{4c}$$

The magnitude of Y_{out} versus frequency is sketched in Fig. 6. For $\omega \ll \omega_z \ll \omega_p$, $Y_{out} \cong G_{out}$. However for $\omega_z \ll \omega_p \ll \omega$, the output admittance becomes $Y_{out} \cong G_{ds2}$. We see that the cascode effect is lost. This can easily be understood since for $\omega_p \ll \omega$, the cascode node 1 is shortened to the ac ground and the voltage controlling the source of M2 is zero leaving the output conductance of M2 only.

1.2 Noise analysis

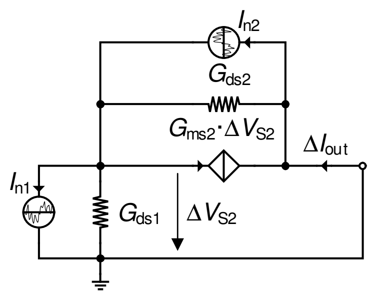

Figure 7: Small-signal schematic of the cascode stage of Fig. 1 for the noise calculation.

To calculate the output noise PSD we can use the schematic shown in Fig. 7. The output noise current is then given by

$$I_{nout} = \frac{G_{ms2} + G_{ds2}}{G_{ms2} + G_{ds1} + G_{ds2}} \cdot I_{n1} + \frac{G_{ds1}}{G_{ms2} + G_{ds1} + G_{ds2}} \cdot I_{n2} \cong I_{n1} + \frac{I_{n2}}{G_{ms2}/G_{ds1}}, \tag{5}$$

for $G_{ms2} \gg G_{ds1}, G_{ds2}$. We see that the contribution to the output noise current of the cascode stage is actually divided by G_{ms2}/G_{ds1}. Provided that this gain can be made sufficiently large and that both transistor have the same noise, the noise of the cascode stage can be made negligible compared to the noise due to M1. Ultimately if $G_{ds1} = 0$, the noise current I_{n2} circulates in the cascode transistor M2 (G_{ms2} in the small-signal schematic) and hence does not reach the output.

The output noise conductance is then given by

$$G_{nout}(f) \cong G_{n1}(f) + \left(\frac{G_{ds1}}{G_{ms2}}\right)^2 G_{n2}(f) \tag{6}$$

where the noise conductances are given by

$$G_{ni}(f) = \gamma_{ni} \cdot G_{mi} + \frac{\rho_n}{f\, W_i\, L_i} \cdot G_{mi}^2 \quad \text{for } i = 1, 2. \tag{7}$$

The input-referred noise is obtained by dividing G_{nout} by G_{meq}^2, resulting in

$$R_{nin} = \frac{G_{nout}(f)}{G_{meq}}. \tag{8}$$

It can be decomposed into the thermal and flicker noise components according to

$$R_{nin} = R_{nt} + R_{nf}(f) \tag{9}$$

where R_{nt} is the total input-referred thermal noise

$$R_{nt} \cong \frac{\gamma_{n1}}{G_{m1}} + \left(\frac{G_{ds1}}{G_{m1}}\right)^2 \frac{\delta_{n2}}{G_{ms2}} = \frac{\gamma_{n1}}{G_{m1}} \left[1 + \frac{G_{ds1}^2}{G_{m1}\, G_{ms2}} \frac{\delta_{n2}}{\gamma_{n1}}\right], \tag{10}$$

and $R_{nf}(f)$ is the input-referred 1/f noise

$$R_{nf}(f) \cong \frac{\rho_n}{f\, W_1\, L_1} + \left(\frac{G_{ds1}}{G_{m1}}\right)^2 \frac{\rho_n}{n_2^2 f\, W_2 L_2} = \frac{\rho_n}{f\, W_1\, L_1} \left[1 + \left(\frac{G_{ds1}}{G_{m1}}\right)^2 \frac{W_1\, L_1}{n_2^2\, W_2\, L_2}\right]. \tag{11}$$

The γ noise factor of the cascode stage is given by

$$\gamma_{cas} \triangleq G_{meq} R_{nt} = G_{m1} R_{nt} = \gamma_{n1} \left[1 + \frac{G_{ds1}^2}{G_{m1}\, G_{ms2}} \frac{\delta_{n2}}{\gamma_{n1}}\right] \cong \gamma_{n1}, \tag{12}$$

since $G_{m1} G_{ms2} \gg G_{ds1}^2$. The contribution of the cascode transistor to the overall noise excess factor is therefore negligible. The same remark holds for the 1/f noise. Indeed, assuming M1 and M2 have the same area and $G_{m1}/G_{ds1} \gg 1$, (11) simplifies to

$$R_{nf}(f) \cong \frac{\rho_n}{f\, W_1\, L_1}, \tag{13}$$

which corresponds to the contribution of M1 only.

