# Fundamentals of Analog & Mixed Signal VLSI Design
# Exercise 1 (18.09.2024)

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## Problem 1 Stacked transistors

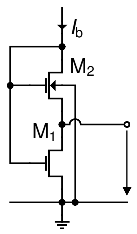


Figure 1: Stacked transistors.

### 1.1 Voltage $V_b$ as a function of transistor's $\beta_1$ and $\beta_2$

Calculate the voltage $V_b$ in circuit shown in Fig. 1 as a function of the betas ($\beta_1$ and $\beta_2$) of transistors and of the thermodynamic voltage (in weak inversion) or the pinch-off voltage (in strong inversion), assuming:

- Both transistors are biased in weak inversion.
- Both transistors are biased in strong inversion.

### 1.2 Channel voltage

If the transistors have the same width $W$, then

$$\frac{\beta_1}{\beta_2} = \frac{L_2}{L_1}, \tag{1}$$

and the circuit can be considered as a single transistor with length $L = L_1 + L_2$. The channel voltage can then be extracted at a fraction $\xi$ along the channel by the common source and drain diffusion of $M_2$ and $M_1$ respectively

$$\xi = \frac{L_1}{L_1 + L_2} = \frac{\beta_2}{\beta_1 + \beta_2}, \tag{2}$$

Knowing that the current $I_b$ is due to diffusion of minority carrier, it is given by,

$$I_b = -\mu \cdot \frac{W}{L} \cdot Q_i \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} \tag{3}$$

with $\xi = x/L$. Show that $V_b$ is actually equal to the channel voltage along the channel $V(x)$.

## Problem 2 The Vittoz current reference [1]

Fig. 2 shows the Vittoz current reference generating a current $I_b$ which for $M_1$ and $M_3$ biased in weak inversion is proportional to absolute temperature (PTAT) [1]. The bias current $I_b$ is available as a source current from $M_6$ or a sink current from $M_5$. Transistors $M_2$ and $M_4$ are assumed to be identical whereas $M_3$ is made $K$-times larger than $M_1$ (i.e. $\beta_3 = K \cdot \beta_1$). All transistors are biased in saturation.

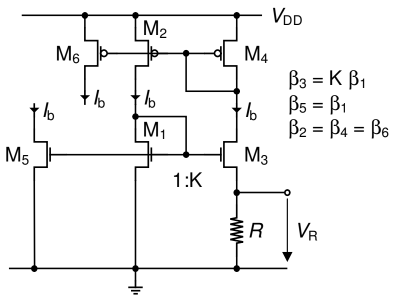


Figure 2: The Vittoz current reference.

### 2.1 Weak inversion

Calculate the value of the current $I_b$ as a function of the transistor ratio $K$ assuming that both $M_1$ and $M_3$ are in weak inversion. Do the transistor $M_2$ and $M_4$ need to be in weak inversion as well?

### 2.2 Strong inversion

Calculate the value of the current $I_b$ as a function of the transistor ratio $K$ assuming that both $M_1$ and $M_3$ are in strong inversion. Do the transistor $M_2$ and $M_4$ need to be in strong inversion as well?

## Problem 3 The Oguey current reference [2]

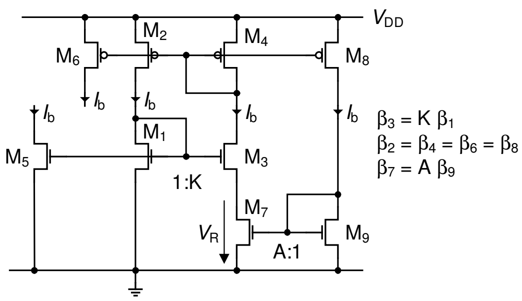


Figure 3: The Oguey current reference.

