Fundamentals of Analog & Mixed Signal VLSI Design

Continuous-Time Filters (CTF)

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Outline

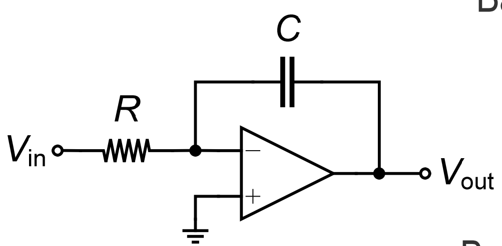
- **Introduction**
- RC-active filters
- MOSFET-C filters
- G_m-C filters
- Source-follower CTFs
- Noise in CTFs
- Automatic tuning

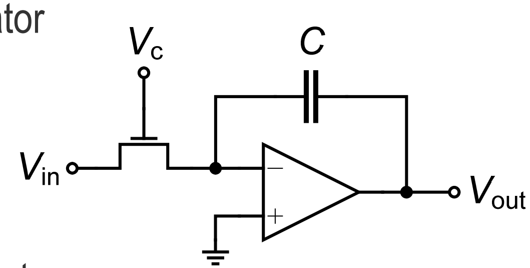
Different Types of CTFs – Active RC and MOSFET-C

Active RC

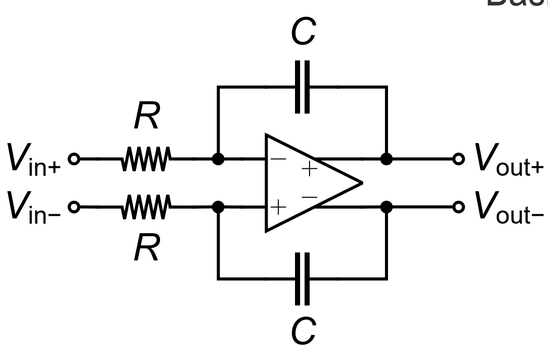
MOSFET-C

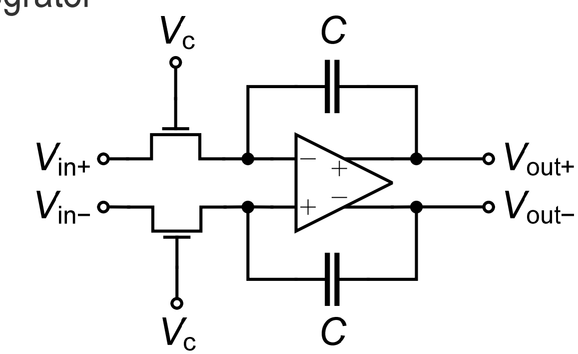
Basic single-ended integrator





Basic fully differential integrator





Active RC

\+ Good linearity
\+ Small number of active elements
\+ Insensitive to parasitics
− Must drive low impedance loads
− No continuous-time tuning mechanism

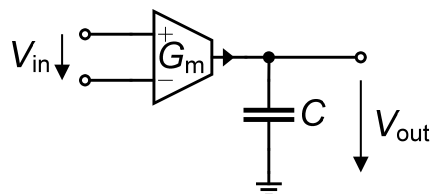
MOSFET-C

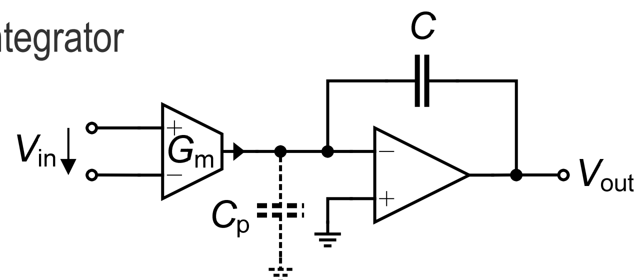
\+ Insensitive to parasitics
\+ Small number of active elements
− Must drive low impedance loads
− Nonlinearity of MOS resistance

Different Types of CTFs – G_m-C and G_m-OPAMP-C

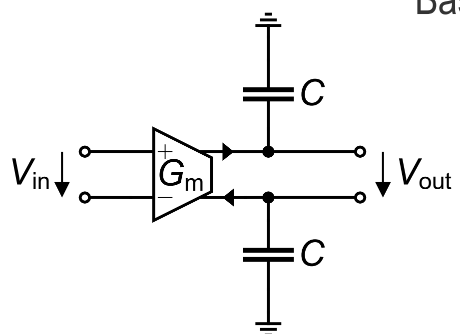
G_m-C | **G_m-OPAMP-C**

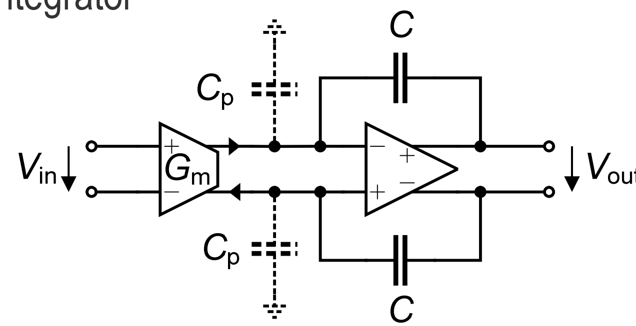
Basic single-ended integrator





Basic fully differential integrator





- \+ Fastest of all approaches
- \+ No need to drive low impedance loads
- − Sensitive to parasitic capacitances
- − Nonlinearity of transconductor

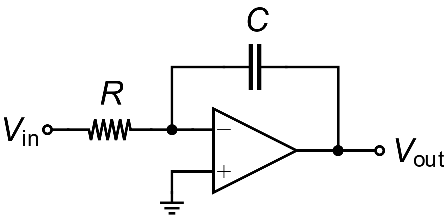
- \+ Insensitive to parasitic capacitances
- \+ No need to drive low impedance loads
- \+ Small output swing transconductors
- − Largest number of active elements
- − Additional poles → slow

Outline

- Introduction
- **RC-active filters**
- MOSFET-C filters
- G_m-C filters
- Source-follower CTFs
- Noise in CTFs
- Automatic tuning

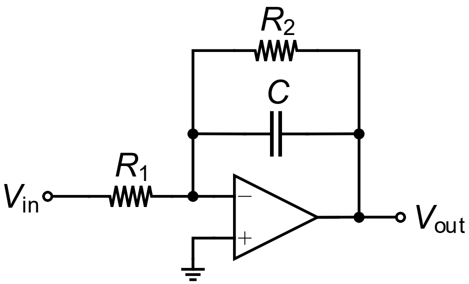
Active-RC Filters

Integrator



$$T(s) \triangleq \frac{V_{out}}{V_{in}} = -\frac{1}{s\tau} \text{ with } \tau = RC$$

1st-order low-pass filter



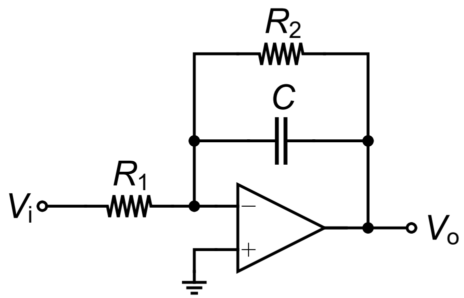
$$T(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{K}{1+\frac{s}{\omega_c}}$$

with $K = -\frac{R_2}{R_1}$ and $\omega_c = \frac{1}{R_2 C}$

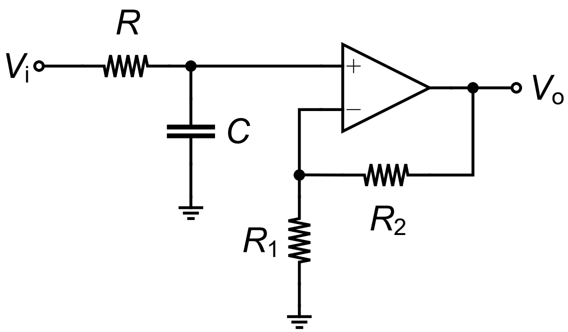
- Assuming the OPAMP gain is large enough, active-RC filters offer
  - **good linearity**
  - **low sensitivity to the parasitic input capacitance**
    (however the cut-off frequency may depend on the load capacitance)
- On the other hand they must drive low impedance loads which leads to **higher power consumption**

First-order LP Sections

Inverting



Noninverting



- The transfer function is given by

$$T(s) \triangleq \frac{V_o}{V_i} = K\,\frac{\omega_c}{s+\omega_c}$$

- with $K = -\frac{R_2}{R_1}$ and $\omega_c = \frac{1}{R_2 C}$
- Notice that the **dc gain K is negative**

- The alternative 1st-order section shown above does not introduce any inversion (**K positive**)

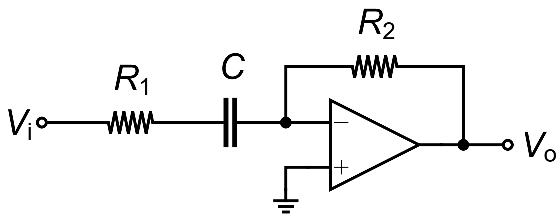
$$T(s) \triangleq \frac{V_o}{V_i} = K\,\frac{\omega_c}{s+\omega_c}$$

- with $K = 1 + \frac{R_2}{R_1}$ and $\omega_c = \frac{1}{RC}$

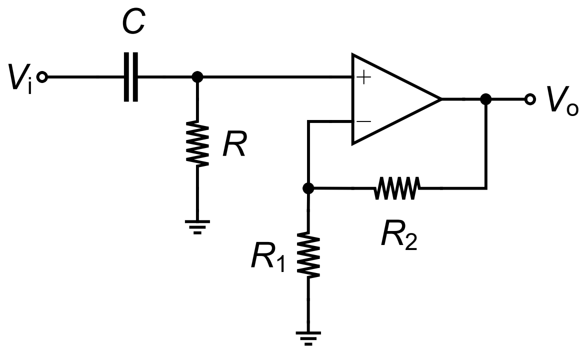
T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

First-order HP Sections

Inverting



Noninverting



- The transfer function is given by

$$T(s) \triangleq \frac{V_o}{V_i} = K \frac{s}{s + \omega_c}$$

- with $K = -\frac{R_2}{R_1}$ and $\omega_c = \frac{1}{R_1 C}$

- The alternative 1st-order section shown above does not introduce any inversion in the passband

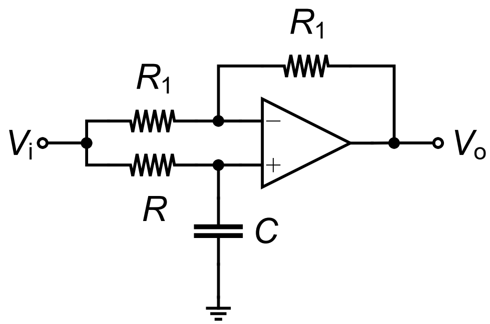
$$T(s) \triangleq \frac{V_o}{V_i} = K \frac{s}{s + \omega_c}$$

- with $K = 1 + \frac{R_2}{R_1}$ and $\omega_c = \frac{1}{RC}$

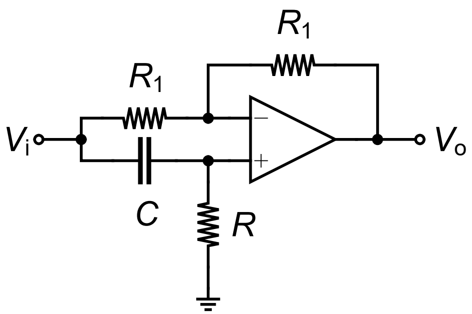
T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

First-order All-pass Sections

Inverting



Noninverting



- The transfer function is given by

$$T(s) \triangleq \frac{V_o}{V_i} = K\,\frac{s-\omega_c}{s+\omega_c}$$

- with $K = -1$ and $\omega_c = \frac{1}{RC}$

- The alternative 1st-order section shown above has the same transfer function

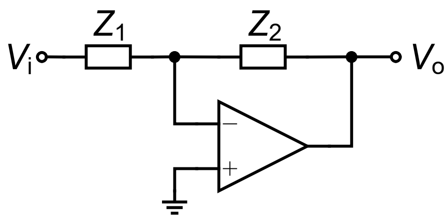
$$T(s) \triangleq \frac{V_o}{V_i} = K\,\frac{s-\omega_c}{s+\omega_c}$$

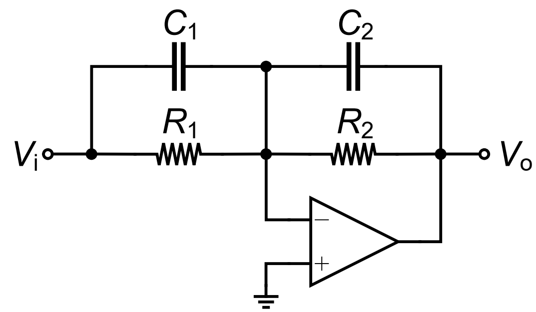
- except that now the gain is positive

$$K = +1 \text{ and } \omega_c = \frac{1}{RC}$$

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

First-order Filters Based on the Inverting Amplifier





- Other 1st-order transfer functions can be obtained from the inverting amplifier using impedances Z_1 and Z_2
- The transfer function is then given by

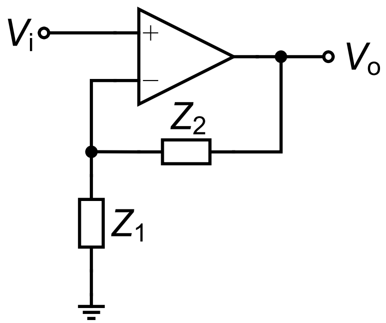
$$T(s) \triangleq \frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2}$$

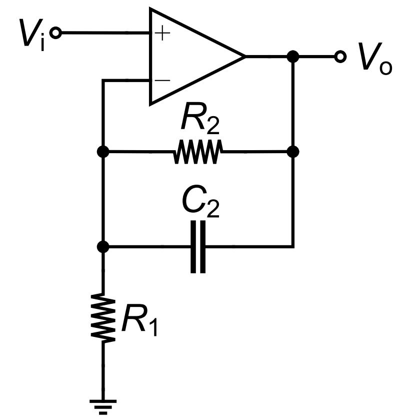
- For example replacing Z_1 and Z_2 by parallel RC leads to

$$T(s) \triangleq \frac{V_o}{V_i} = K\frac{s+\omega_z}{s+\omega_c}$$

- with $K = -\frac{C_1}{C_2}$, $\omega_z = \frac{1}{R_1 C_1}$ and $\omega_c = \frac{1}{R_2 C_2}$

First-order Filters Based on the Noninverting Amplifier





- The noninverting amplifier can be used if positive gain is need
- The transfer function is then given by

$$T(s) \triangleq \frac{V_o}{V_i} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{Y_1}{Y_2}$$

- The transfer function of the above circuit is given by

$$T(s) \triangleq \frac{V_o}{V_i} = K\,\frac{s+\omega_z}{s+\omega_c}$$

- with $K = 1 + \frac{R_2}{R_1}$, $\omega_z = \frac{1}{C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ and $\omega_c = \frac{1}{R_2 C_2}$

The General Second-order Filter Function

- The second-order filter function, in its general form, is the following

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

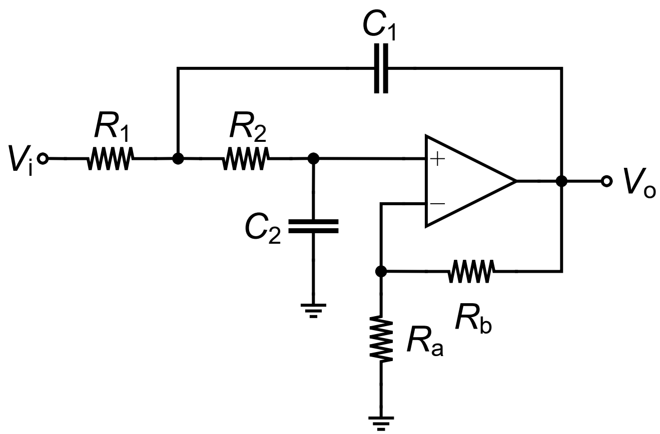
- Realization of this function using active RC networks is of interest only in the case that $\sqrt{b_0} > 0.5\, b_1$, i.e. when the poles of $T(s)$ are **complex conjugate**
- $T(s)$ can be written alternatively in the general "**biquadratic**" (or **biquad**) form as

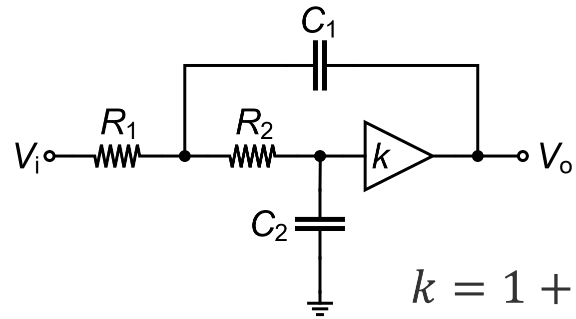
$$T(s) = K \frac{s^2 + \frac{\omega_z}{Q_z} s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

- where ω_z and ω_p are the undamped **natural** (or **resonance**) **frequencies** of the zeros and poles respectively, while Q_z and Q_p are the corresponding **quality factors**, or Q-factors
- The zero or pole frequency is the magnitude of the zero or pole, respectively, while their quality is a measure of how near the corresponding zero or pole are to the jω-axis in the s-plane

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Sallen-Key Single OPAMP Second-order LP Filter





$$k = 1 + \frac{R_b}{R_a}$$

- The most popular single OPAMP 2nd-order **LP** circuit is that of **Sallen-Key** shown above, which uses a noninverting amplifier with a **single OPAMP**
- Its transfer function is given by

$$T(s) \triangleq \frac{V_o}{V_i} = \frac{k/(R_1 R_2 C_1 C_2)}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-k}{R_2 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}} = \frac{K}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

- where $k = 1 + R_b/R_a$ is the dc gain of the noninverting amplifier

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Sallen-Key Single OPAMP Second-order LP Filter

- Identifying the coefficients gives the following **design equations**

$$K = \frac{k}{R_1 R_2 C_1 C_2},\ \omega_p = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}},\ Q_p = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2(R_1+R_2)+R_1 C_1(1-k)}$$

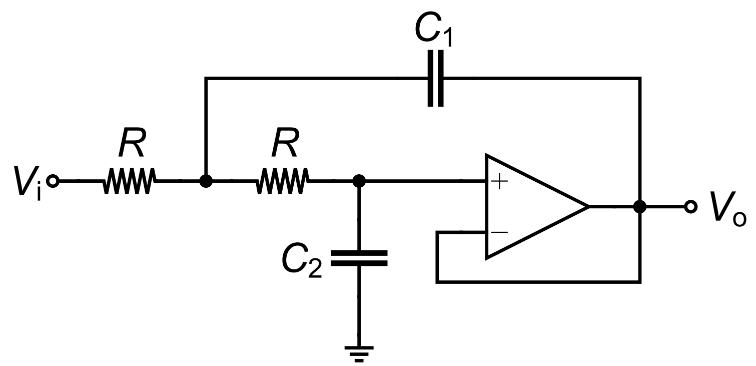
- with $k = 1 + \frac{R_b}{R_a}$
- Since the number of unknowns is larger than the number of equations, some components will have to be selected arbitrarily (or from other constraints)
- One popular choice is to choose $k = 1$ ($R_a = \infty$) which corresponds to operating the OPAMP as a **voltage follower** and hence use its **full bandwidth**
- The other resistances can then be selected equal, leading to

$$R_1 = R_2 = R,\ K = \omega_p^2 = \frac{1}{R^2 C_1 C_2},\ \omega_p = \frac{1}{R\sqrt{C_1 C_2}},\ Q_p = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