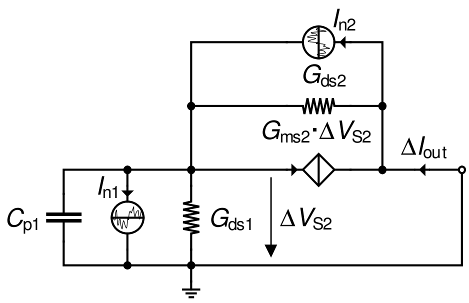

Figure 8: Small-signal schematic of the cascode stage of Fig. 1 for the calculation of the output conductance including the effect of the parasitic capacitance C_{p1}.

Similarly, the impact of the parasitic capacitance on the noise can be calculated from Fig. 8. The output noise conductance is then given by

$$G_{nout} = |H_{n1}(\omega)|^2 \cdot G_{n1} + |H_{n2}(\omega)|^2 \cdot G_{n2}, \tag{14}$$

where

$$H_{n1}(s) = \frac{1}{1 + s/\omega_p} \tag{15a}$$

$$H_{n2}(s) = \frac{G_{ds1}}{G_{ms2}} \cdot \frac{1 + s/\omega_z}{1 + s/\omega_p}. \tag{15b}$$

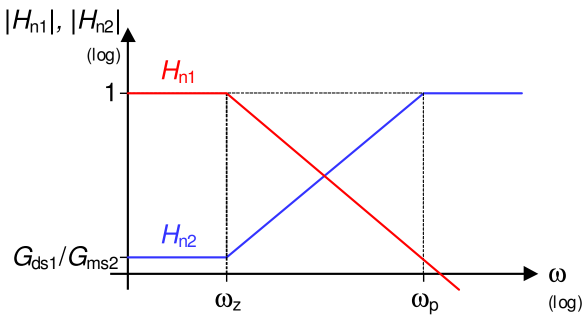

Figure 9: Noise transfer functions H_{n1} and H_{n2} versus frequency.

The magnitude of H_{n1} and H_{n2} versus frequency are sketched in Fig. 9. For $\omega \ll \omega_z \ll \omega_p$, $H_{n1} \cong 1$ and $H_{n2} \cong G_{ds1}/G_{ms2}$ which is the result obtained above. However, for $\omega_z \ll \omega_p \ll \omega$, $H_{n1} \cong \omega_p/s$ and $H_{n2} \cong 1$. We see that the cascode effect is lost since the noise of M2 is no more divided by the cascode gain G_{ms2}/G_{ds1} but is entirely transferred to the output.

As a conclusion, adding a cascode stage reduces the output conductance without penalty on the noise, but at the cost of a slight voltage overhead for maintaining M2 in saturation. This is only true for $\omega < \omega_z = G_{ds1}/C_{p1}$. For frequencies $\omega \gg \omega_p = G_{ms2}/C_{p1}$ the cascode effect is lost. Note that in order to maximize G_{ms2} at a given current and minimize its saturation voltage, M2 should be biased in weak inversion.

Problem 2 The Simple OTA

2.1 Small-signal Analysis

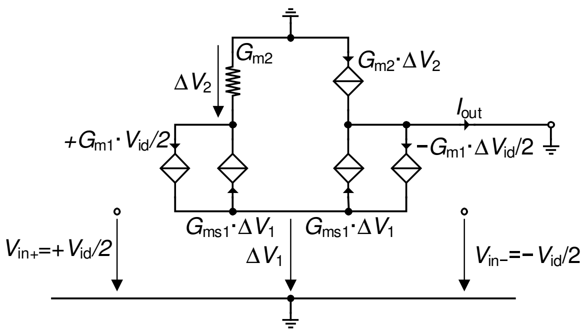

Figure 10: Small-signal schematic of the simple OTA of Fig. 2.

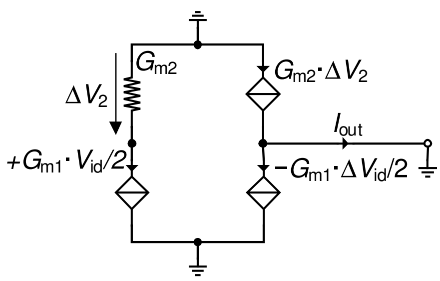

Figure 11: Simplified circuit of Fig. 10.