The resistor in the Vittoz current reference of Fig. 2 can actually be replaced by a transistor $M_7$ which is biased in the linear region by making it $A$-times larger than $M_9$ [2]. All other transistors are assumed to be biased in saturation. $M_1$ and $M_3$ are biased in weak inversion, whereas $M_7$ and $M_9$ are biased in strong inversion. Find the expression of the bias current $I_b$ in terms of ratios $K$ and $A$ and the specific current $I_{spec9}$ of $M_9$ and $I_{spec7}$ of $M_7$.

# Solutions to Exercise 1 (18.09.2024)

## Problem 1 Stacked Transistor

### 1.1 Voltage $V$ as a function of transistor's $\beta_1$ and $\beta_2$

#### 1.1.1 Both transistors in weak inversion

Both transistors $M_1$ and $M_2$ share the same gate and have therefore the same gate voltage $V_{G1} = V_{G2} = V_G$. They have therefore also the same pinch-off voltage $V_{P1} = V_{P2} = V_P$ and slope factors $n_1 = n_2 = n$. Transistor $M_2$ is in saturation because $V_{D2} = V_G \cong V_{T0} \gg U_T$. However, $M_2$ is in the linear region because $V_{D1} = V_b \cong V_G - V_{T0}$ which is only slightly larger than zero. If both transistors are in weak inversion ($I_{D1} = I_{D2} \ll I_{spec}$), the current going through both transistors is then given by

$$I_{D1} = I_{spec1} \cdot e^{\frac{V_P}{U_T}} \cdot \left(1 - e^{\frac{-V_b}{U_T}}\right), \tag{4a}$$

$$I_{D2} = I_{spec2} \cdot e^{\frac{V_P}{U_T}} \cdot \left(e^{\frac{-V_b}{U_T}} - e^{\frac{-V_{D2}}{U_T}}\right), \tag{4b}$$

where $I_{spec1} \triangleq 2n\beta_1 U_T^2$ and $I_{spec2} \triangleq 2n\beta_2 U_T^2$ with $\beta_1 \triangleq \mu C_{ox} W/L_1$ and $\beta_2 \triangleq \mu C_{ox} W/L_2$. In saturation $V_{D2} \gg U_T$ and we have

$$I_{D1} = I_{spec1} \cdot e^{\frac{V_P}{U_T}} \cdot \left(1 - e^{\frac{-V_b}{U_T}}\right), \tag{5a}$$

$$I_{D2} = I_{spec2} \cdot e^{\frac{V_P}{U_T}} \cdot e^{\frac{-V_b}{U_T}}. \tag{5b}$$

Since $I_{D1} = I_{D2}$, we have

$$I_{spec1} \cdot \left(1 - e^{\frac{-V_b}{U_T}}\right) = I_{spec2} \cdot e^{\frac{-V_b}{U_T}}, \tag{6}$$

from which we obtain the bias voltage $V_b$

$$V_b = U_T \cdot ln\left(1 + \frac{I_{spec2}}{I_{spec1}}\right). \tag{7}$$

Since both transistor have the same pinch-off voltage, they also have the same slope factor $n$. (7) can then be simplified as

$$V_b = U_T \cdot \ln\left(1 + \frac{\beta_2}{\beta_1}\right) = U_T \cdot \ln\left(1 + \frac{L_1}{L_2}\right), \tag{8}$$

From (8), we see that the bias voltage $V_b$ does not depend on the current $I_D$, but only depends on the channel length ratio.