- The dc gain K/ω_p^2 is restricted to unity which is usually not a problem for filter design

📖 T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Sallen-Key Single OPAMP Second-order LP Filter



- The circuit simplifies to the one shown above and the capacitances are given by

$$C_1 = \frac{2Q_p}{R\omega_p} \text{ and } C_2 = \frac{1}{2R\omega_p Q_p}$$

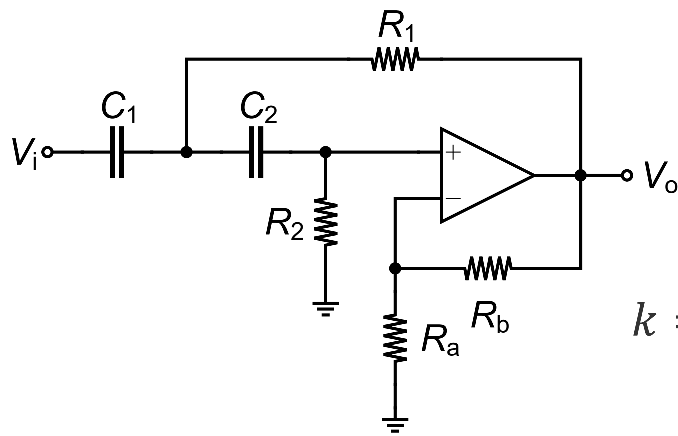
- The capacitance ratio can be calculated as

$$\frac{C_1}{C_2} = 4Q_p^2$$

- implying that for high Q values, the capacitors value spread can be undesirably high, presenting difficulties in the implementation
- This limits the usefulness of this design to **low Q filters**

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Sallen-Key Single OPAMP Second-order HP Filter



$$k = 1 + \frac{R_b}{R_a}$$

- Applying the LP-to-HP transformation we get the 2nd-order HP filter having a transfer function given by

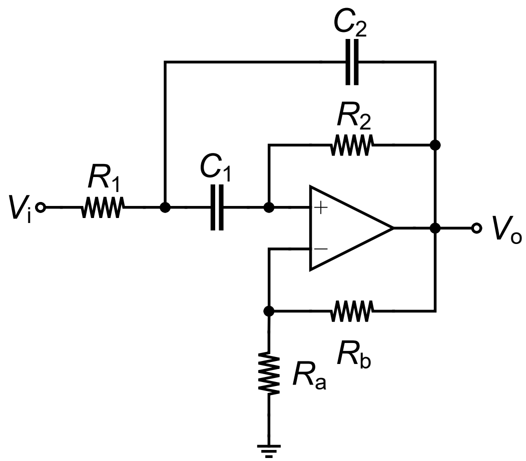
$$T(s) \triangleq \frac{V_o}{V_i} = \frac{ks^2}{s^2 + \left(\frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} + \frac{1-k}{R_1 C_1}\right) s + \frac{1}{R_1 R_2 C_1 C_2}} = \frac{ks^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

- where $k = 1 + R_b / R_a$ is the dc gain of the noninverting amplifier
- The corresponding design equations are given below

$$\omega_p = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \text{ and } Q_p = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2) + R_2 C_2 (1-k)}$$

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Deliyannis Single OPAMP Second-order BP Filter



- A BP Sallen-Key 2nd-order filter is shown above with the following transfer function

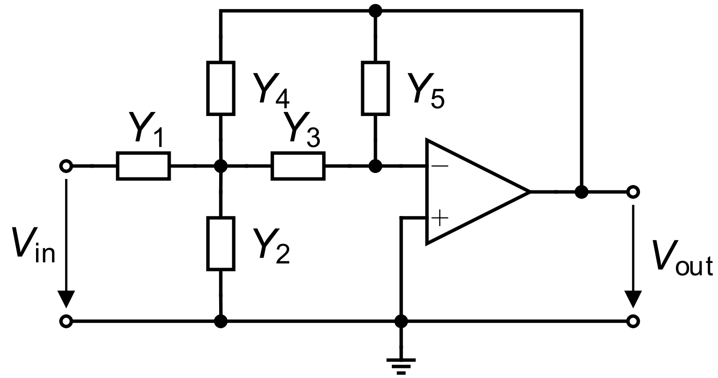
$$T(s) \triangleq \frac{V_o}{V_i} = \frac{\frac{1+K}{R_1 C_2} s}{s^2 + \left(\frac{C_1 + C_2}{R_2 C_1 C_2} - \frac{K}{R_1 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}} = \frac{h\ s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

- where $K = R_a/R_b$ and $h = (1+K)/(R_1 C_2)$
- The corresponding design equations are given below

$$\omega_p = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \text{ and } Q_p = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2) - R_2 C_1 K}$$

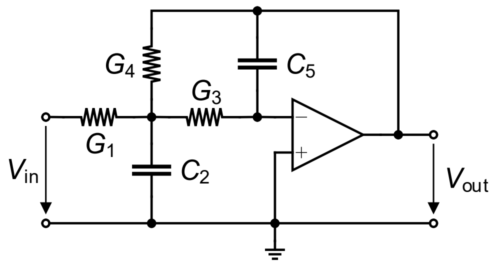
📖 H. G. Dimopoulos, *Analog Electronic Filters*, Springer, 2012.
📖 T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

Multiple Feedback (MF) Single OPAMP Biquads



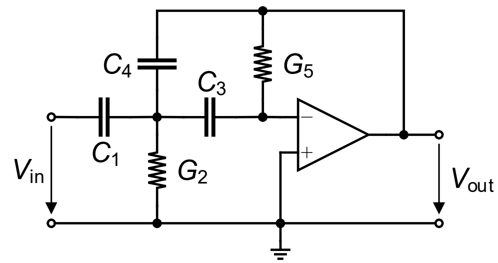
$$T(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{-Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

Low-pass



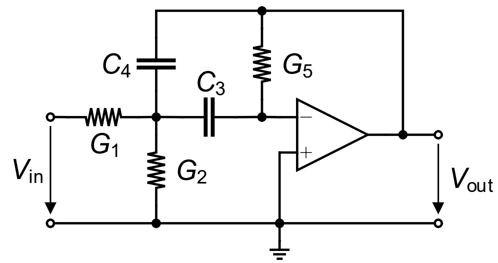
$$\frac{-G_1 G_3}{C_2 C_5 \cdot s^2 + C_5(G_1 + G_3 + G_4) \cdot s + G_3 G_4}$$

High-pass



$$\frac{-C_1 C_3 \cdot s^2}{C_3 C_4 \cdot s^2 + G_5(C_1 + C_3 + C_4) \cdot s + G_2 G_5}$$

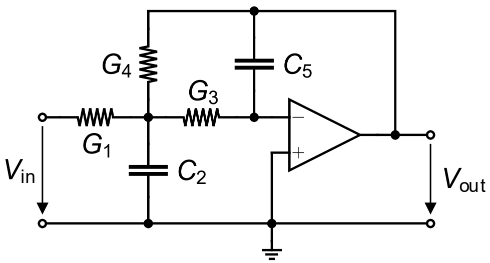
Band-pass



$$\frac{-G_1 C_3 \cdot s}{C_3 C_4 \cdot s^2 + G_5(C_3 + C_4) \cdot s + G_5(G_1 + G_2)}$$

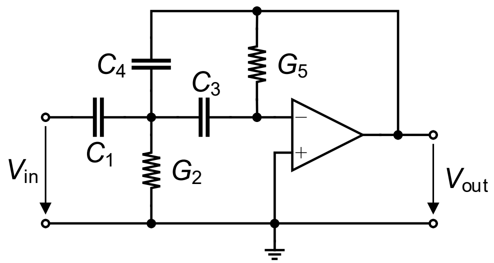
Multiple Feedback (MF) Single OPAMP Biquads

Low-pass



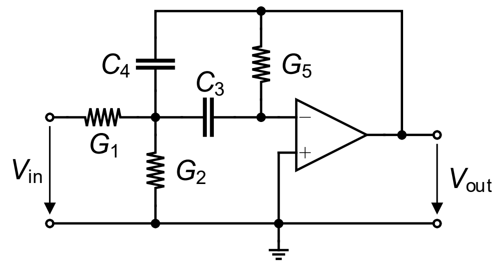
$$H(s) = \frac{K}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

High-pass



$$H(s) = \frac{K \cdot s^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

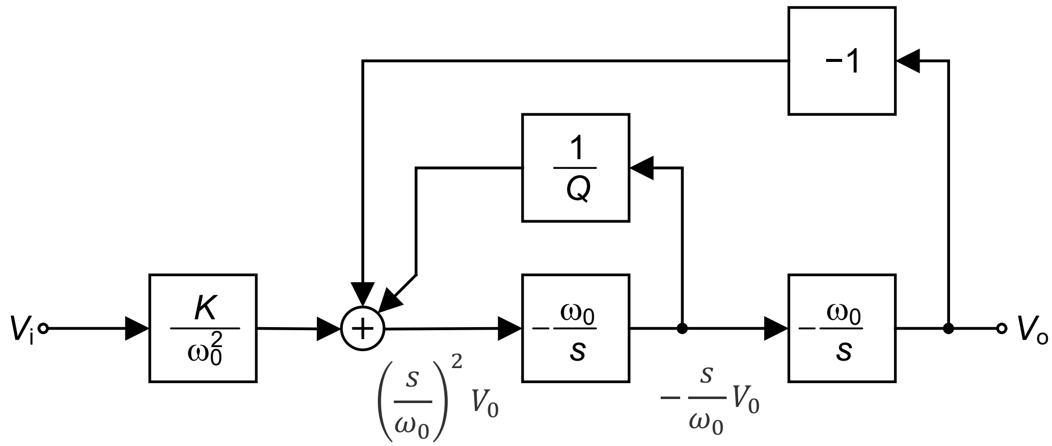
Band-pass



$$H(s) = \frac{K \cdot s}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

Elements	Low-pass	High-pass	Band-pass
Y_1	$G_1 = K/\omega_p$	$C_1 = K$	$G_1 = K$
Y_2	$C_2 = \frac{Q_p\left(2\omega_p^2 + K\right)}{\omega_p^2}$	$G_2 = \omega_p(2+K)Q_p$	$G_2 = 2\omega_p Q_p - K$
Y_3	$G_3 = \omega_p$	$C_3 = 1$	$C_3 = 1$
Y_4	$G_4 = G_3$	$C_4 = C_3$	$C_4 = C_3$
Y_5	$C_5 = \frac{\omega_p^2}{Q_p\left(2\omega_p^2 + K\right)}$	$G_5 = \frac{\omega_p}{Q_p(2+K)}$	$G_5 = \frac{\omega_p}{2Q_p}$

Block Diagram of a 2nd-order LP Transfer Function



- The 2nd-order LP transfer function

$$T_{LP}(s) \triangleq \frac{V_o}{V_i} = \frac{K}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} = \frac{\frac{K}{\omega_0^2}}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\frac{s}{\omega_0} + 1}$$

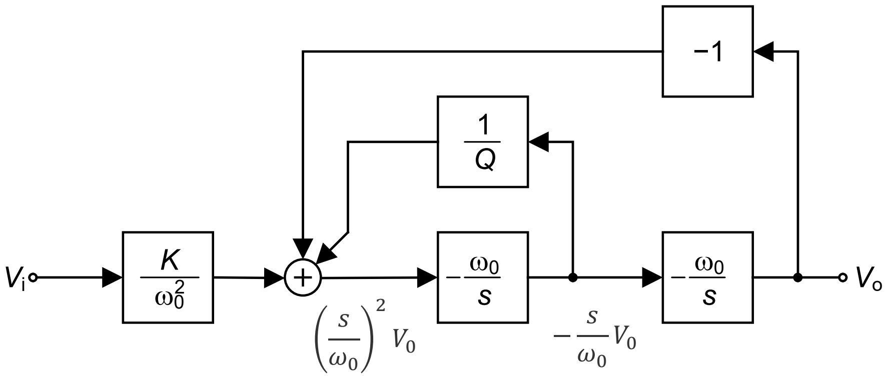
- can be rewritten as

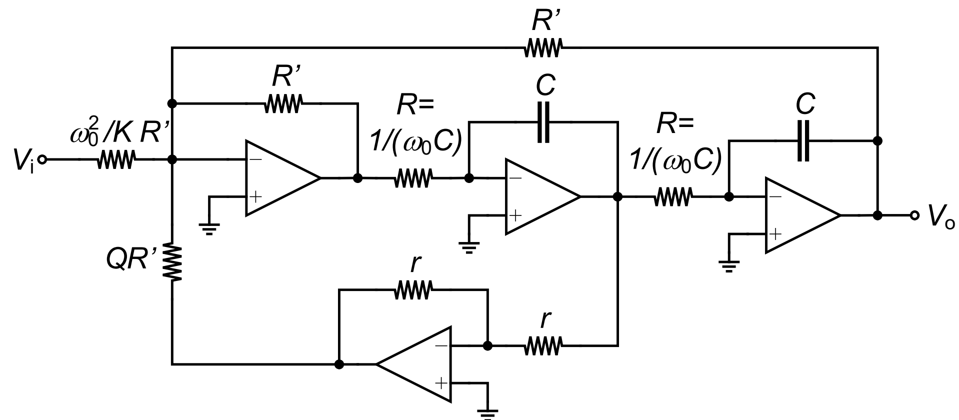
$$\left(\frac{s}{\omega_0}\right)^2 V_o = \frac{K}{\omega_0^2} V_i - \frac{1}{Q}\frac{s}{\omega_0} V_o - V_o$$

- which can be implemented with the block diagram shown above
- This block diagram can easily be implemented with OPAMPS

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

From Block Diagram to Circuit Implementation

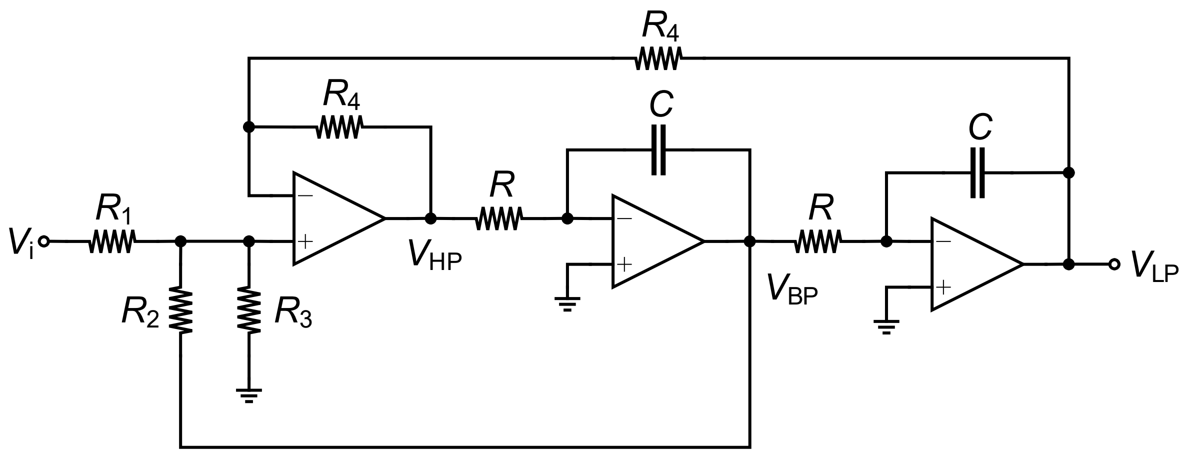




- Using OPAMPs to perform the summing and integration operations, we obtain the circuit shown on the right
- Straightforward analysis of this circuit shows that the function is realized with a minus sign
- To avoid this, and also the use of a fourth amplifier, we make use of the OPAMP difference circuit leading to the KHN (Kerwin, Huelsman, Newcomb) biquad shown in the next slide

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Kerwin, Huelsman, Newcomb (KHN) Biquad



- The KHN offers the 3 different transfer functions LP, BP and HP

$$T_{LP}(s) \triangleq \frac{V_{LP}}{V_i} = K' \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

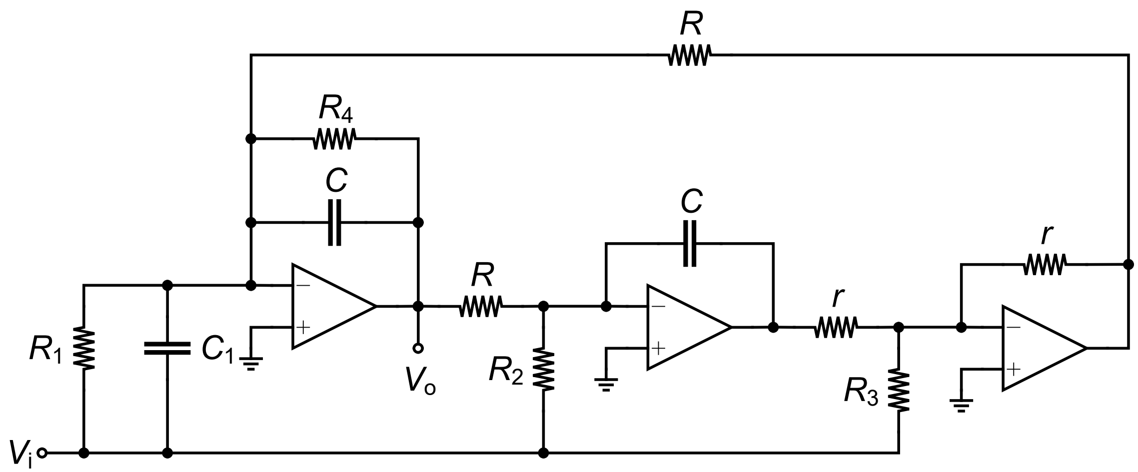
$$T_{BP}(s) \triangleq \frac{V_{BP}}{V_i} = K' \frac{\omega_0 s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$T_{HP}(s) \triangleq \frac{V_{HP}}{V_i} = K' \frac{s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

- with $\omega_0 = \frac{1}{RC}$, $K' = \frac{2R_2R_3}{R_1R_2+R_1R_3+R_2R_3}$ and $Q = \frac{1}{2}\left(1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}\right)$

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Generalized Tow-Thomas Biquad



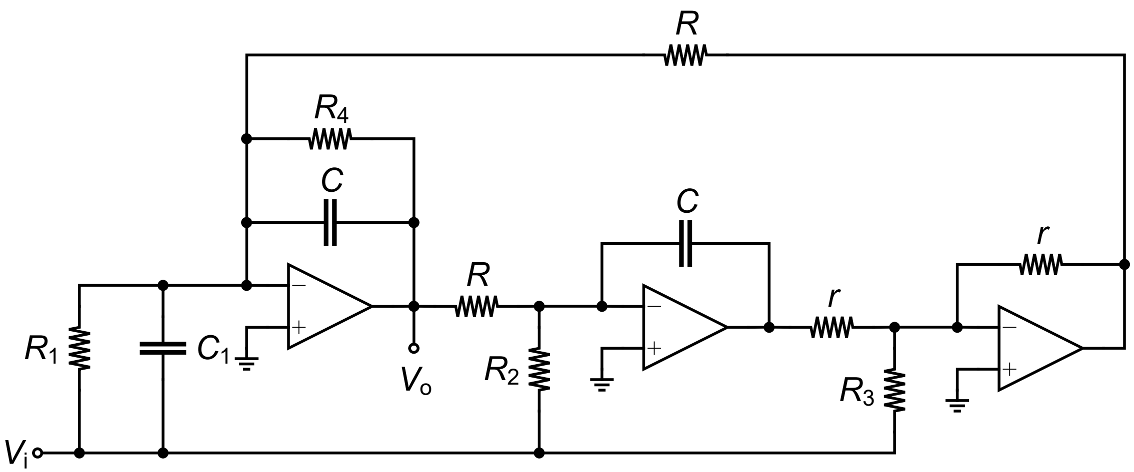
- A more versatile 3 OPAMPs biquad is the generalized Tow-Thomas shown above
- The transfer function is given by

$$T(s) \triangleq \frac{V_o}{V_i} = -\frac{\frac{C_1}{C}s^2 + \frac{1}{RC}\left(\frac{R}{R_1} - \frac{r}{R_3}\right)s + \frac{1}{RR_2C^2}}{s^2 + \frac{s}{R_4 C} + \frac{1}{(RC)^2}}$$