In the case of a differential input voltage $V_{G1a} = V_{ic} + V_{id}/2$ and $V_{G1b} = V_{ic} - V_{id}/2$ and assuming a constant input common-mode voltage V_{ic}, we have $\Delta V_{G1a} = +V_{id}/2$ and $\Delta V_{G1b} = -V_{id}/2$. If transistors M_{1a}-M_{1b} and M_{2a}-M_{2b} are assumed to be perfectly matched, then $G_{m1a} = G_{m1b} = G_{m1}$, $G_{ms1a} = G_{ms1b} = G_{ms1}$ and $G_{m2a} = G_{m2b} = G_{m2}$. The resulting small-signal equivalent circuit is then shown in Fig. 10. The current $+G_{m1} \cdot V_{id}/2$ in M_{1a} is then compensated by the current $-G_{m1} \cdot V_{id}/2$ in M_{1b} so that the sum of the currents $G_{ms1} \cdot \Delta V_1 + G_{ms1} \cdot \Delta V_1 = 2G_{ms1} \cdot \Delta V_1 = 0$ so that $\Delta V_1 = 0$. This means that under differential input voltage and assuming a perfect matching, the common source node can be considered as an ac ground and the small-signal circuit of Fig. 10 simplifies to that of Fig. 11.

The above statement can be proved in a more formal way by writing the KCL equation for the common source node

$$+G_{m1} \cdot V_{id}/2 - G_{ms1} \cdot \Delta V_1 - G_{ms1} \cdot \Delta V_1 - G_{m1} \cdot V_{id}/2 = 0 \tag{16}$$

resulting in

$$-2G_{ms1} \cdot \Delta V_1 = 0 \tag{17}$$

which results in $\Delta V_1 = 0$.

From the simplified schematic of Fig. 11, we can write the output current I_{out} as

$$I_{out} = G_{m1} \cdot \frac{V_{id}}{2} + G_{m2} \cdot \Delta V_2, \tag{18}$$

where the voltage ΔV_2 is given by

$$\Delta V_2 = \frac{G_{m1}}{G_{m2}} \cdot \frac{V_{id}}{2}. \tag{19}$$

Replacing (21) in (18) results in

$$I_{out} = G_{m1} \cdot V_{id}. \tag{20}$$

The equivalent transconductance is therefore equal to the gate transconductance of M_{1a} and M_{1b}

$$G_{md} \triangleq \frac{I_{out}}{V_{id}} = G_{m1}. \tag{21}$$

If we neglect the internal capacitance C_2 at the gate node of the current mirror M_{2a}-M_{2b} (node 2), the transfer function including the load capacitance C_L is then

$$A_{vd}(s) \triangleq \frac{\Delta V_{out}}{V_{id}} = \frac{G_{meq}}{s\,C_L} = \frac{G_{m1}}{s\,C_L}. \tag{22}$$

The unity gain frequency or gain-bandwidth product ω_u is given by

$$\omega_u = \frac{G_{m1}}{C_L}. \tag{23}$$

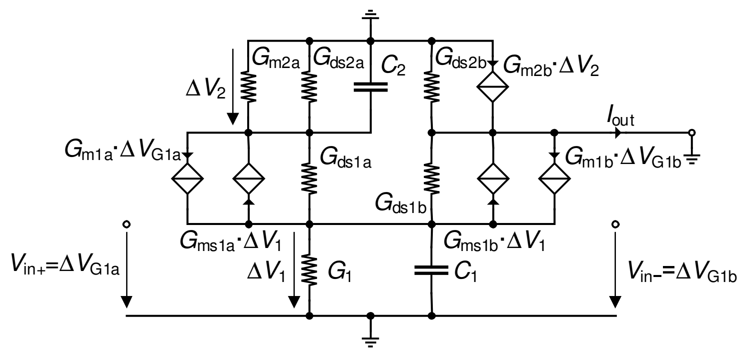

Figure 12: Small-signal schematic of the simple OTA of Fig. 2.

A more detailed analysis can be done using the she small-signal circuit corresponding to the schematics of Fig. 2 shown in Fig. 12, where capacitances C_1 and C_2 correspond to the total parasitic capacitances at node 1 and 2. They depend on the parasitic capacitances of the different transistors connected to these nodes. Conductance G_1 is actually the output conductance of the current source.