#### 1.1.2 Both transistors in strong inversion

Again, both transistors $M_1$ and $M_2$ share the same gate and have therefore the same gate, pinch-off voltages and slope factors $V_{G1} = V_{G2} = V_G$, $V_{P1} = V_{P2} = V_P$ and $n_1 = n_2 = n$. Transistor $M_2$ is in saturation because $V_{D2} = V_G = n \cdot V_P + V_{T0} > V_P$ since $V_{T0} > 0$ and $n > 1$. However, transistor $M_1$ cannot be biased in saturation. If it would be in saturation then $V_{D1} = V_{S2} > V_P$, which is contradiction with the fact that $M_2$ is in saturation and hence $V_{S2} < V_P$. The drain currents of transistors $M_1$ and $M_2$ are then given by

$$I_{D1} = \frac{n\beta_1}{2} \cdot \left[V_P^2 - (V_P - V_b)^2\right], \tag{9a}$$

$$I_{D2} = \frac{n\beta_2}{2} \cdot (V_P - V_b)^2. \tag{9b}$$

By equating the currents $I_{D1} = I_{D2} = I_b$, we obtain

$$\beta_1 \cdot \left[V_P^2 - (V_P - V_b)^2\right] = \beta_2 \cdot (V_P - V_b)^2, \tag{10}$$

or

$$\beta_1 \cdot V_P^2 = (\beta_1 + \beta_2) \cdot (V_P - V_b)^2\,, \tag{11}$$

and

$$\frac{V_P - V_b}{V_P} = 1 - \frac{V_b}{V_P} = \sqrt{\frac{\beta_1}{\beta_1 + \beta_2}}. \tag{12}$$

Finally we obtain the channel voltage as

$$V_b = V_P \cdot \left(1 - \sqrt{\frac{\beta_1}{\beta_1 + \beta_2}}\right) = V_P \cdot \left(1 - \sqrt{\frac{L_2}{L_1 + L_2}}\right). \tag{13}$$

## 1.2 Channel voltage in weak inversion

The current in weak inversion is a diffusion current given by

$$I_D = W \cdot \mu_n \cdot U_T \cdot \frac{\mathrm{d}Q_i}{\mathrm{d}x} \tag{14}$$

Since the current is constant along the channel, the inversion charge $Q_i(x)$ is a linear function of the position and since transistor $M_2$ is in saturation we have $Q_i(x = L) = Q_{iD} = 0$. It follows that

$$Q_i(x) = Q_{iS} \cdot \left(1 - \frac{x}{L}\right) = Q_{iS} \cdot (1 - \xi), \tag{15}$$

where $Q_{iS} = Q_i(x = 0)$ and $\xi = x/L$. The drain current can then be written as

$$I_D = -W \cdot \mu_n \cdot U_T \cdot \frac{Q_{iS}}{L}. \tag{16}$$

The current is also given as a function of the channel voltage $V$ by

$$I_D = \mu_n \cdot W \cdot (-Q_i) \cdot \frac{\mathrm{d}V}{\mathrm{d}x} = \mu_n \cdot \frac{W}{L} \cdot (-Q_i) \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} \tag{17}$$

Replacing $Q_i$ in (17) by (15), results in

$$I_D = \mu_n \cdot \frac{W}{L} \cdot (-Q_{iS}) \cdot (1 - \xi) \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} \tag{18}$$

Equating (18) to (16), leads to

$$(1 - \xi) \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} = U_T, \tag{19}$$

and

$$\frac{\mathrm{d}V}{\mathrm{d}\xi} = \frac{U_T}{1 - \xi}, \tag{20}$$

which can be integrated from $\xi = 0$ ($x = 0$) to $\xi$ ($x$), leading to

$$V(\xi) - V(\xi = 0) = -U_T \cdot ln(1 - \xi). \tag{21}$$

But $V(\xi = 0) = V_S = 0$ and hence

$$V(\xi) = -U_T \cdot ln(1 - \xi) = -U_T \cdot ln\left(1 - \frac{x}{L}\right). \tag{22}$$

Now,

$$\frac{x}{L} = \frac{L_1}{L_1 + L_2} = \frac{\beta_2}{\beta_1 + \beta_2}, \tag{23}$$

and therefore

$$V_b = V(x = L_1) = -U_T \cdot ln\left(1 - \frac{L_1}{L_1 + L_2}\right) = -U_T \cdot ln\left(\frac{L_2}{L_1 + L_2}\right) = U_T \cdot ln\left(1 + \frac{L_1}{L_2}\right) = U_T \cdot ln\left(1 + \frac{\beta_2}{\beta_1}\right), \tag{24}$$

which is identical to (8). Therefore, in weak inversion, the voltage $V_b$ at the intermediate node between $M_1$ and $M_2$ is equal to the channel voltage $V$ at position $x = L_1$.