- Clearly, it is possible to obtain **any kind of second-order filter function** by a proper choice of the component values
- The choice is summarized in the table on the next slide

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

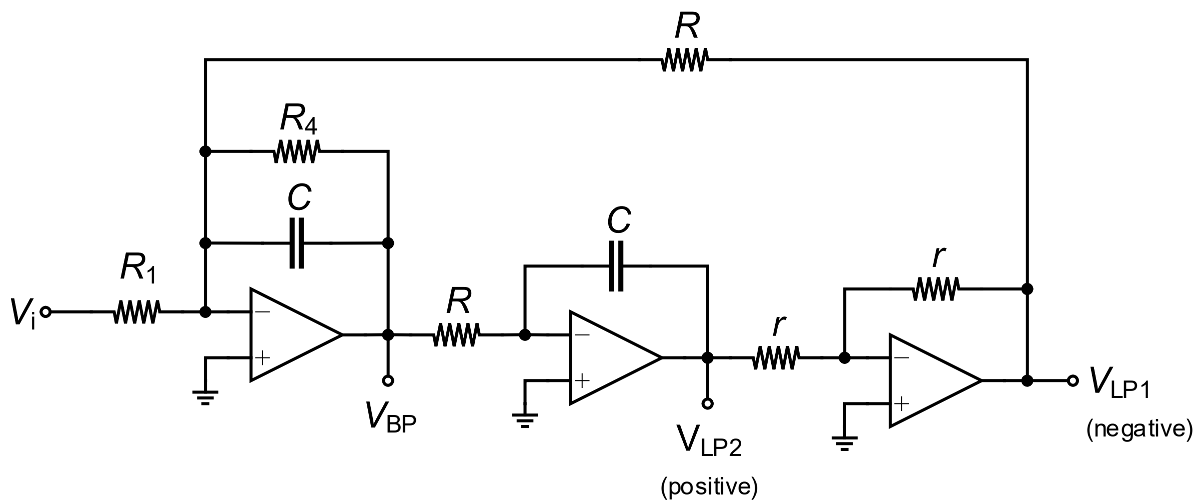
The Generalized Tow-Thomas Biquad



Type	Conditions	Comments
LP	$C_1 = 0 \qquad R_1 = R_3 = \infty$	
BP	$C_1 = 0 \qquad R_1 = R_2 = \infty$	Positive sign
BP	$C_1 = 0 \qquad R_2 = R_3 = \infty$	Negative sign
HP	$C_1 = C \quad R_1 = R_2 = R_3 = \infty$	
Notch	$C_1 = C \qquad R_1 = R_3 = \infty$	
Allpass	$C_1 = C \quad R_1 = \infty, r = R_3/Q$	

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The LP and BP Tow-Thomas Biquad



- The particular **LP** and **BP** cases of the Tow-Thomas biquad are shown above

$$T_{BP}(s) \triangleq \frac{V_{BP}}{V_i} = K\frac{\omega_0 s}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \text{ and } T_{LP}(s) \triangleq \frac{V_{LP1}}{V_i} = \frac{K}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

- Where $\omega_0 = \frac{1}{RC}$, $Q = \frac{R_4}{R}$ and $K = -\frac{R}{R_1}$
- We see that R_4 sets the Q-factor **independently** of the center frequency ω_0 and R_1 sets the gain in the passband

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

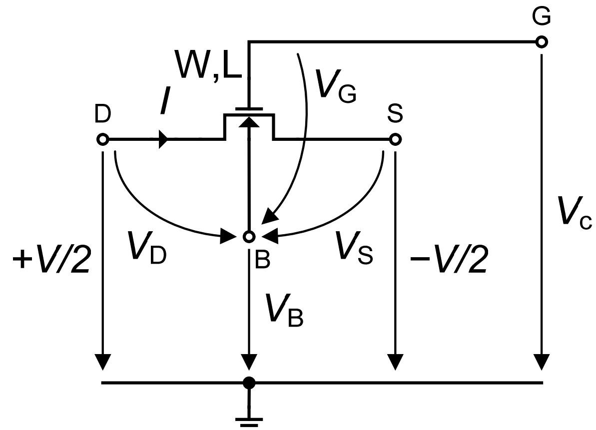
Outline

- Introduction
- RC-active filters
- **MOSFET-C filters**
- G_m-C filters
- Source-follower CTFs
- Noise in CTFs
- Automatic tuning

MOSFET-C Filters – Linear MOSFET Resistors

- MOSFET-C filters are derived from **active-RC filter** where the resistors are replaced by one or several MOS transistors
- MOSFET-C hence require linear resistors made of one or several MOS transistors usually operated in their linear range and in SI
- Linear operation is achieved by **cancellation of even-order nonlinearities** using balanced signals with differential circuits
- Remaining odd-order terms are usually very small
- Several basic circuit principle for the implementation of linear MOSFET resistors will be examined:
  - **Single** device with balanced source and drain voltages
  - **Two** devices with balanced drains and common source
  - **Four** devices (bridge)

Linear MOSFET Resistors – Circuit Principle 1



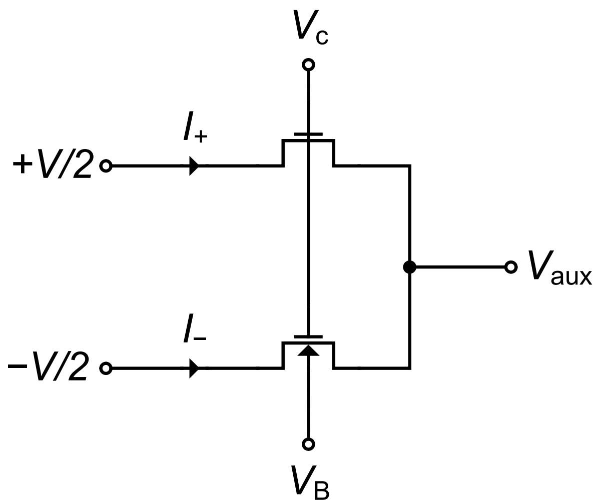
$$\beta \triangleq \mu C_{ox} \frac{W}{L}$$

- Balanced operation of a single MOST biased in linear region and in SI
- Drain current: $I = n\beta \left(V_P - \frac{V_D + V_S}{2}\right)(V_D - V_S)$ for $V_S, V_D < V_P \cong \frac{V_G - V_{T0}}{n}$
- Linear operation if $V_D - V_S = V$ and $V_D + V_S = -2V_B$ resulting in

$$I = n\beta(V_P + V_B) \cdot V = G \cdot V = V/R$$

- with $G = n\beta(V_P + V_B) \cong \beta(V_G - V_{T0} + nV_B) = \beta(V_c - V_{T0} + (n-1)V_B)$
- provided $|V| < \max\left\{2V_B; 2\frac{V_c - V_{T0} + (n-1)V_B}{n}\right\}$
- Nonlinearity cancellation relies on **precise voltage balancing**

Linear MOSFET Resistors – Circuit Principle 2



$$\beta \triangleq \mu C_{ox} \frac{W}{L}$$

- **Two** MOSFETs in differential configuration biased in the linear region and in SI and with balanced input

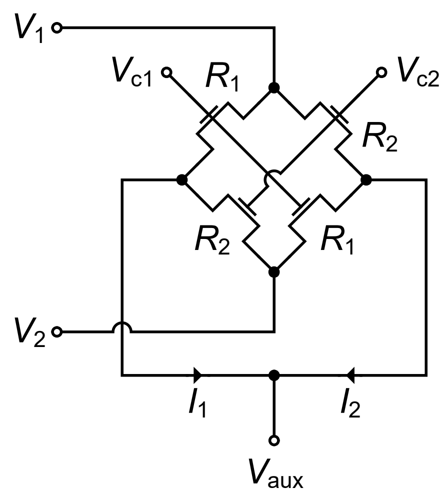
$$\Delta I \triangleq I_+ - I_- = n\beta(V_P + V_B) \cdot V = G \cdot V = V/R$$

- with $G = n\beta(V_P + V_B) \cong \beta(V_G - V_{T0} + nV_B) = \beta(V_c - V_{T0} + (n-1)V_B)$
- independently of the value of V_{aux} provided

$$|V| < \max\left\{2V_B; 2\,\frac{V_c - V_{T0} + (n-1)V_B}{n}\right\} \text{ and } V_B < V_{aux} < \frac{V_c - V_{T0} + (n-1)V_B}{n}$$

- Nonlinearity cancellation relies on proper **voltage balancing** and **device matching**

Linear MOSFET Resistors – Circuit Principle 3



- **Four** MOSFETs in bridge configuration with **all transistors identical** and biased in the linear region and in SI
- Input voltage includes a common mode component V_{ic} in addition to the differential term V with $V_1 = V_{ic} + V/2$ and $V_2 = V_{ic} - V/2$ resutling in

$$\Delta I \triangleq I_2 - I_1 = \beta(V_{c1} - V_{c2}) \cdot V = G \cdot V = V/R$$

- with $\quad G = G_1 - G_2 = \beta(V_{c1} - V_{c2})$ where $G_i = 1/R_i$ for $i = 1,2$
- Does not depend on input common mode voltage V_{ic} neither on threshold voltage
- Depends on the difference between the control voltages $V_{c1} - V_{c2}$

Linear MOSFET Resistors – Circuit Principle 3

Pros

+ Both even and odd harmonics are cancelled (assuming perfect device matching and constant mobility)
+ Input signals V_1 and V_2 do not need to be fully balanced (they can have a non zero common mode term)
+ Resistance R is controlled by a differential voltage $V_{c1} - V_{c2}$
  + R can be negative as well as positive
  + R is insensitive to V_{T0} and common mode noise

Cons

– Complexity, requires 4 matched devices and two control voltages

– Highly sensitive to mismatches ($\Delta\beta$ and ΔV_T) for $R \gg R_1, R_2$ (i.e. V_{c1} close to V_{c2})

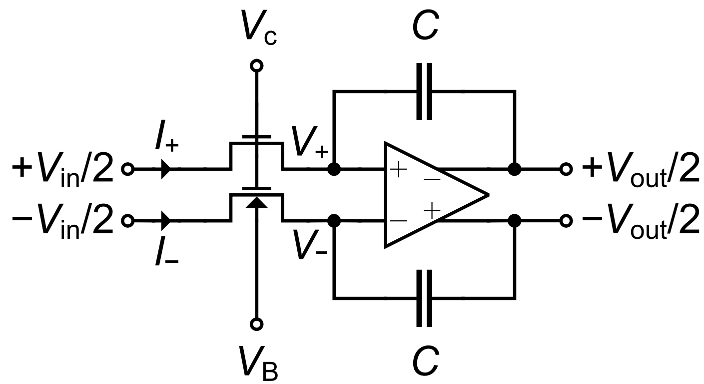
– Higher noise than for circuits 1 and 2 excess noise factor γ given by

$$\gamma \triangleq R \cdot G_n = 1 + \frac{2R}{R_2}$$

where G_n is the noise conductance of the resistances

- This is higher compared to $\gamma = 1$ for circuits 1 and 2
- Example with $R = R_2$ $(R_1 = R/2)$
  $\gamma = 3$

MOSFET-C Filters – Basic Fully Differential Integrator



- Combination of a linear MOSFET resistor using the circuit principle 2 and a fully differential OPAMP with a balanced output (incl. an output common-mode feedback)
- Assuming OPAMP with infinite input impedance and matched capacitances

$$+\frac{V_{out}}{2} = V_+ - \frac{1}{C}\int_{-\infty}^{t} I_+(\tau)d\tau \text{ and } -\frac{V_{out}}{2} = V_- + \frac{1}{C}\int_{-\infty}^{t} I_-(\tau)d\tau$$

- Assuming infinite differential gain → $V_+ = V_-$

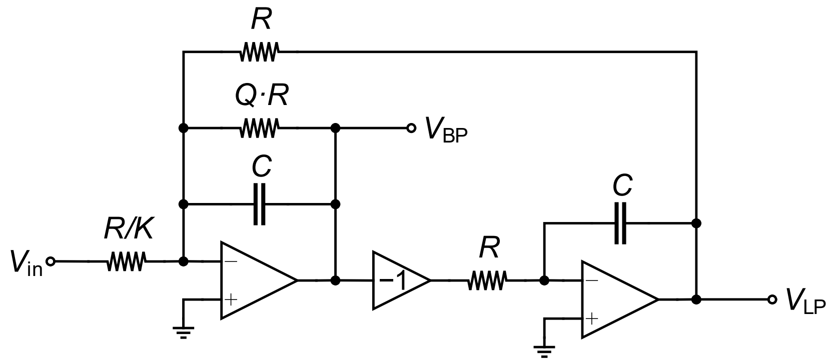
$$V_{out} = -\frac{1}{C}\int_{-\infty}^{t}\big(I_+(\tau) - I_-(\tau)\big)d\tau = -\frac{1}{RC}\int_{-\infty}^{t} V_{in}(\tau)d\tau$$

- where $R^{-1} = G = \beta(V_c - V_{T0} + (n-1)V_B)$
- Note that fully differential OPAMP requires a common-mode feedback (CMFB) circuit to set the output common mode voltage

MOSFET-C Filters – OPAMP Requirements

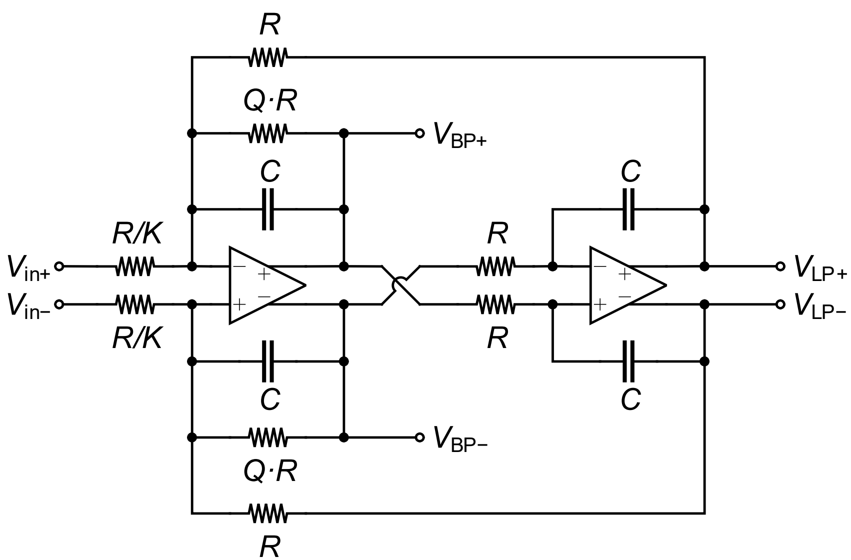
- Low output impedance ($R_{out} \ll R_{MOSFET}$)
- High gain-bandwidth product in order to achieve:
  - small errors in the filter frequency response
  - accurate nonlinearity cancellation
- Fast and accurate output CMFB to maintain precise voltage balancing over the entire frequency range where nonlinearity cancellation are important
- Modest input common-mode range and rejection capabilities (input common-mode voltage is an even nonlinear function of the input signal)

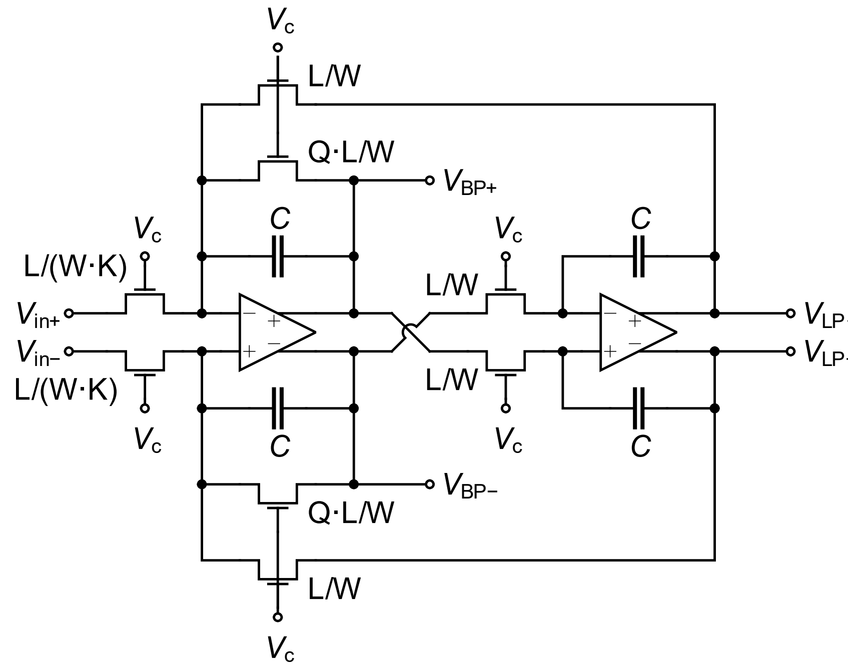
MOSFET-C Filters – The Tow-Thomas Biquad



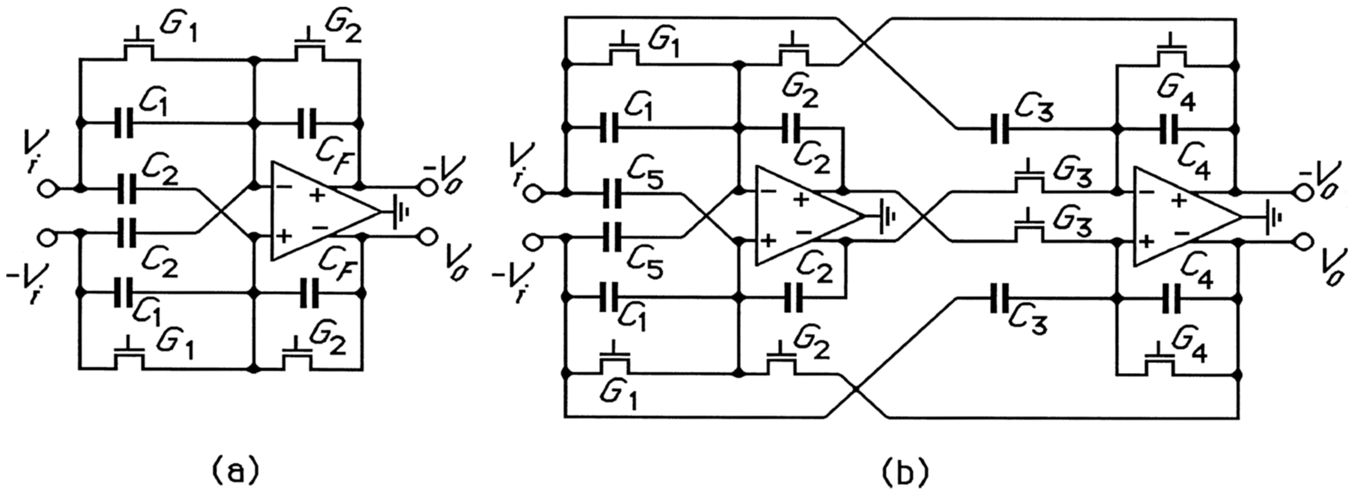
$$H_{BP}(s) \triangleq \frac{V_{BP}}{V_{in}} = K\,\frac{\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H_{LP}(s) \triangleq \frac{V_{LP}}{V_{in}} = \frac{K}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$