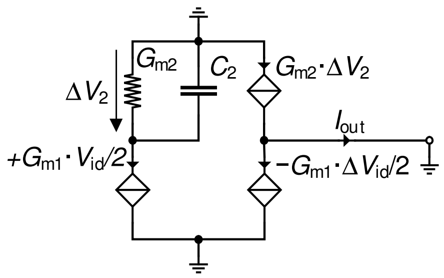

Figure 13: Simplified small-signal schematic of the simple OTA of Fig. 12.

In order to calculate the differential transadmittance Y_{md}, the output node is ac grounded and the input common mode $V_{ic} \triangleq (V_{in+} + V_{in-})/2$ is maintained constant (i.e. the smal-signal common-mode voltage $\Delta V_{ic} = 0$. In a 1st-order analysis we can neglect all the output conductances except G_1 of the bottom bias current source. Assuming a perfect matching, results in $G_{m1a} = G_{m1b} = G_{m1}$, $G_{ms1a} = G_{ms1b} = G_{ms1}$, $G_{m2a} = G_{m2b} = G_{m2}$. As shown above, it is easy to show that for a differential input voltage, the small-signal voltage of the common-source node 1 is zero $\Delta V_1 = 0$. There is therefore no current flowing into G_1 and the source transconductances have no effect. The small-signal circuit of Fig. 12 then simplifies to that of Fig. 13 from which it is easy to derive the differential transadmittance accounting for the parasitic capacitance at node 2

$$Y_{dm} = G_{m1}\frac{1 + s\tau_2/2}{1 + s\tau_2} = G_{m1}\frac{1 + s/(2\omega_2)}{1 + s/\omega_2} \tag{24}$$

where $\tau_2 = 1/\omega_2 \triangleq C_2/G_{m2}$ is the time constant introduced by the current mirror M_{2a}-M_{2b} due to the parasitic capacitance C_2 at node 2. A transfer function like (24) with a zero at double the value of the pole is called a *doublet*.

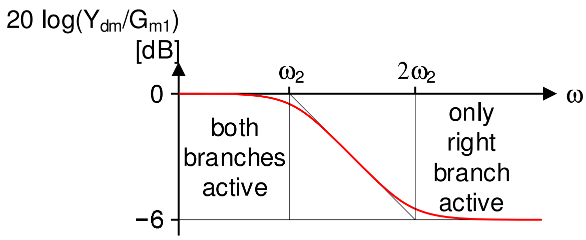

Figure 14: Bode plot of the small-signal differential transadmittance.

Fig. 14 shows the magnitude of Y_{md} normalized to the low-frequency value G_{m1} versus the frequency. For $\omega < \omega_2$, both current branches of the differential pair are active. On the other hand, for $\omega > \omega_2$, the voltage at node 2 is low-pass filtered and ac grounded and therefore the current coming from transistor M1a is not copied to the output. The output current is hence only coming from transistor M_{1b}, resulting in half the low-frequency transconductance.

The differential mode voltage transfer function A_{vd} is simply given by

$$A_{vd} \triangleq \frac{V_{out}}{V_{id}} = Y_{md} \cdot Z_L \tag{25}$$

where Z_L is the output load. In the case the output load is only capacitive

$$Y_L \triangleq 1/Z_L = G_o + sC_L \tag{26}$$

where G_o is the total conductance at the output node $G_o = G_{ds1} + G_{ds2}$. This results in

$$A_{vd} = \frac{G_{m1}}{G_o + sC_L} \cdot \frac{1 + s/(2\omega_2)}{1 + s/\omega_2} = A_0 \cdot \frac{1 + s/(2\omega_2)}{(1 + s/\omega_0)(1 + s/\omega_2)} \tag{27}$$

where $A_0 \triangleq G_{m1}/G_o$ is the *dc gain*, $\omega_2 \triangleq G_{m2}/C_2$ the *non-dominant pole* and $\omega_0 \triangleq G_0/C_L$ the *dominant pole*. The Bode plot of the differential voltage transfer function is plotted in Fig. 15. Assuming that the non-dominant pole ω_2 is much higher than the unity gain frequency ω_u (or gain-bandwidth product *GBW*), the latter is then given by

$$\omega_u = GBW = A_0 \cdot \omega_0 = \frac{G_{m1}}{G_o} \cdot \frac{G_o}{C_L} = \frac{G_{m1}}{C_L}. \tag{28}$$