# Problem 2 The Vittoz current reference [1]

## 2.1 Weak inversion

Both transistors $M_1$ and $M_3$ share the same gate and have therefore the same gate voltage $V_{G1} = V_{G3} = V_G$. They have therefore also the same pinch-off voltage $V_{P1} = V_{P3} = V_P$ and the same slope factor $n_1 = n_3 = n$. Assuming that transistors $M_1$ and $M_3$ are biased in weak inversion and in saturation, we have

$$I_{D1} = I_{spec1} \cdot e^{\frac{V_P}{U_T}}, \tag{25a}$$

$$I_{D3} = I_{spec3} \cdot e^{\frac{V_P - V_R}{U_T}}, \tag{25b}$$

where $I_{spec1} \triangleq 2n\beta_1 U_T^2$ and $I_{spec3} \triangleq 2n\beta_3 U_T^2$ with $\beta_3 = K \cdot \beta_1$. Assuming a perfect matching between transistors $M_1$ and $M_3$ and between $M_2$ and $M_4$, we have $I_R = I_1 = I_3$, resulting in

$$e^{\frac{V_R}{U_T}} = \frac{I_{spec3}}{I_{spec1}} = \frac{\beta_3}{\beta_1} = K. \tag{26}$$

and hence

$$V_R = U_T \cdot \ln(K) \tag{27}$$

and

$$I_R = I_b = \frac{U_T}{R} \cdot \ln(K). \tag{28}$$

This circuit provides a reference voltage $V_R$ proportional to absolute temperature or PTAT. The temperature dependence of the reference current $I_b$ depends on the temperature dependence of the resistance $R$. So the reference current $I_b$ is PTAT only if the resistance $R$ can be considered as temperature-independent.

## 2.2 Strong inversion

As above, both transistors $M_1$ and $M_3$ share the same gate and have therefore the same gate voltage $V_{G1} = V_{G3} = V_G$, pinch-off voltage $V_{P1} = V_{P3} = V_P$ and slope factor $n_1 = n_3 = n$. Assuming that transistors $M_1$ and $M_3$ are long-channel transistors biased in strong inversion and in saturation, we have

$$I_{D1} = I_R = \frac{n\beta_1}{2} \cdot V_P^2, \tag{29a}$$

$$I_{D3} = \frac{n\beta_3}{2} \cdot (V_P - V_R)^2. \tag{29b}$$

Assuming a perfect matching between transistors $M_1$ and $M_3$ and between $M_2$ and $M_4$, we have $I_R = I_{D1} = I_{D3}$, resulting in

$$V_P = \sqrt{K} \cdot (V_P - V_R) \tag{30}$$

and

$$V_P = \frac{\sqrt{K}}{\sqrt{K} - 1} \cdot V_R = \frac{\sqrt{K}}{\sqrt{K} - 1} \cdot R \cdot I_R. \tag{31}$$

Introducing (31) into (29a) and solving for $I_R$ results in

$$I_R = I_b = \frac{2}{n \cdot \beta_3 \cdot R^2} \cdot (\sqrt{K} - 1)^2 \tag{32}$$

and

$$V_R = \frac{2}{n \cdot \beta_3 \cdot R} \cdot (\sqrt{K} - 1)^2. \tag{33}$$

The current reference in strong inversion is actually set by $1/(\beta_3 \cdot R)$, which is strongly technology dependent.