Generic 1st- and 2nd-order MOSFET-C Section



$$H_1(s) = \frac{s\left(C_1 - C_2\right) + G_1}{sC_F + G_2}$$

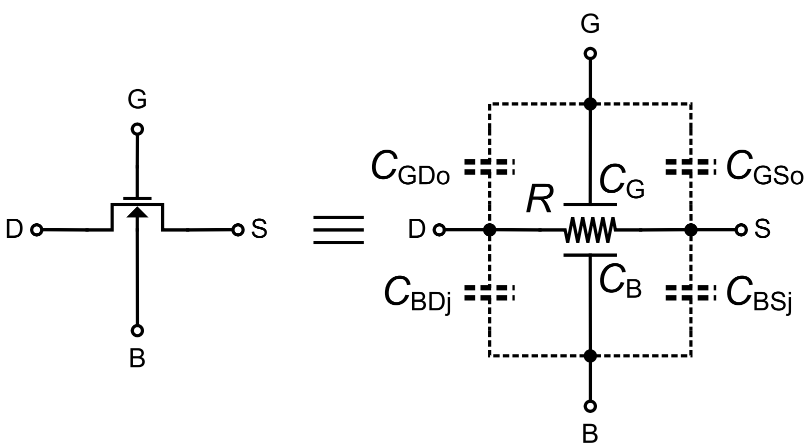
$$H_2(s) = \frac{s^2 C_2 C_3 + s\left(C_1 - C_5\right) G_3 + G_1 G_3}{s^2 C_2 C_4 + s C_2 G_4 + G_2 G_3}$$

Function	Choice of element
Bandpass	$G_1 = C_3 = 0$; $C_5 = 0$ if non-inverting, $C_1 = 0$ if inverting
Low-pass	$C_1 = C_3 = C_5 = 0$
High-pass	$C_1 = C_5 = G_1 = 0$
Notch	$C_1 = C_5$
All-pass	$(C_1 - C_5) \cdot G_3 = -C_2 \cdot G_4$

R. Schaumann, Continuous-Time Integrated Filters in The Circuits and Filters Handbook, Ed. W-K. Chen, 2nd Edition, CRC 2003

MOSFET-C Filters – Non-Ideal Effects

- Nonlinear distortions
  - Device mismatch and signal imbalance → imperfect even-order distortions cancellation
  - Departure from simple square-law model (due to mobility variation, moderate inversion effects, etc...) → odd-order distortions
- Errors in the frequency response
  - Extra phase shift in the integrators due to:
    - finite OPAMP bandwidth
    - distributed gate-to-channel and channel-to-bulk capacitances



1st-order modeling of distributed RC effect by a time constant in the V/I transfer function

$$\tau_{RC} = \frac{1}{6} \cdot R \cdot \left(C_G + C_B\right) = \frac{1}{6} \cdot R \cdot n \cdot W \cdot L \cdot C_{ox} =$$

$$= \frac{1}{6} \cdot \frac{L^2}{\mu \cdot \left(V_P + V_B\right)} \cong \frac{1}{6} \cdot \frac{n \cdot L^2}{\mu \cdot \left(V_G - V_{T0} + nV_B\right)}$$

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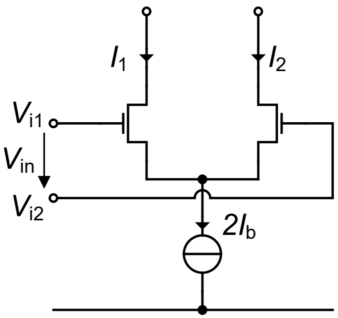
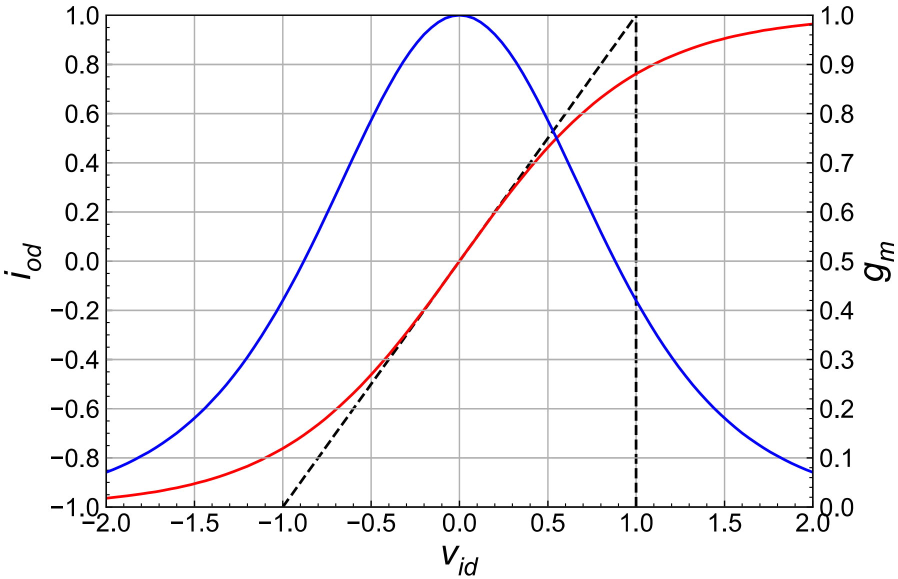
Differential Pair in Weak Inversion



$$I_{od} \triangleq I_1 - I_2$$
$$2I_b = I_1 + I_2$$
$$V_{id} \triangleq V_{i1} - V_{i2}$$
$$V_{ic} \triangleq \frac{V_{i1} + V_{i2}}{2}$$



- In **weak inversion**, the normalized differential output current is given by

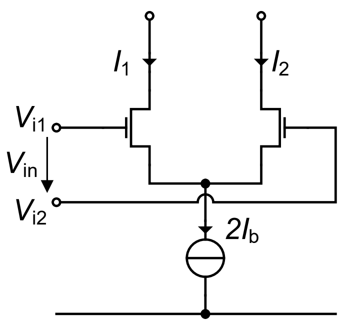
$$i_{od} \triangleq \frac{I_{od}}{2I_b} = \tanh(v_{id}) \text{ with } v_{id} \triangleq \frac{V_{id}}{2nU_T}$$

- and the normalized transconductance by

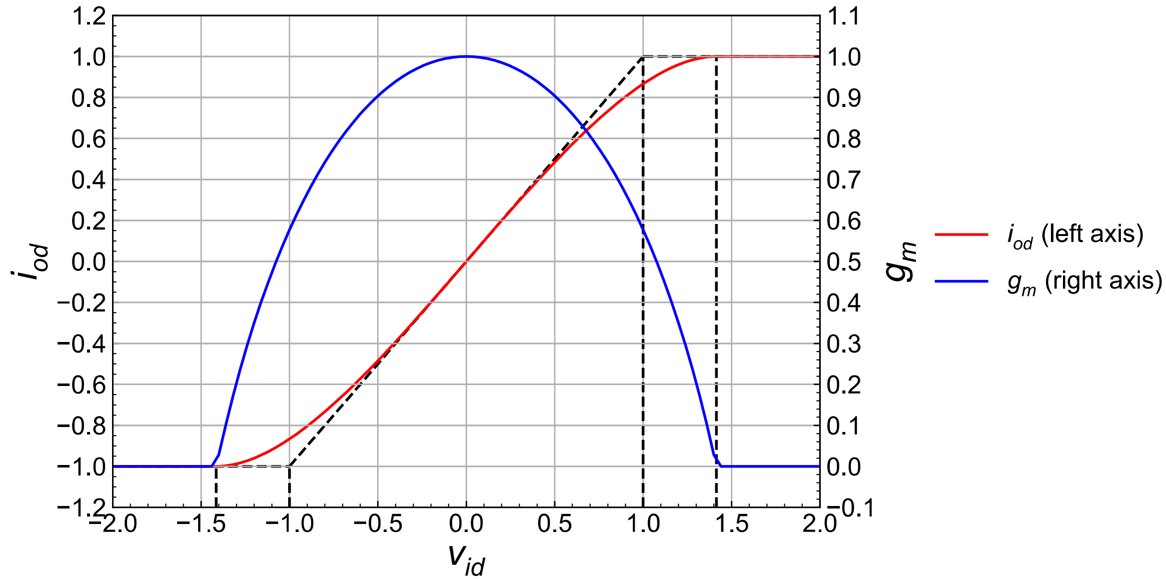
$$g_m \triangleq \frac{G_m}{G_{m0}} = 1 - \tanh^2(v_{id}) \text{ with } G_{m0} \triangleq G_m(0) = \frac{I_b}{nU_T}$$

- Although it offers the **highest current efficiency** $G_{m0}/(2I_b)$, it has the **smallest linear range** ($\cong 4nU_T \cong 155\ mV$)

Differential Pair in Strong Inversion



$$I_{od} \triangleq I_1 - I_2$$
$$2I_b = I_1 + I_2$$
$$V_{id} \triangleq V_{i1} - V_{i2}$$
$$V_{ic} \triangleq \frac{V_{i1} + V_{i2}}{2}$$



- In **strong inversion**, the normalized differential output current is given by

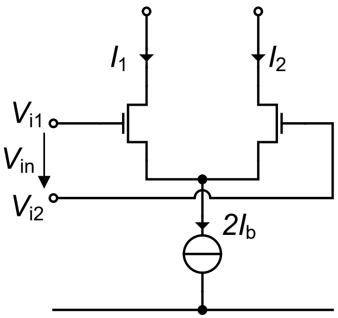
$$i_{od} \triangleq \frac{I_{od}}{2I_b} = v_{id} \cdot \sqrt{1 - \left(\frac{v_{id}}{2}\right)^2} \text{ for } |v_{id}| \le \sqrt{2} \text{ with } v_{id} \triangleq \frac{V_{id}}{n\,V_{DSsat}} = \frac{V_{id}}{n(V_p - V_S)} = \frac{V_{id}}{V_G - V_{T0} - nV_S}$$

- and the normalized transconductance by

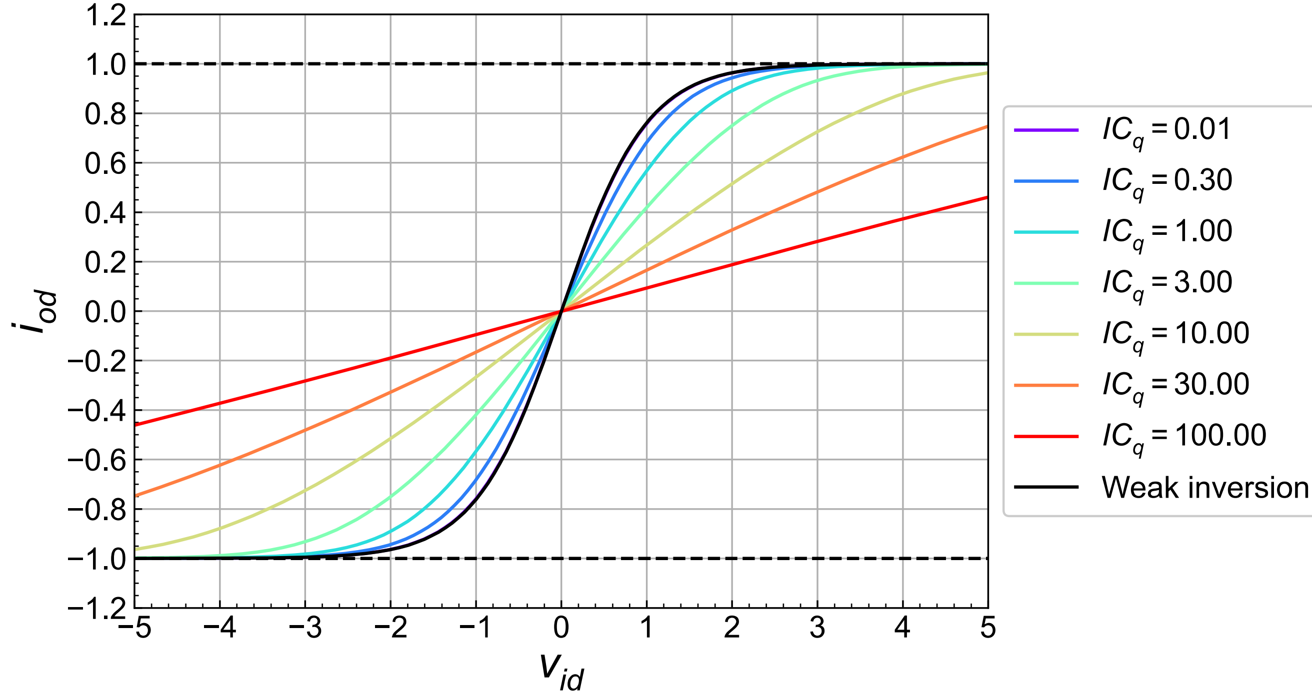
$$g_m \triangleq \frac{G_m}{G_{m0}} = \frac{2 - v_{id}^2}{\sqrt{4 - v_{id}^2}} \text{ with } G_{m0} \triangleq G_m(0) = \sqrt{\frac{2\beta I_b}{n}} = \frac{2I_b}{n\,V_{DSsat}} = \frac{2I_b}{n(V_p - V_S)} = \frac{2I_b}{V_G - V_{T0} - nV_S}$$

- The linear range ($\cong V_G - V_{T0} - nV_S$) can be extended by increasing the overdrive voltage $V_G - V_{T0}$ at the cost of a lower current efficiency

Differential Pair in All Regions of Inversion

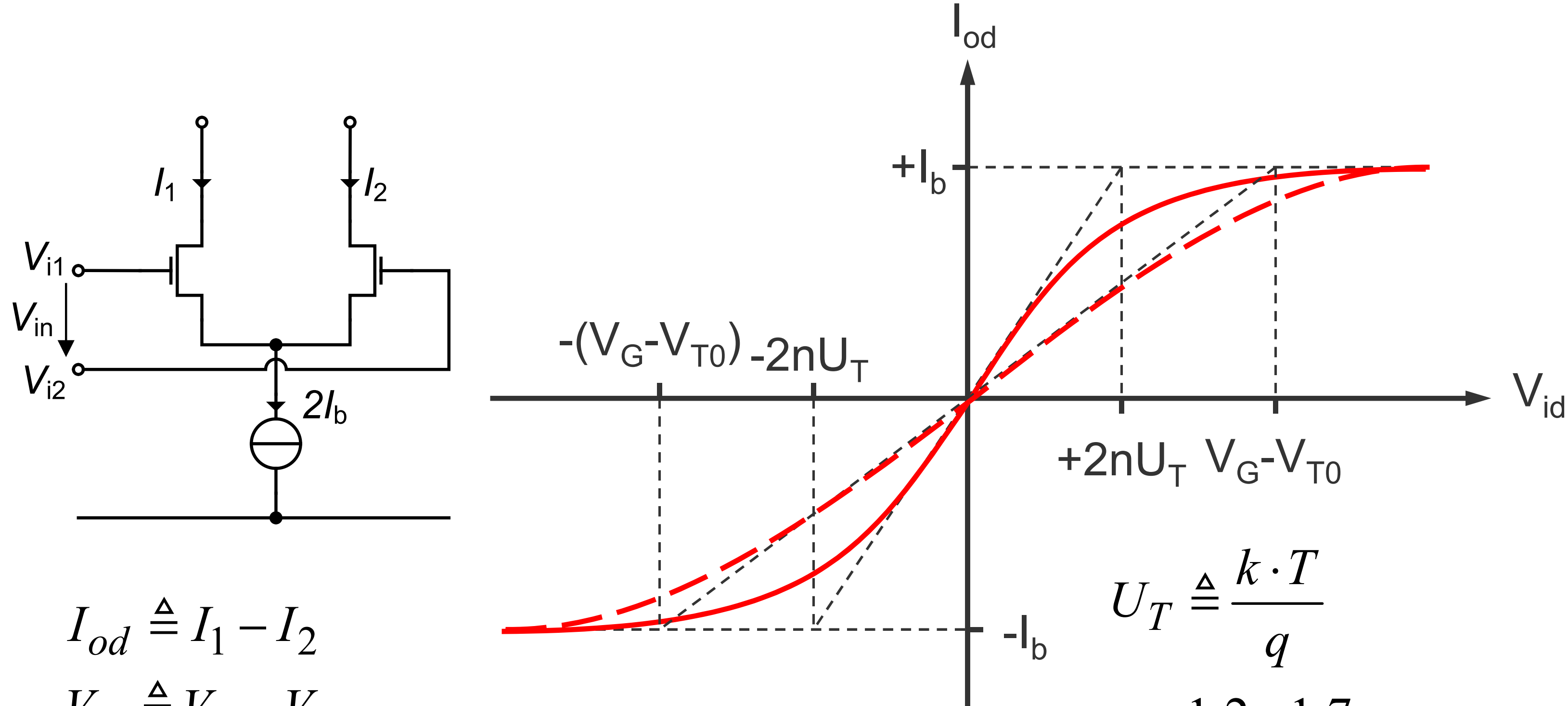


$$I_{od} \triangleq I_1 - I_2$$
$$2I_b = I_1 + I_2$$
$$V_{id} \triangleq V_{i1} - V_{i2}$$
$$V_{ic} \triangleq \frac{V_{i1} + V_{i2}}{2}$$



- The normalized differential current $i_{od} \triangleq I_{od}/(2I_b)$ can be calculated versus the differential input voltage $v_{id} \triangleq V_{id}/(2nU_T)$ in all regions of inversion defined by the quiescent inversion coefficient $IC_q = I_b/I_{spec}$
- The above plot illustrates how moving to strong inversion extends the linear range