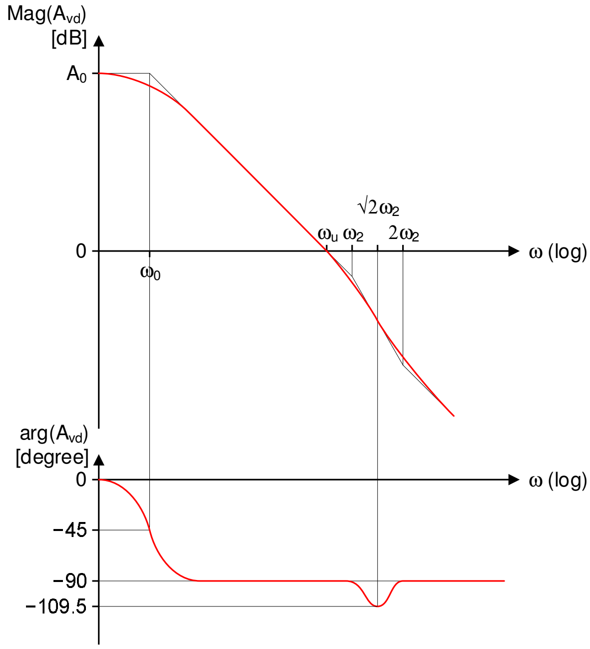

Figure 15: Bode plot of the differential voltage transfer function A_{vd}.

Note that the phase reaches a minimum for $\omega = \sqrt{2}\omega_2$

$$\Phi_{min} = -\frac{\pi}{2} + arctg(\sqrt{2}/2) - arctg(\sqrt{2}) \cong -109.5°. \tag{29}$$

2.2 Noise Analysis

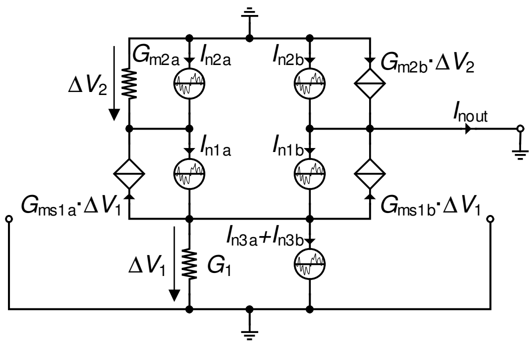

Figure 16: Small-signal circuit including the noise sources.

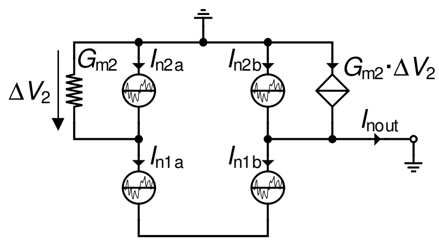

Figure 17: Simplified small-signal circuit including the noise sources.

In order to calculate the noise output current I_{nout}, the input terminals are grounded. The small-signal equivalent circuit including the noise sources of all the transistors is shown in Fig. 16. Note that all the output conductances have been neglected. Since we want to calculate the noise at low-frequency (meaning for $\omega < \omega_2$), we can neglect the parasitic capacitances C_1 and C_2.

If a perfect symmetry is assumed, then the two currents generated by transconductances G_{ms1a} and G_{ms1b} in Fig. 16 are equal. If the current mirror is also assumed to be perfectly matched, then the current coming from M_{1a} is mirrored at the output and compensated by the current coming directly from M_{1b}. Therefore, the source transconductances G_{ms1a} and G_{ms1b} have no effect on the output current and can therefore be neglected. Assuming again perfect symmetry, the noise currents I_{n3a} and I_{n3b} coming from M_{3a} and M_{3b} split equally between the two branches and produce no net current at the output neither. It can therefore also be neglected. Finally, the small-signal circuit of Fig. 16 simplifies to that of Fig. 17 and the output noise current at low frequency is simply given by simply given by

$$I_{nout} = I_{n1a} - I_{n1b} - I_{n2a} + I_{n2b}. \tag{30}$$

This means that the transfer functions at low-frequency from each of the noise sources to the output current is simply equal to ± 1. Note that the sign is not relevant since for noise we are only interested by the square magnitude of the transfer functions.