## 2.3 General Remark

Note that if the current $I_b$ is used to bias another n-channel transistor with factor $\beta$ and operating also in saturation with the same level of inversion than transistor $M_3$, its source transconductance $G_{ms}$ is then inversely proportional to $R$ according to

$$G_{ms} = \frac{B}{R} \tag{34}$$

where factor $B$ is given by

$$B = \begin{cases} \ln(K) & \text{in weak inversion} \\ 2 \cdot \sqrt{\frac{\beta}{\beta_3}} \cdot (\sqrt{K} - 1) & \text{in strong inversion.} \end{cases} \tag{35}$$

and is to first-order independent of the technology and of the temperature.

## Problem 3 The Oguey current reference [2]

From Problem 2 we get $V_R = U_T \cdot \ln(K)$. Since $M_7$ and $M_9$ share the same gate voltage $V_{G7} = V_{G9} = V_G$ they also have the same pinch-off voltage $V_{P7} = V_{P9} = V_P$. Assuming $M_7$-$M_9$ are biased in strong inversion with $M_7$ in the linear region and $M_9$ in saturation, we can write

$$I_b = I_{D7} = A \cdot I_{spec9} \cdot \left[ \left( \frac{V_P}{2U_T} \right)^2 - \left( \frac{V_P - V_R}{2U_T} \right)^2 \right], \tag{36a}$$

$$I_b = I_{D9} = I_{spec9} \cdot \left( \frac{V_P}{2U_T} \right)^2. \tag{36b}$$

Solving (36a) and (36b) for $I_b$ results in

$$I_b = I_{spec9} \cdot \left( \frac{A \cdot \ln K}{2} \right)^2 \cdot \left( 1 + \sqrt{1 + \frac{1}{A}} \right)^2. \tag{37}$$

We see that the bias current $I_b$ is now proportional to the specific current of $M_9$. If $A$ can be made much larger than 1, (37) reduces to

$$I_b \cong I_{spec9} \cdot (A \cdot \ln K)^2 \quad \text{for } A \gg 1. \tag{38}$$

Since $I_{spec7} = A \cdot I_{spec9}$, $I_b$ is also proportional to $I_{spec7}$

$$I_b \cong I_{spec7} \cdot A \cdot (\ln K)^2 \quad \text{for } A \gg 1. \tag{39}$$

This circuit provides a current which is proportionnal to the specific current $I_{spec9}$ of $M_9$ or $I_{spec7}$ of $M_7$. This allows to set the inversion coefficient of a given transistor. Say we want to set the inversion coefficient of transistor $M_x$ to $IC_x$. We can then use a multiple $N$ of the bias current $I_b$

$$I_{Dx} = I_{specx} \cdot IC_x = N \cdot I_b = N \cdot I_{spec7} \cdot IC_7 \tag{40}$$

which leads to

$$I_{specx} = N \cdot I_{spec7} \cdot \frac{IC_7}{IC_x}. \tag{41}$$

or

$$I_{specn\square} \cdot \frac{W_x}{L_x} = N \cdot I_{specn\square} \cdot \frac{W_7}{L_7} \cdot \frac{IC_7}{IC_x}. \tag{42}$$

The aspect ratio $W/L$ of $M_x$ is then given by

$$\frac{W_x}{L_x} = N \cdot \frac{W_7}{L_7} \cdot \frac{IC_7}{IC_x}. \tag{43}$$

Choosing the aspect ratio to $W_x/L_x$ given by (43) and knowing $IC_7$ and $W_7/L_7$ will set the inversion coefficient to the desired $IC_x$.

## References

[1] E. Vittoz and J. Fellrath, “CMOS analog integrated circuits based on weak inversion operations,” *Solid-State Circuits, IEEE Journal of*, vol. 12, no. 3, pp. 224–231, June 1977.

[2] H. J. Oguey and D. Aebischer, “CMOS current reference without resistance,” *Solid-State Circuits, IEEE Journal of*, vol. 32, no. 7, pp. 1132–1135, July 1997.