Linearization Based on the Overdrive Voltage $V_G - V_{T0}$



$$I_{od} \triangleq I_1 - I_2$$

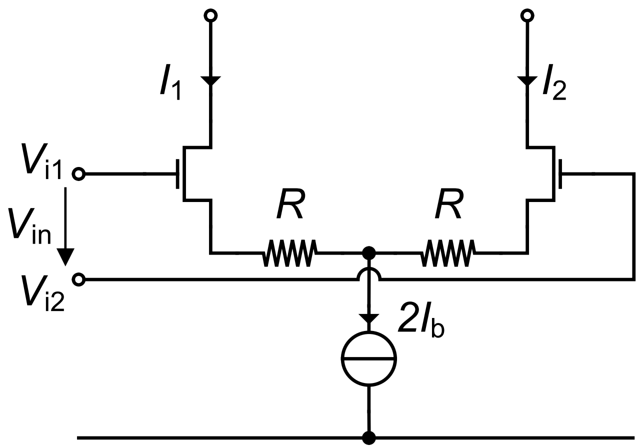
$$V_{id} \triangleq V_{i1} - V_{i2}$$

$$U_T \triangleq \frac{k \cdot T}{q}$$

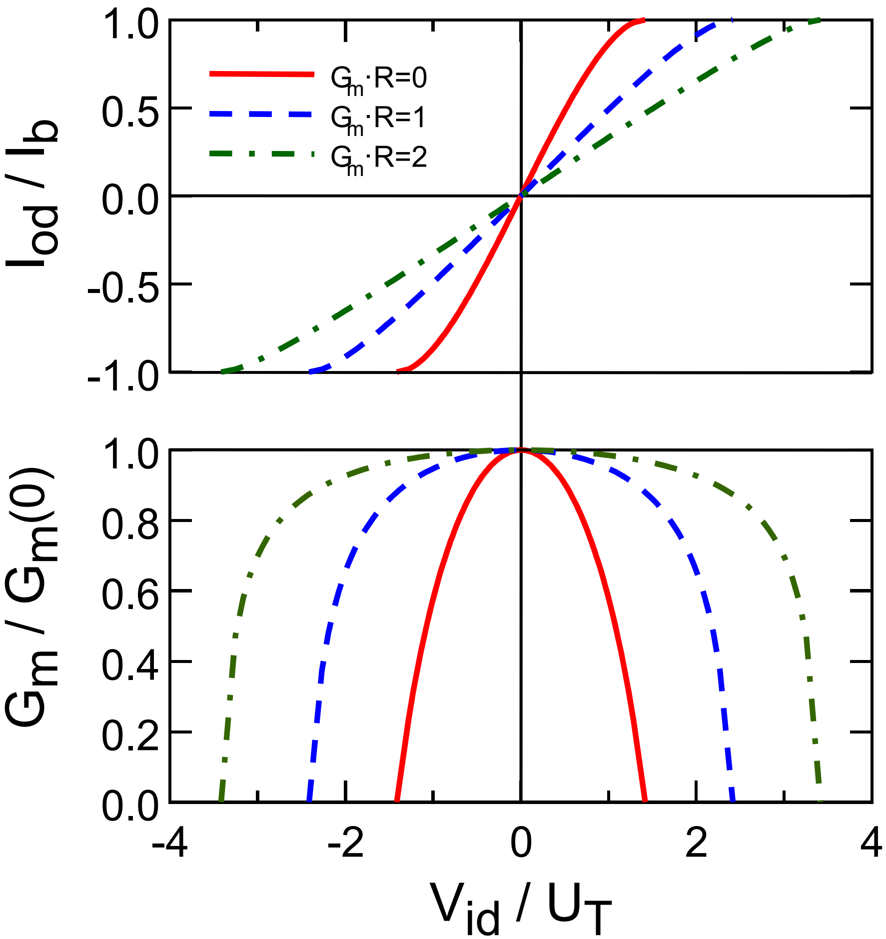
$$n \cong 1.2 \ldots 1.7$$

- Basically does not change the shape of the I-V characteristic, but scales the input voltage according to $V_G - V_{T0}$

Differential Pair Degenerated with Resistor

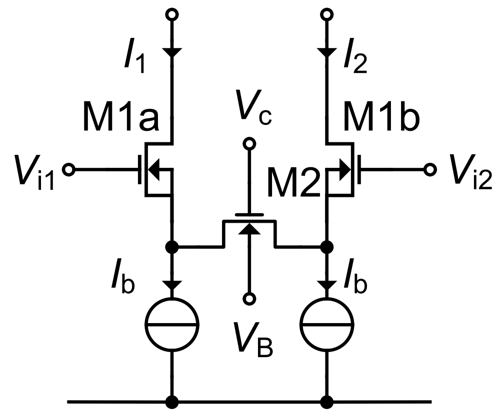


$$I_{od} \triangleq I_1 - I_2$$
$$V_{id} \triangleq V_{i1} - V_{i2}$$



- Not appropriate for low-voltage

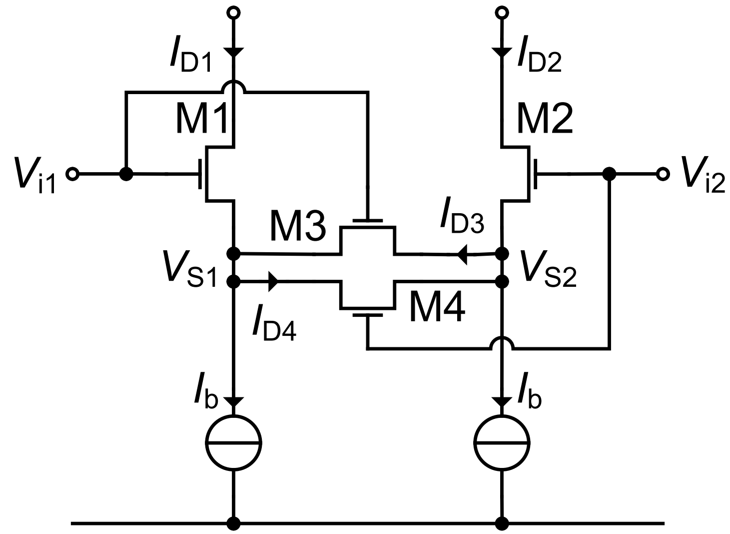
Differential Pair Degenerated with MOSFET



- Linearization based on degenerating MOS resistor using **circuit principle 1**
- Can achieve good linearity provided
  - Input voltage V_{in} is balanced
  - M1a and M1b are in separate wells connected to their own source to avoid substrate modulation ($G_{ms1} = 0$)
  - The transconductance G_{m1} of M1a and M1b is much larger than the drain-to-source conductance $G_{md2} = G_{ms2}$ of M2 ($G_{m1} \gg G_{ms2}$)

$$G_m \triangleq \frac{I_{od}}{V_{id}} = \frac{I_1 - I_2}{V_{i1} - V_{i2}} \cong 2G_{ms2} \text{ for } G_m \gg G_{ms2}$$

The Krummenacher Differential Pair – Principle



$$I_{od} \triangleq I_{D1} - I_{D2}$$
$$V_{id} \triangleq V_{i1} - V_{i2}$$

- Degenerating resistor implemented with two MOSFETs operating in the linear region and controlled by the inputs
- M1 and M2 in saturation
- For V_{id} close to 0, M3 and M4 are in the linear region
- When $V_{id} > 0$ becomes larger, M4 enters in saturation whereas M3 remains in the linear region

F. Krummenacher and N. Joel, JSSC, June 1988.

The Krummenacher Differential Pair – Strong Inversion

- M1 and M2 in saturation and M3 and M4 in the linear region for $v_{id} < v_k$

$$i_{od} = v_{id} \cdot \sqrt{1 - \frac{v_{id}^2}{4}} \text{ for } v_{id} < v_k \text{ where } v_k \triangleq \sqrt{\frac{2a^2+2a+1}{2a^4+\frac{1}{2}}} \text{ with a} \triangleq 1 + \frac{\beta_1}{4\beta_3}$$

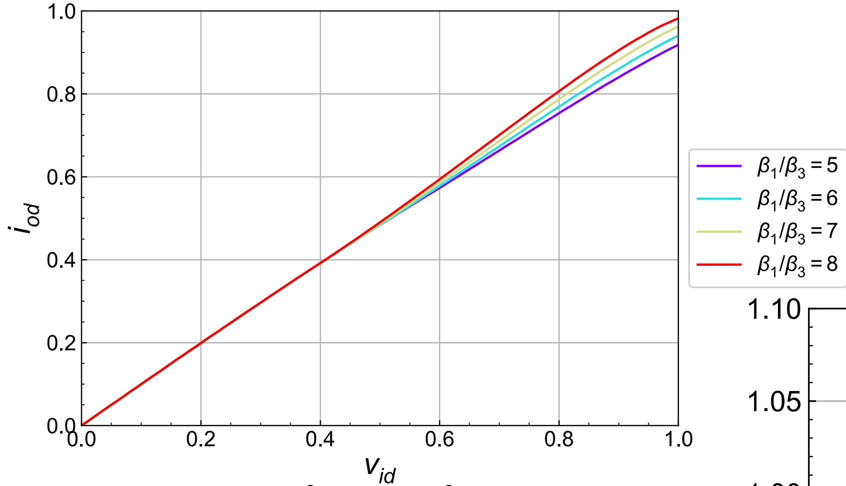
- where $i_{od} \triangleq \frac{I_{od}}{2I_b}$, $v_{id} \triangleq \frac{G_{m0}}{2I_b} \cdot V_{id}$, $G_{m0} \triangleq \frac{G_{m1}}{1+\frac{\beta_1}{4\beta_3}}$, $G_{m1} = \sqrt{\frac{2\beta_1 I_b}{n}}$
- M1, M2 and M4 in saturation, M3 in the linear region for $v_{id} \geq v_k$

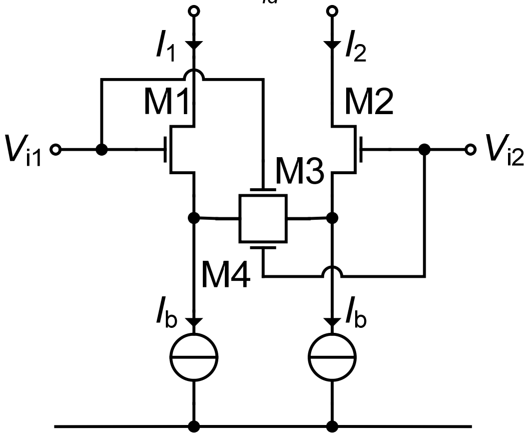
$$i_{od} = \frac{1}{b}\left[1 + (av_{id})^2\left(1 - \frac{2}{b}\right) + 2av_{id}\sqrt{1 - \frac{1+(av_{id})^2\left(1-\frac{1}{b}\right)}{b}}\right] \text{ with } b \triangleq 3 + \frac{\beta_1}{\beta_3}$$

- The transconductance G_{m0} defined at $V_{id} = 0$ is $1 + \beta_1/(4\beta_3)$ times smaller than the transconductance G_{m1} of the normal differential pair

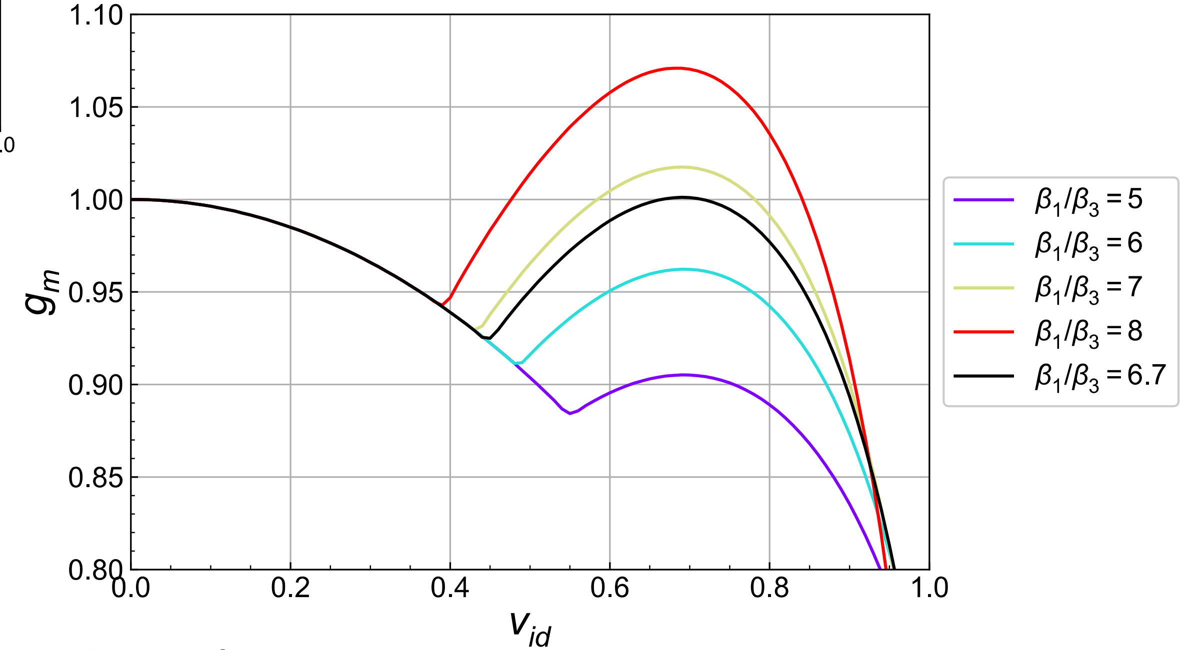
N. Joel, PhD Thesis, EPFL, 1992.

Krummenacher Differential Pair – Strong Inversion



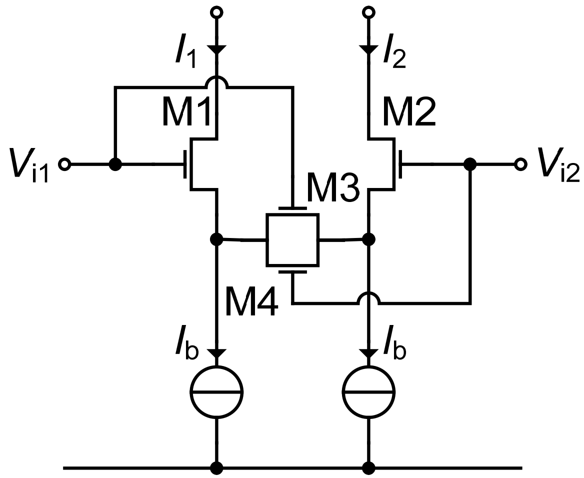


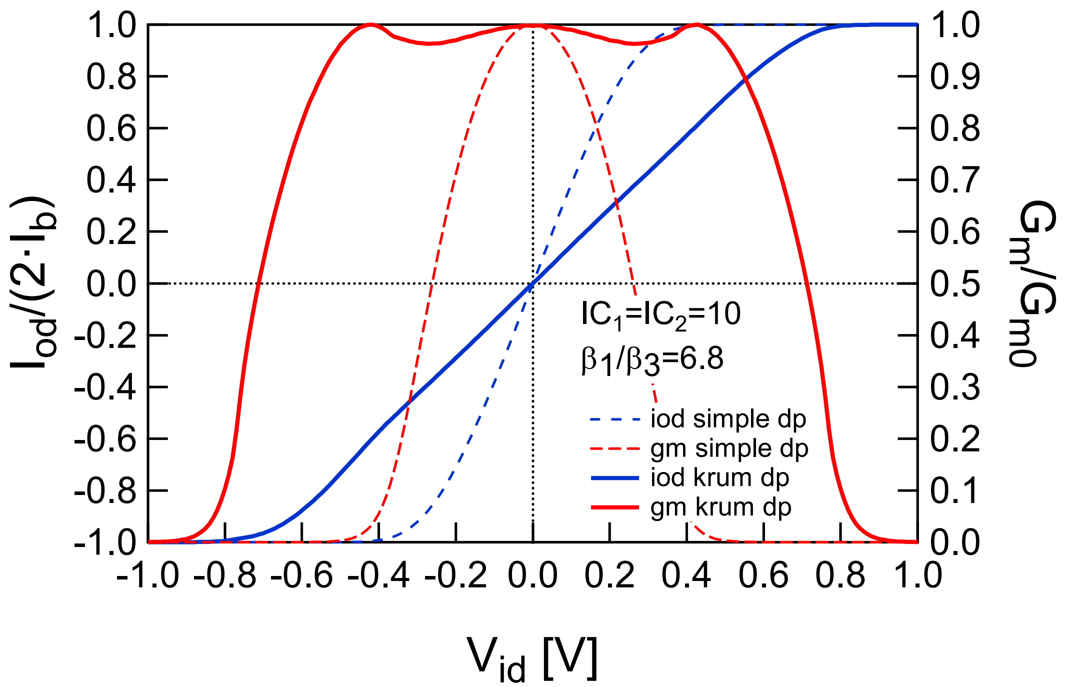
$$g_m = \frac{di_{od}}{dv_{id}} = \frac{G_m}{G_{m0}}$$



- Residual G_m ripple is minimum for $\beta_1/\beta_3 \approx 6.7$
- Linear range extended by ~2x without degradation of the current efficiency compared to normal differential pair with the same bias

The Krummenacher Differential Pair – Strong Inversion

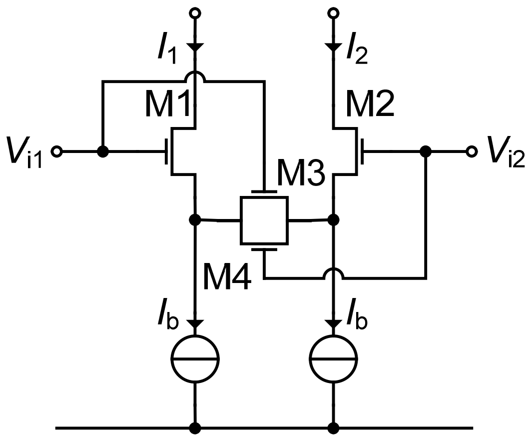




- Simulation with full EKV model for $IC = 10$
- Residual G_m ripple is minimum for $\beta_1/\beta_3 \approx 6.8$
- High current efficiency since no additional current branch
- Good noise performance

F. Krummenacher and N. Joel, JSSC, June 1988.

The Krummenacher Differential Pair – Weak Inversion



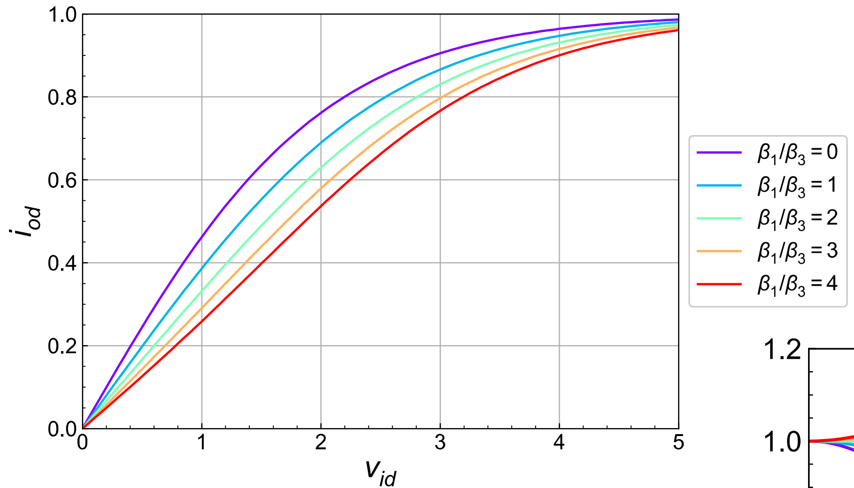
$$I_{od} \triangleq I_{D1} - I_{D2}$$
$$V_{id} \triangleq V_{i1} - V_{i2}$$

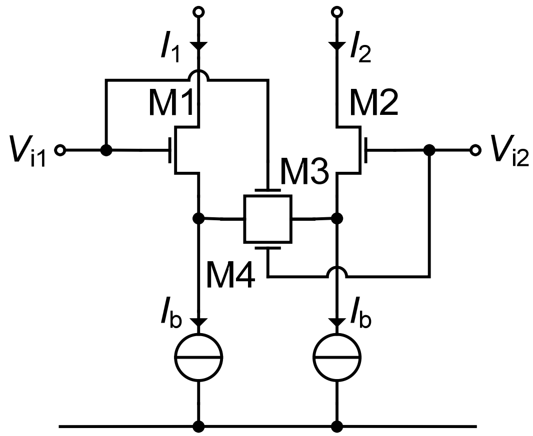
- In WI, the differential current is then given by

$$i_{od} \triangleq \frac{I_{od}}{2I_b} = \frac{e^{v_{id}} - 1}{e^{v_{id}} + 1 + K\dfrac{e^{v_{id}}}{e^{v_{id}} + 1}} = \frac{e^{2v_{id}} - 1}{e^{2v_{id}} + (2+K)e^{v_{id}} + 1}$$