The power spectral density (PSD) of the output noise current is then given by

$$S_{nout}(f) = 4kT \cdot G_{nout}(f), \tag{31}$$

with

$$G_{nout}(f) = G_{n1}(f) + G_{n2}(f) + G_{n3}(f) + G_{n4}(f) = 2\,[G_{n1}(f) + G_{n2}(f)]\,. \tag{32}$$

The noise conductances $G_{ni}(f)$ with $i = 1, 2$, are frequency dependent since they include both the thermal noise and the 1/f noise. They are given by

$$G_{ni}(f) = \gamma_{ni} G_{mi} + G_{mi}^2 \frac{\rho_i}{W_i L_i f} \quad \text{for } i = 1, 2, \tag{33}$$

where $\rho_1 = \rho_n$ and $\rho_2 = \rho_p$ and

$$\gamma_{ni} = \begin{cases} \frac{n_i}{2} & \text{in weak inversion and saturation} \\ \frac{2}{3}\, n_i \cong 1 & \text{in strong inversion and saturation.} \end{cases} \tag{34}$$

At low-frequency, the noise can be referred to the differential input by dividing G_{nout} by G_{m1}^2, resulting in

$$R_{nin}(f) \triangleq \frac{G_{nout}}{G_{m1}^2} = R_{nt} + R_{nf}(f) \tag{35}$$

where R_{nt} is the part of the input-referred noise resistance corresponding to the thermal noise

$$R_{nt} = 2\left(\frac{\gamma_{n1}}{G_{m1}} + \gamma_{n2}\frac{G_{m2}}{G_{m1}^2}\right) = \frac{2\gamma_{n1}}{G_{m1}}\,(1 + \eta_{th}), \tag{36}$$

where

$$\eta_{th} = \frac{\gamma_{n2}}{\gamma_{n1}}\frac{G_{m2}}{G_{m1}} \tag{37}$$

represents the contribution to the input-referred thermal noise of the current mirror relative to the differential pair.

$R_{nf}(f)$ is the part corresponding to the 1/f noise

$$R_{nf}(f) = 2\left[\frac{\rho_n}{W_1 L_1 f} + \left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{\rho_p}{W_2 L_2 f}\right] = \frac{2\rho_n}{W_1 L_1 f}\,(1 + \eta_{fl}), \tag{38}$$

where

$$\eta_{fl} = \left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{\rho_p}{\rho_n}\frac{W_1 L_1}{W_2 L_2} \tag{39}$$

represents the contribution to the input-referred flicker noise of the current mirror relative to the differential pair.

In the same way a noise excess factor γ_n has been defined for a single transistor, a thermal noise excess factor can also be defined for the complete OTA as

$$\gamma_{ota} \triangleq G_m \cdot R_{nt} = \frac{G_{nout(thermal)}}{G_m} = 2\gamma_{n1}\,(1 + \eta_{th}) \tag{40}$$

where $G_m = G_{m1}$ is the OTA differential transconductance. The total input-referred thermal noise resistance then writes

$$R_{nt} = \frac{\gamma_{ota}}{G_{m1}}. \tag{41}$$

The minimum value of the OTA noise excess factor is equal to that of the differential pair only, namely $\gamma_{ota,min} = 2\gamma_{n1}$. In order to limit the contribution of the current mirror to a minimum, η_{th} should be made much smaller than one. This can be achieved by setting the transconductance ratio $G_{m2}/G_{m1} \ll 1$. This can be done by biasing M_{1a}-M_{1b} in weak inversion and M_{2a}-M_{2b} in strong inversion, respectively. Replacing G_{m2}/G_{m1} results in

$$\eta_{th} = \frac{\gamma_{n2}}{\gamma_{n1}} \frac{2n_1 U_T}{V_{G2} - V_{T0p}} = \frac{8n_2}{3} \frac{U_T}{V_{G2} - V_{T0p}}, \tag{42}$$

where $\gamma_{n1} = n_1/2$ and $\gamma_{n2} = n_2 2/3$. The OTA thermal noise excess factor is therefore minimized by setting the transconductance ratio $G_{m2} \ll G_{m1}$, which is realized by choosing an overdrive voltage $V_{G2} - V_{T0p}$ of M_{2a}-M_{2b} much larger than $8n_2/3U_T \cong 4U_T$ where it has been assumed that $n_2 \cong 1.5$.

The 1/f noise corner frequency f_k is defined as the frequency at which the 1/f noise becomes equal to the thermal noise

$$R_{nf}(f_k) = R_{nt}. \tag{43}$$

It is given by

$$f_k = \frac{1}{R_{nt}} \frac{2\rho_n}{W_1 L_1}(1 + \eta_{fl}) = \frac{G_{m1}}{\gamma_{ota}} \frac{2\rho_n}{W_1 L_1}(1 + \eta_{fl}) = \frac{G_{m1}}{\gamma_{n1}} \frac{\rho_n}{W_1 L_1} \frac{1 + \eta_{fl}}{1 + \eta_{th}}. \tag{44}$$