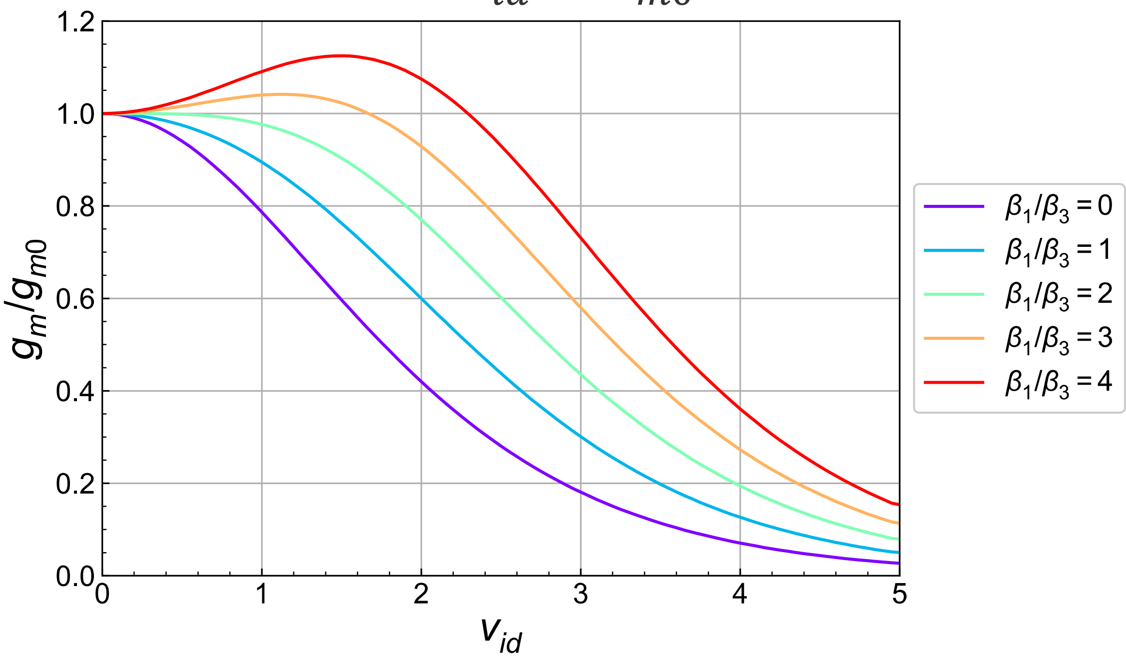
- with $K \triangleq \beta_1/\beta_3$
- and $i_{od} \triangleq \frac{I_{od}}{2I_b}$, $I_{od} \triangleq I_1 - I_2$, $v_{id} \triangleq \frac{G_{m0}}{2I_b} \cdot V_{id}$, $V_{id} \triangleq V_{i1} - V_{i2}$

Krummenacher Differential Pair – Weak Inversion



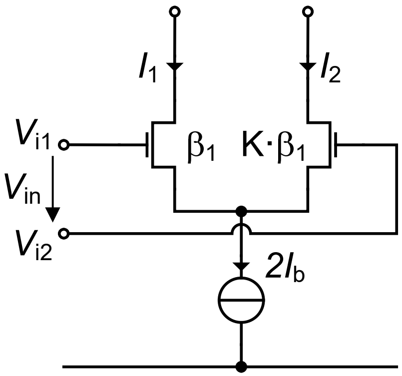


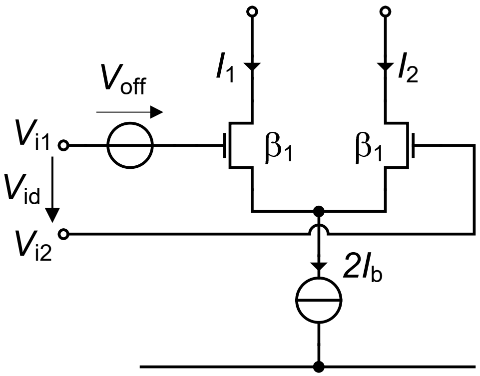
$$g_m = \frac{di_{od}}{dv_{id}} = \frac{G_m}{G_{m0}}$$

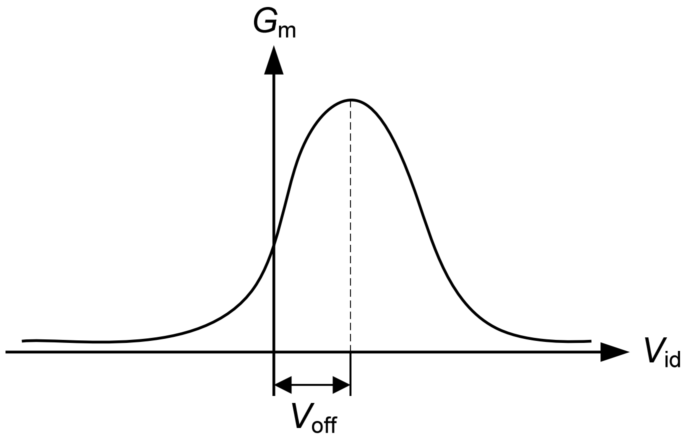


- Residual G_m ripple is minimum for $\beta_1/\beta_3 \approx 2$
- Linear range extended by ~2x

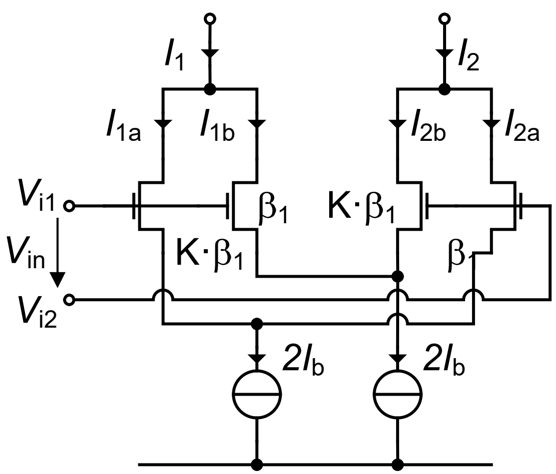
Multi-tanh Linearization – Principle

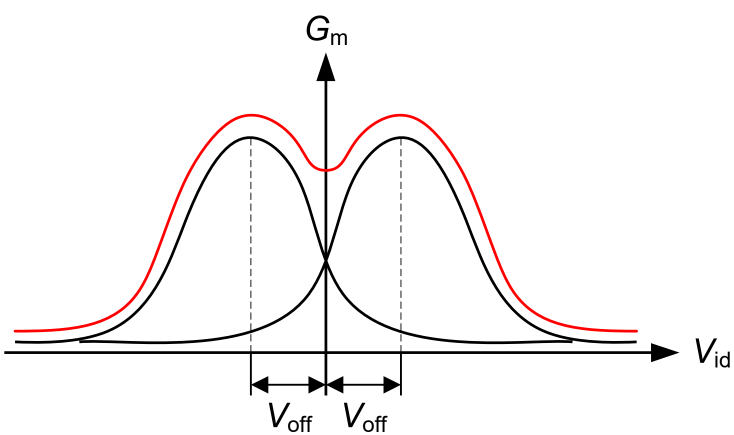






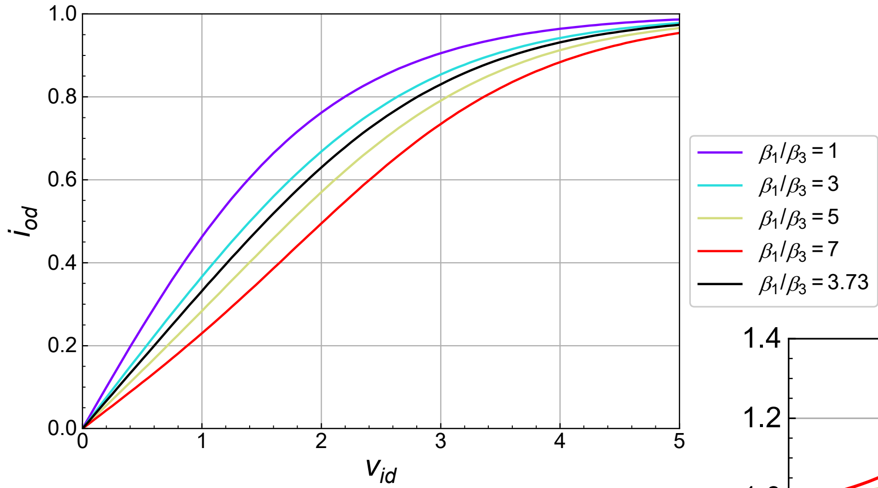
- Controlled offset voltage in weak inversion $V_{off} = nU_T \cdot \ln K$
- Combination of two pairs with opposite offset voltages

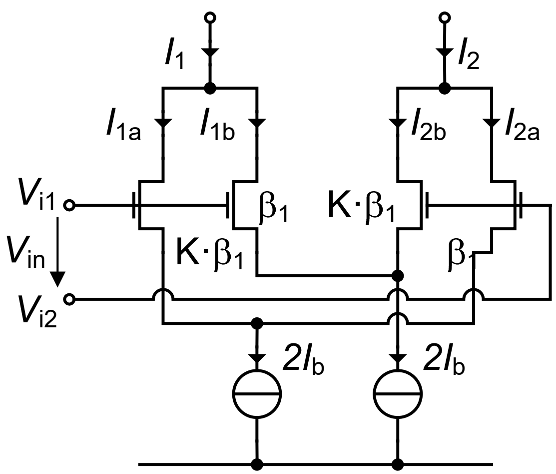




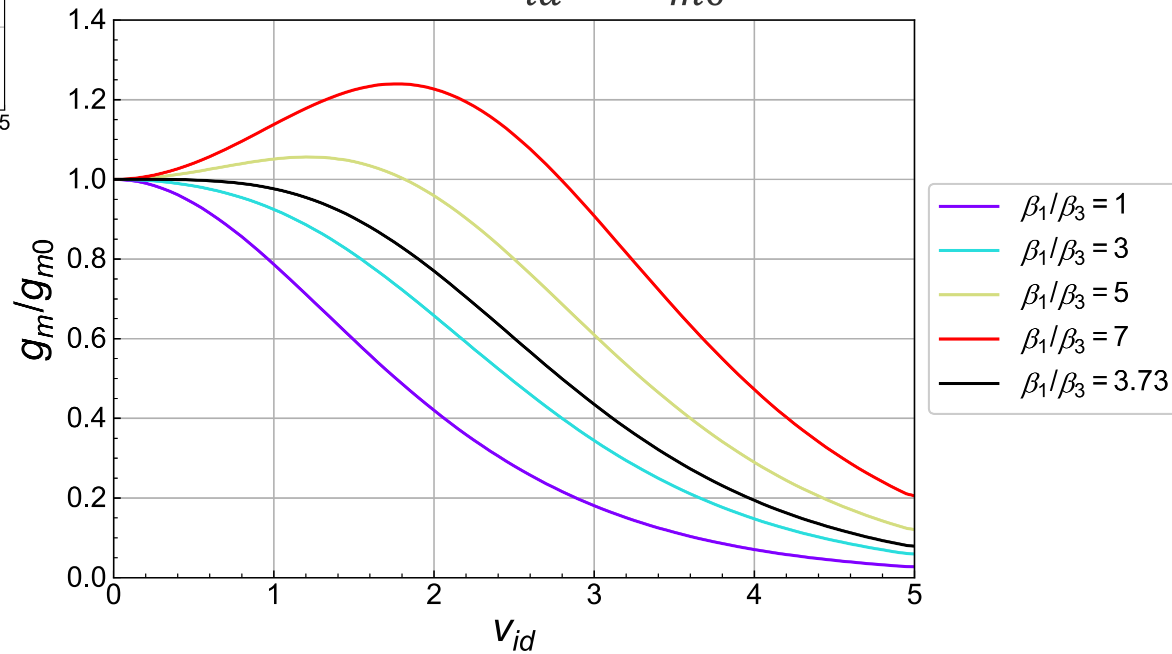
H. Tanimoto, M. Koyama and Y. Yoshida, JSSC, July 1991.

Multi-tanh Linearization – Example





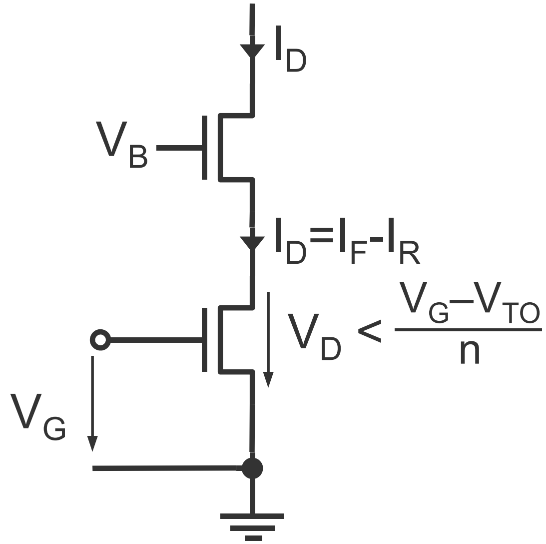
$$g_m = \frac{di_{od}}{dv_{id}} = \frac{G_m}{G_{m0}}$$

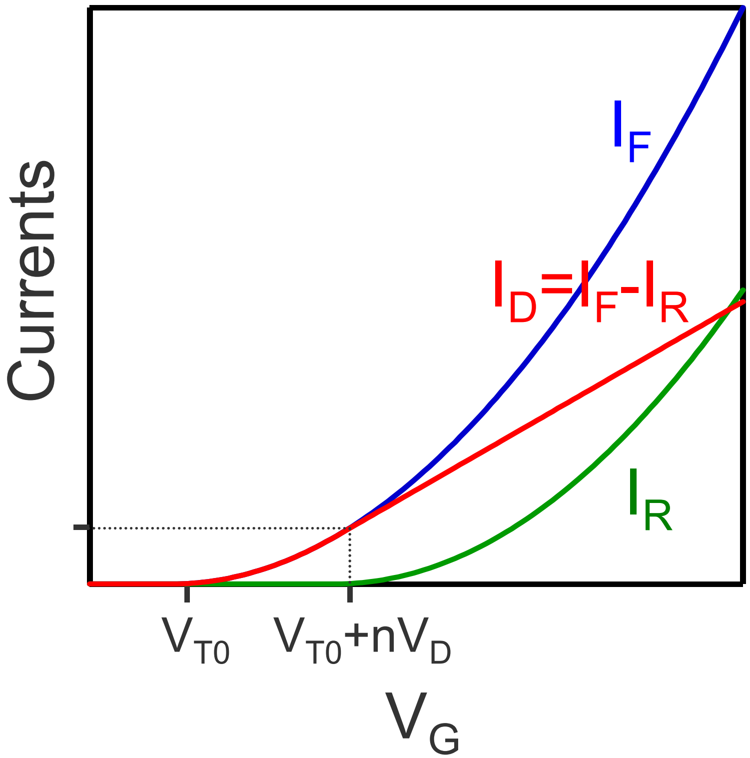


- For $N = 2$ the residual G_m ripple is minimum for $K = \beta_1/\beta_3 = 3.73$
- Linear range extended by almost 2x

Linearization Based on MOSFET in Linear Region

- Particular case of square-law difference where the difference is done inside the same MOS transistor
- Achieves excellent linearity at the cost of higher noise factor





Linearization Based on MOS in Linear Region

- M1 in the linear region $I_D = \beta V_D \left(V_G - V_{T0} - \frac{n}{2} V_D\right)$ for $V_D < V_P \cong \frac{V_G - V_{T0}}{n}$
- The transconductance is hence given by $G_m = \beta V_D$
- Linear range defined by $V_{Gmin} \geq V_{T0} + n \cdot V_D$ and V_{Gmax} limited by mobility degradation
- V_D must be maintained constant to avoid any distortion
  - Cascode transistor and maximum G_{ms} for given current (i.e. weak inversion)
  - Distortion due to residual variations of V_D
- The PSD of current fluctuations due to thermal noise (non-stationary) is given by

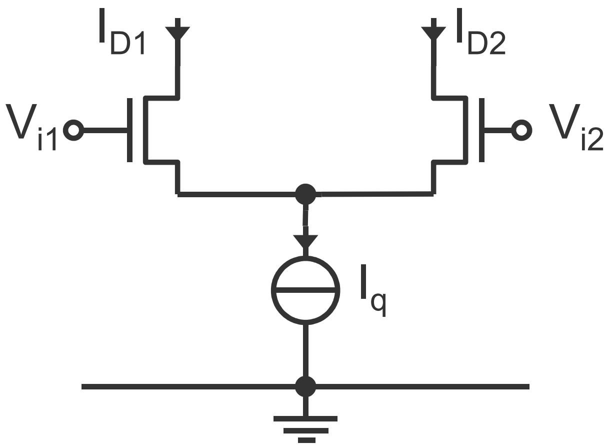
$$S_{\Delta I} = 4kT\, G_n \text{ with } G_n \cong G_{ms} = \beta(V_G - V_{T0})$$

- resulting in a **noise excess factor** given by

$$\gamma = \frac{G_n}{G_m} = \frac{\beta(\overline{V_G} - V_{T0})}{\beta V_D} = \frac{V_{Gmin} + \Delta V - V_{T0}}{V_D} \geq 1$$

- Usually much larger than one since $V_D < V_{Gmin} + \Delta V - V_{T0}$ where ΔV is the signal amplitude resulting in a linearity versus noise trade-off

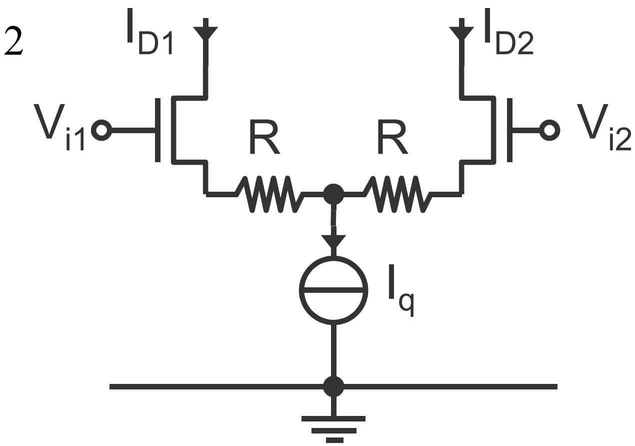
Linearization vs Current Efficiency (1/2)



$$I_{od} \triangleq I_{D1} - I_{D2}$$

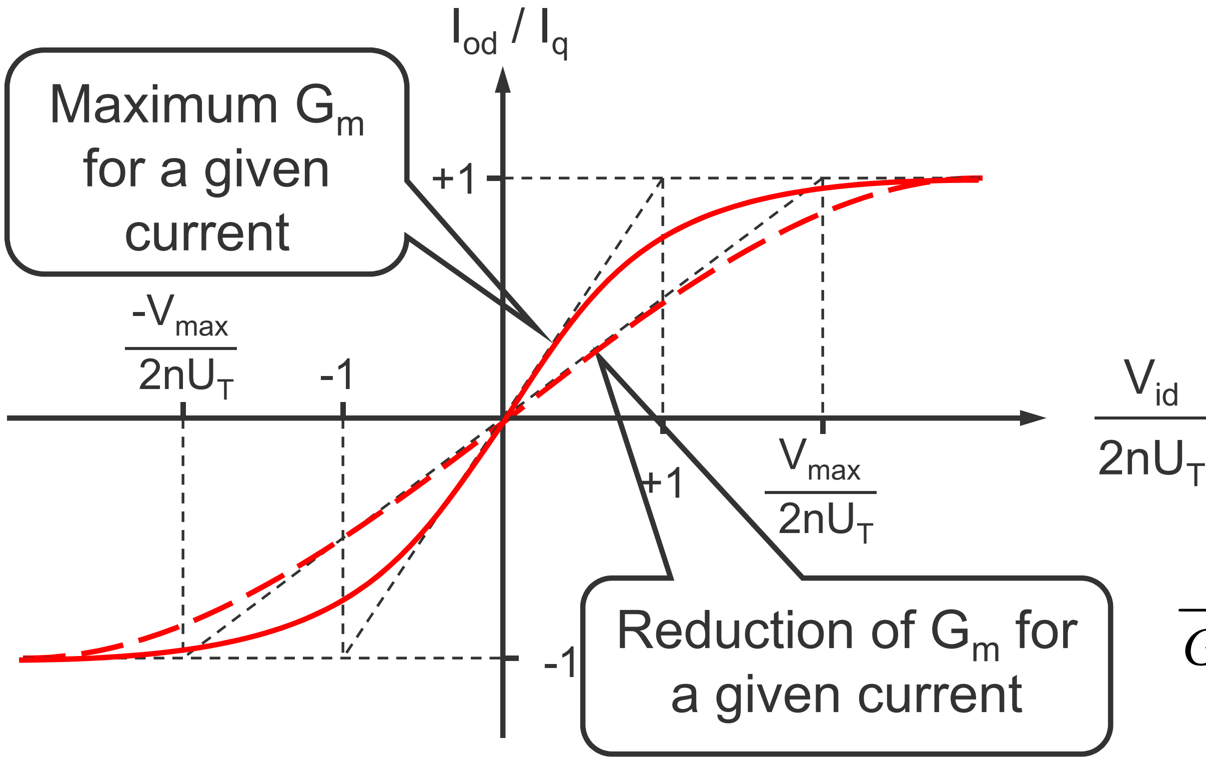
$$V_{id} \triangleq V_{i1} - V_{i2}$$

$$G_m = \frac{I_q}{V_{\max}}$$



	Linear range V_{max}	Current efficiency G_m/I_q
Weak inversion	$2nU_T$	$\frac{1}{2nU_T}$
Strong inversion	$V_G - V_{T0}$	$\frac{1}{V_G - V_{T0}}$
Degenerated	$R \cdot I_q$	$\frac{1}{R \cdot I_q}$

Linearization vs Current Efficiency (2/2)



$$G_{m\text{-}max} = \frac{I_q}{2nU_T}$$

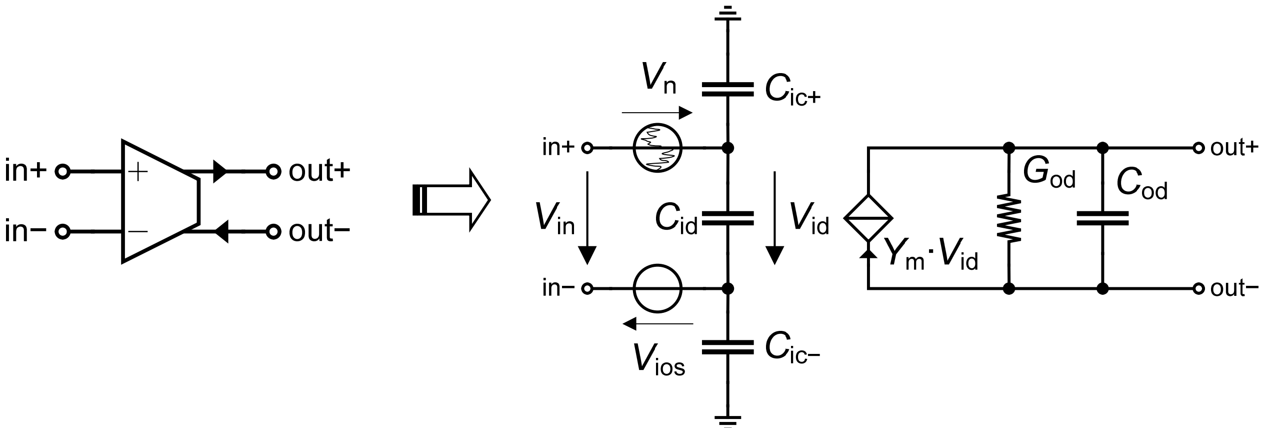
$$\xi \triangleq \frac{V_{max}}{2nU_T}$$

$$\frac{G_m}{G_{m\text{-}max}} = \frac{2nU_T}{V_{max}} = \frac{1}{\xi}$$

- Current efficiency G_m/I_q degrades proportionally to the increase of the linear range
- Results in power consumption increase for a required G_m

Linear Model of OTAs

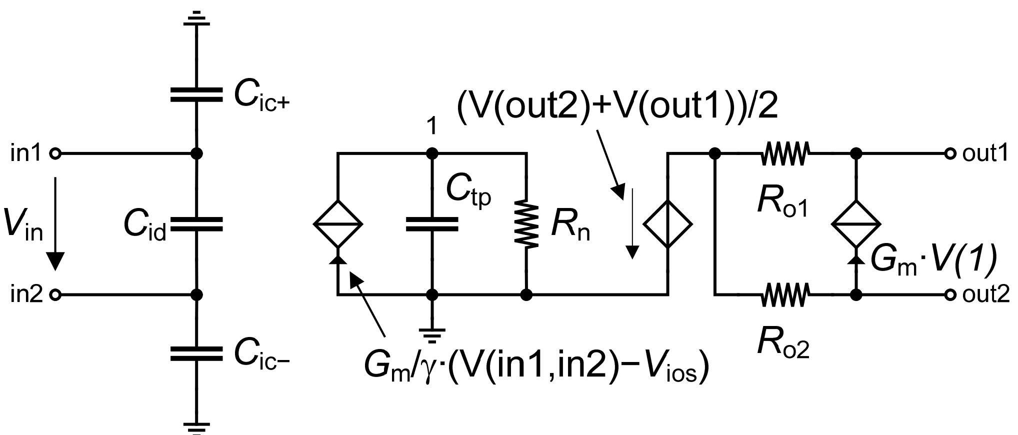
- An OTA can be modeled by the simple linear model given below



$$Y_m(s) = \frac{G_m}{1 + s \cdot \tau_p}$$

$$S_{V_n} = 4kT \cdot \frac{\gamma}{G_m}$$

- Simple Spice implementation



$$R_n = \frac{\gamma}{G_m}$$

$$C_{tp} = \tau_p \cdot \frac{G_m}{\gamma}$$

First-order OTA LP Filters

- The transfer function of the above circuit is given by

$$T(s) \triangleq \frac{V_o}{V_i} = K\frac{\omega_c}{s+\omega_c}$$

- with $K = 1$ and $\omega_c = \frac{G_m}{C_1}$

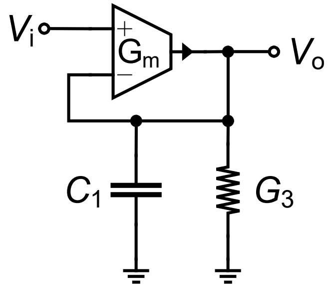
- The above alternative offers a dc gain different than one

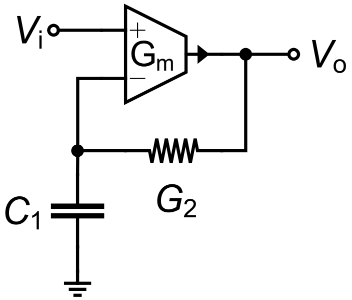
$$T(s) \triangleq \frac{V_o}{V_i} = K\frac{\omega_c}{s+\omega_c}$$

- with $K = \frac{G_m}{G_2}$ and $\omega_c = \frac{G_2}{C_3}$

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

First-order OTA LP Filters





- Another 1st-order LP section is shown above with a transfer function given by

$$T(s) \triangleq \frac{V_o}{V_i} = K \frac{\omega_c}{s + \omega_c}$$

- with $K = \frac{G_m}{G_m + G_3}$ and $\omega_c = \frac{G_m + G_3}{C_3}$

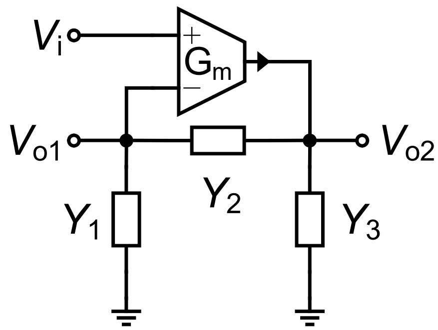
- The above circuit introduces an additional zero

$$T(s) \triangleq \frac{V_o}{V_i} = K \frac{s + \omega_z}{s + \omega_c}$$

- with $K = \frac{G_m}{G_2}$, $\omega_z = \frac{G_2}{C_1}$ and $\omega_c = \frac{G_m}{C_1}$

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

Generic Single OTA Filter



- All the previous circuits can be derived from the generic circuit shown above which has the following transfer functions

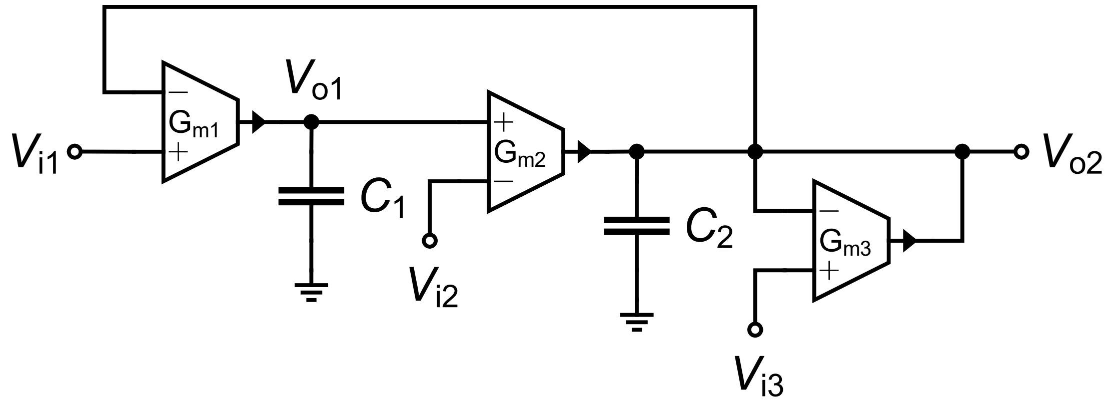
$$T_1(s) \triangleq \frac{V_{o1}}{V_i} = \frac{G_m Y_2}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3 + G_m Y_2}$$

$$T_2(s) \triangleq \frac{V_{o2}}{V_i} = \frac{G_m (Y_1 + Y_2)}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3 + G_m Y_2}$$

- The above 1st-order sections are obtained by replacing one of the admittances by a capacitor, one by a conductance and the remaining one either by a short or an open resulting in two passive component 1st-order sections

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Tow-Thomas OTA-C Biquad



- The **Tow-Thomas** filter can also be realized with **OTAs** as shown above
- The various transfer functions are given by

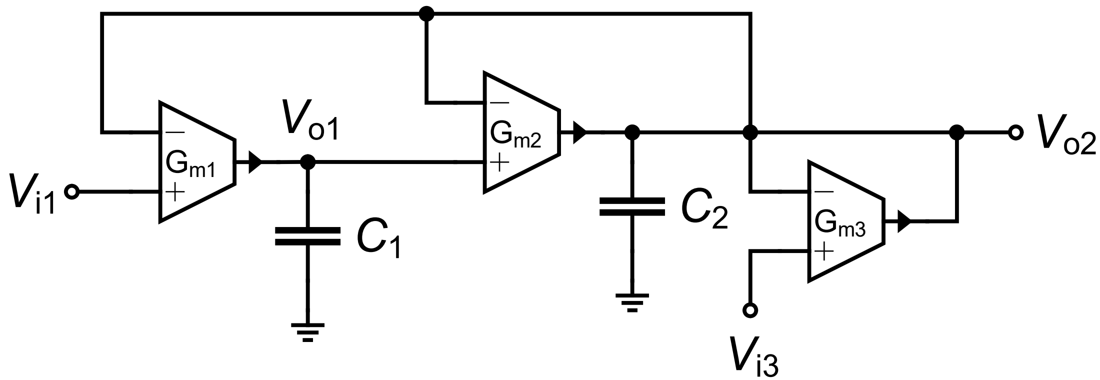
$$V_{o1} = \frac{(s\tau_2 + k_{22})V_{i1} + V_{i2} - k_{22}V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1}$$

$$V_{o2} = \frac{V_{i1} - s\tau_1 V_{i2} + k_{22}s\tau_1 V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1}$$

- with $k_{22} = \frac{G_{m3}}{G_{m2}}$, $\tau_1 = \frac{C_1}{G_{m1}}$ and $\tau_2 = \frac{C_2}{G_{m2}}$
- This circuit is simple and has very **low sensitivity** and **low parasitic effects**

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Feedback Lossy Integrator OTA-C Biquad



- This is a variant of the previous Tow-Thomas lossy integrators OTA biquad where the input of G_{m2} has been connected to its output (making it lossy)
- The output is then given by

$$V_{o1} = \frac{(s\tau_2 + k_{22})V_{i1} - (k_{22} - 1)V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1}$$

$$V_{o2} = \frac{V_{i1} + (k_{22} - 1)s\tau_1 V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1}$$

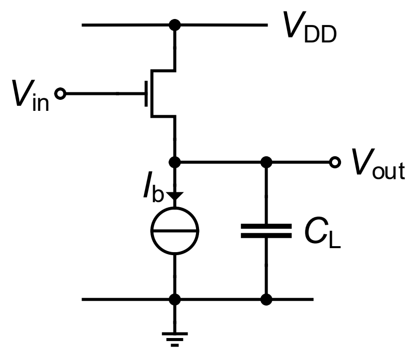
- with $k_{22} = 1 + \frac{G_{m3}}{G_{m2}}$, $\tau_1 = \frac{C_1}{G_{m1}}$ and $\tau_2 = \frac{C_2}{G_{m2}}$

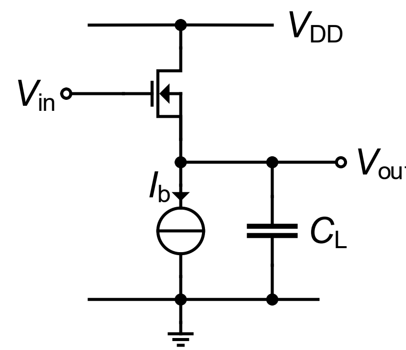
T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

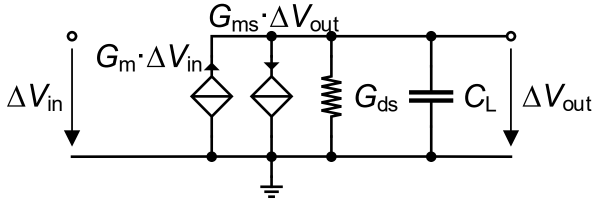
Outline

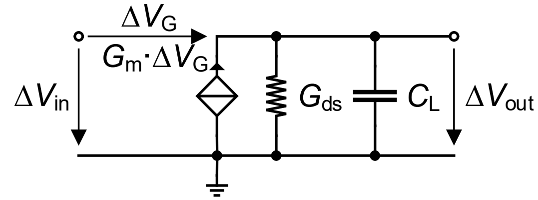
- Introduction
- RC-active filters
- MOSFET-C filters
- G_m-C filters
- **Source-follower CTFs**
- Noise in CTFs
- Automatic tuning

Source-Follower Continuous-Time Filters









$$H(s) \triangleq \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{A_{dc}}{1 + \frac{s}{\omega_c}}$$

$$A_{dc} = \frac{G_m}{G_{ms} + G_{ds}} \cong \frac{G_m}{G_{ms}} = \frac{1}{n}$$

$$\omega_c = \frac{G_{ms} + G_{ds}}{C_L} \cong \frac{G_{ms}}{C_L} = \frac{nG_m}{C_L}$$

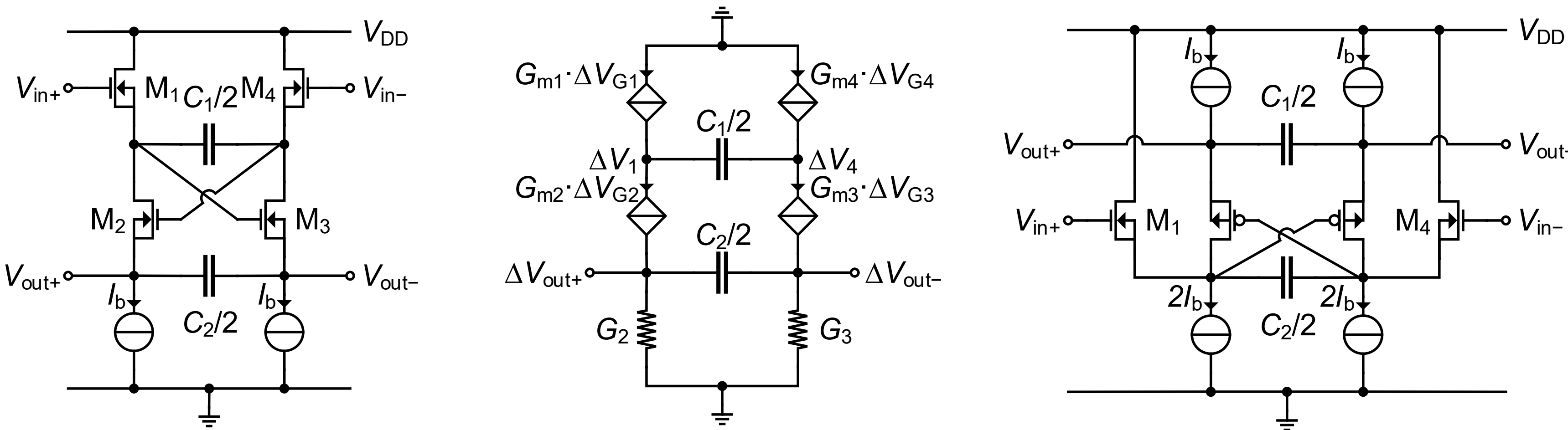
$$H(s) \triangleq \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{A_{dc}}{1 + \frac{s}{\omega_c}}$$

$$A_{dc} = \frac{G_m}{G_m + G_{ds}} \cong 1$$

$$\omega_c = \frac{G_m + G_{ds}}{C_L} \cong \frac{G_m}{C_L}$$

S. D'Amico, M. Conta, and A. Baschirotto, JSSCC, Dec. 2006.

Source-Follower 2nd-order LP Filter



Folded structure
(double well process)

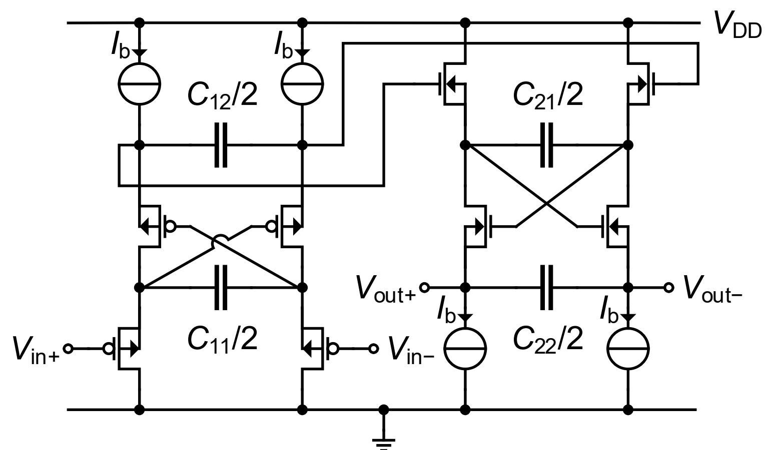
- The differential transfer function assuming $G_{m1} = G_{m2} = G_{m3} = G_{m4} = G_m$ and neglecting the output conductances gives a 2nd-order LP filter

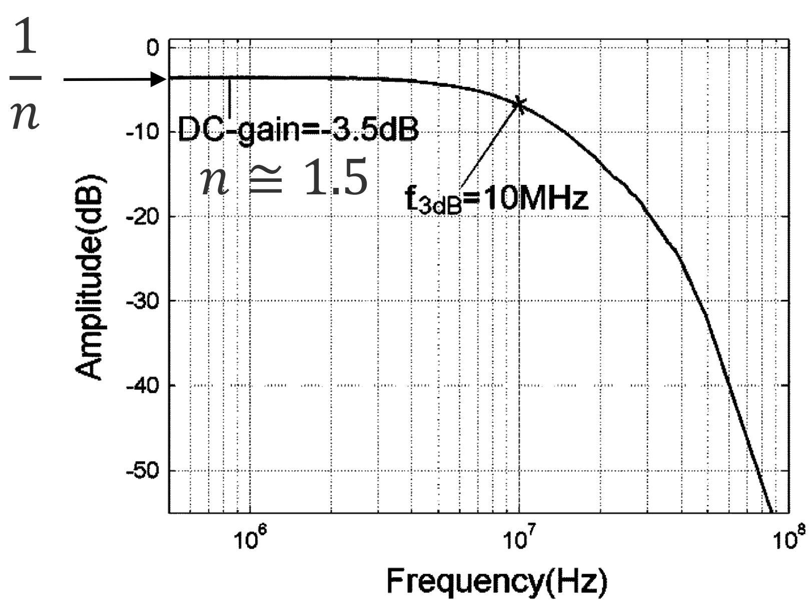
$$H_d(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} \text{ with } \omega_0 = \frac{G_m}{\sqrt{C_1 C_2}} \text{ and } Q = \sqrt{\frac{C_2}{C_1}}$$

- The transfer function slightly changes if the transistors are in the common substrate due to the substrate effect

S. D'Amico, M. Conta, and A. Baschirotto, JSSC, Dec. 2006.

A Fourth-Order SF Continuous-Time Filter

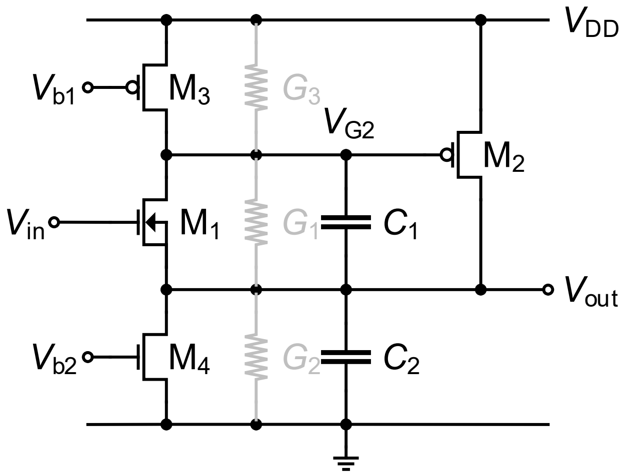




Technology	CMOS 0.18µm
Die Area	0.26mm²x2
Power supply	1.8V
Current consumption	2.28mA
Power consumption	4.1mW
DC-gain	-3.5dB
f_{-3dB}	10MHz
f_{-3dB} tuning range	±40%
IRN	7.5nV/ Hz
$V_{in,noise}$	$24\mu V_{rms}$
DR (HD3=-40dB)	79dB
in-band IIP3 (f_1=3MHz, f_2=4MHz)	17.5dBm
1dBcp	5dBm
HD3 (600mV_{pp}@3MHz)	-40dB

S. D'Amico, M. Conta, and A. Baschirotto, JSSCC, Dec. 2006.

Super-Source-Follower (SSF) Filters



$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1}$$

with $\omega_0 = \sqrt{\frac{G_{m1} G_{m2}}{C_1 C_2}}$ and $Q = \sqrt{\frac{G_{m1}}{G_{m2}} \frac{C_2}{C_1}}$

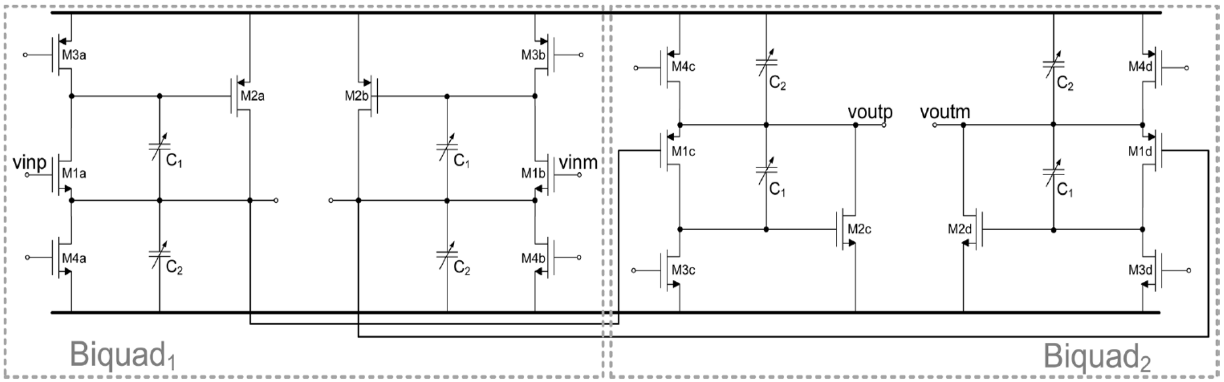
- It is important to put M1 into a separate well to avoid any substrate effects
- The dc gain accounting for the output conductances is given by

$$H_{dc} \cong \frac{1 + \frac{G_3}{G_{m2}}}{1 + \frac{G_3}{G_{m2}} + \frac{G_1}{G_{m1}}} \cong 1$$

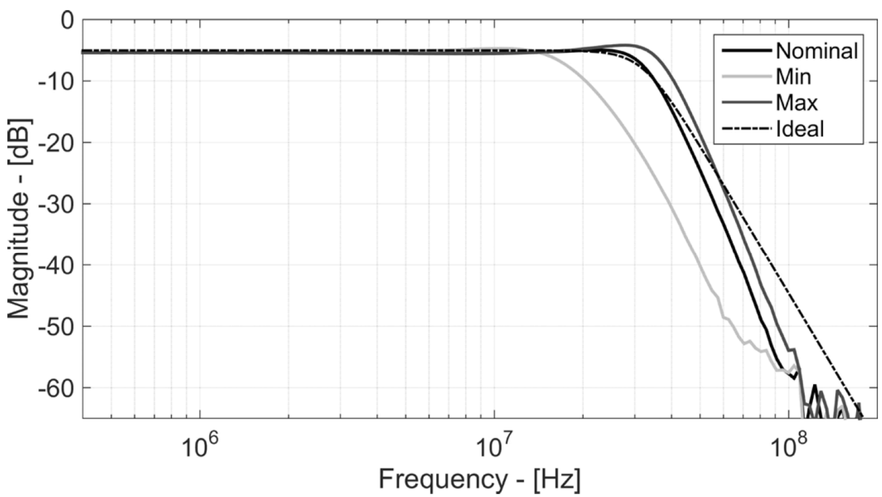
- for $G_1 \ll G_{m1}$ and $G_3 \ll G_{m2}$

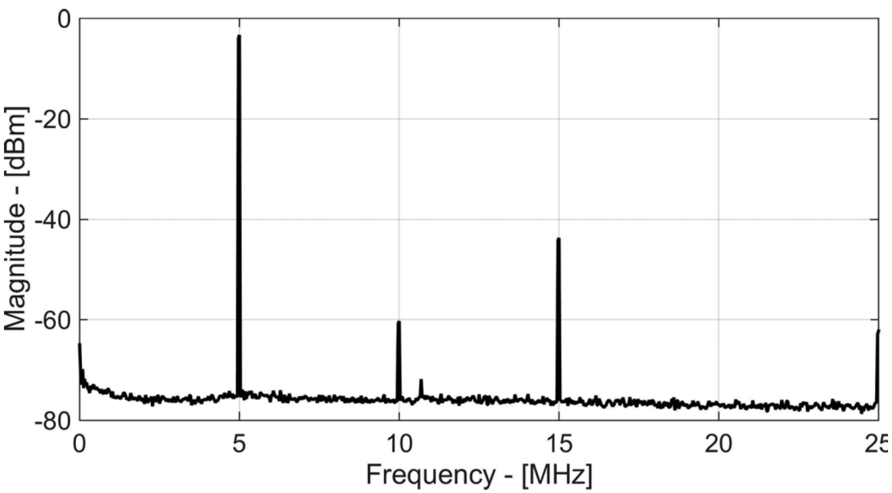
M. De Matteis, A. Pezzotta, S. D'Amico and A. Baschirotto, JSSCC, July. 2015.

Fourth-order Low-pass SSF Filter



Parameter	Biquad_1	Biquad_2
Poles Frequency – f_0	33 MHz	33 MHz
Poles Quality Factor – Q	0.5412	1.3066
g_{m_1}	1.8 mA/V	1.8 mA/V
g_{m_2}	1.8 mA/V	1.8 mA/V
C_1	15 pF	6.2 pF
C_2	4.4 pF	10 pF

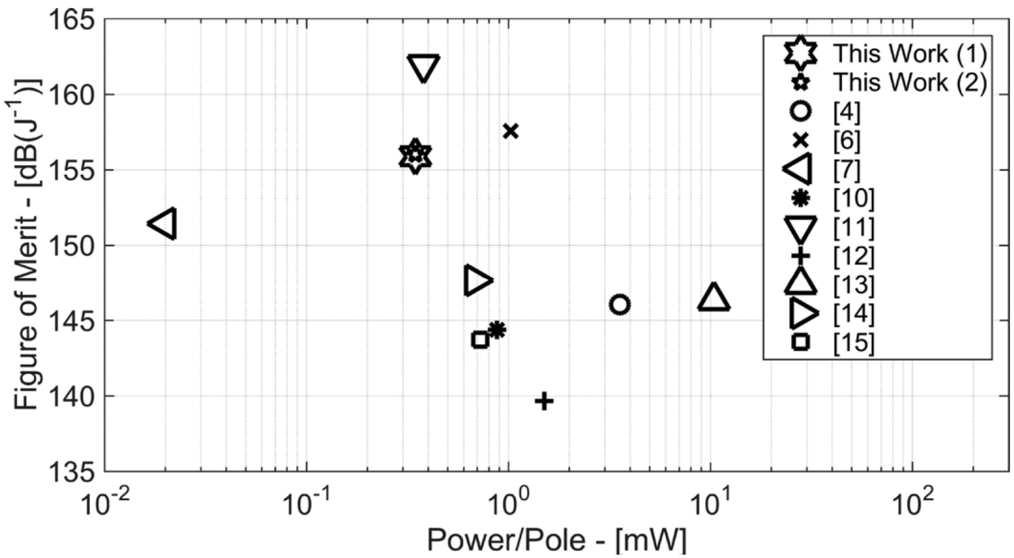




M. De Matteis, A. Pezzotta, S. D'Amico and A. Baschirotto, JSSCC, July. 2015.

Benchmark

Parameters	This work	[4]	[6]	[7]	[10]	[11]	[12]	[13]	[14]	[15]
Technology [nm]	180	130	180	130	130	90	130	130	180	180
Supply voltage [V]	1.8	1.2	1.8	1.2	0.55	1.2	1	1.2	1.8	1.5
Order	4	4	4	6	4	6	5	2	6	6
Topology	SSF-based	Active-g_m-RC	SF-based	SF-based	Active-g_m-RC	Active-RC	Active-RC	OTA-C	Active-RC	g_m-C
f_0 [MHz]	33	11	10	280	11.3	255	20	200	500	13.5
Power [mW]	1.38	14.2	4.1	0.12	3.5	2.28	7.5	20.8	4.1	4.35
Power/Pole [mW]	0.34	3.55	0.88	0.26	1	0.02	1.5	10.4	0.68	0.73
IIP3 [dBm]	18 - 1	21	7.5	11	10	14	26	14	15.9	22
f_{IM3L} [MHz]	1 - 14	0.5	2	1	2	60	0.27	149.7	5	1
f_{IM3L}/f_0	0.03 - 0.42	0.04	0.2	0.004	0.18	0.24	0.014	0.75	0.01	0.074
In-Band Integrated Noise [μV_{RMS}]	45	36	23.7	368	110	200	232	494	293	335
Area [mm^2]	0.14	0.9	0.43	0.06	0.52	0.018	1.53	0.5	0.23	1
FoM_{conv} [$dB(J^{-1})$]	171.1 - 159.8	159.5	164.6	175.9	151.9	168.2	158.4	147.6	167.7	155
FoM [$dB(J^{-1})$]	155.9 - 156	146.1	157.6	151.4	144.4	162	139.7	146.3	147.7	143.7



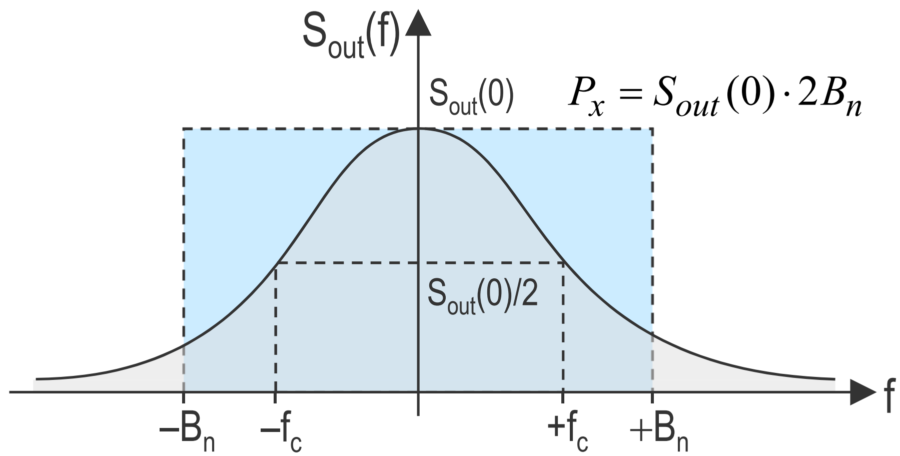
$$FoM = 10\log_{10}\left(\frac{IMFDR_3\ f_{@-3\text{ dB}}\ N}{P_W}\cdot\left[\frac{\mathbf{f}_{\mathbf{IM3_L}}}{\mathbf{f}_{\mathbf{poles}}}\right]\right)$$

[4] S. D'Amico, V. Giannini, and A. Baschirotto, "A 4th-order active-Gm-RC reconfigurable (UMTS/WLAN) filter," *IEEE J. Solid-State Circuits*, vol. 41, no. 7, pp. 1630–1637, Jul. 2006.
[5] S. D'Amico, V. Giannini, and A. Baschirotto, "A 1.2 V–21 dBm OIP3 4th-order active-gm-RC reconfigurable (UMTS/WLAN) filter with on-chip tuning designed with an automatic tool," in *Proc. 31st European Solid-State Circuits Conf., ESSCIRC 2005*, Sep. 2005, pp. 315–318.
[6] S. D'Amico, M. Conta, and A. Baschirotto, "A 4.1-mW 10-MHz fourth-order source-follower-based continuous-time filter with 79-dB DR," *IEEE J. Solid-State Circuits*, vol. 41, no. 12, pp. 2713–2719, Dec. 2006.
[7] S. D'Amico, M. D. Matteis, and A. Baschirotto, "A 6th-order 100 μA 280 MHz source-follower-based single-loop continuous-time filter," in *IEEE Int. Solid-State Circuits Conf. Dig. Tech. Papers, ISSCC 2008*, Feb. 2008, pp. 72–596.
[8] P. Wambacq, V. Giannini, K. Scheir, W. Van Thillo, and Y. Rolain, "A fifth-order 880 MHz/1.76 GHz active lowpass filter for 60 GHz communications in 40 nm digital CMOS," in *Proc. ESSCIRC 2010*, Sep. 2010, pp. 350–353.
[9] H. Khorramabadi and P. Gray, "High-frequency CMOS continuous-time filters," *IEEE J. Solid-State Circuits*, vol. 19, no. 6, pp. 939–948, Dec. 1984.
[10] M. D. Matteis, S. D'Amico, and A. Baschirotto, "A 0.55 V 60 dB-DR fourth-order analog baseband filter," *IEEE J. Solid-State Circuits*, vol. 44, no. 9, pp. 2525–2534, Sep. 2009.
[11] S. D'Amico, M. De Blasi, M. De Matteis, and A. Baschirotto, "A 255 MHz programmable gain amplifier and low-pass filter for ultra low power impulse-radio UWB receivers," *IEEE Trans. Circuits Syst. I: Reg. Papers*, vol. 59, no. 2, pp. 337–345, Feb. 2012.
[12] H. Amir-Aslanzadeh, E. Pankratz, and E. Sanchez-Sinencio, "A 1-V +31 dBm IIP3, reconfigurable, continuously tunable, power-adjustable active-RC LPF," *IEEE J. Solid-State Circuits*, vol. 44, no. 2, pp. 495–508, Feb. 2009.
[13] M. Mobarak, M. Onabajo, J. Silva-Martinez, and E. Sanchez-Sinencio, "Attenuation-predistortion linearization of CMOS OTAs with digital correction of process variations in OTA-C filter applications," *IEEE J. Solid-State Circuits*, vol. 45, no. 2, pp. 351–367, Feb. 2010.
[14] L. Ye, H. Liao, C. Shi, J. Liu, and R. Huang, "A 2.3 mA 240-to-500 MHz 6th-order active-RC low-pass filter for ultra-wideband transceiver," in *Proc. IEEE Asian Solid State Circuits Conf., A-SSCC 2010*, Nov. 2010, pp. 1–4.
[15] M. Oskooei, N. Masoumi, M. Kamarei, and H. Sjoland, "A CMOS 4.35-mW +22-dBm IIP3 continuously tunable channel select filter for WLAN/WiMAX receivers," *IEEE J. Solid-State Circuits*, vol. 46, no. 6, pp. 1382–1391, Jun. 2011.

Outline

- Introduction
- RC-active filters
- MOSFET-C filters
- G_m-C filters
- Source-follower CTFs
- **Noise in CTFs**
- Automatic tuning

Equivalent Noise Bandwidth



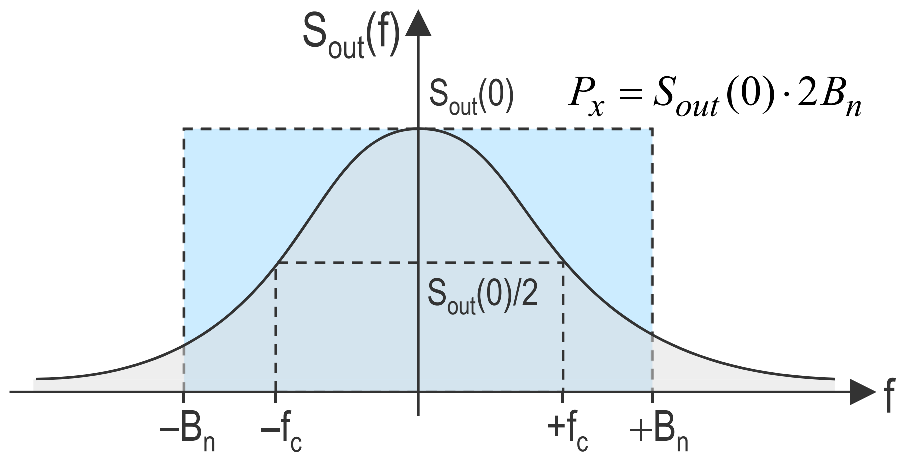
- The **equivalent noise bandwidth** of a low-pass noise PSD is defined as the bandwidth B_n of an ideally low-pass filtered **white noise** having the same value $S_{out}(0)$ (at $f = 0$) and the same power P_x than the original noise

$$B_n = \frac{1}{2} \cdot \frac{1}{S_{out}(0)} \cdot \int_{-\infty}^{+\infty} S_{out}(f) \cdot df = \frac{P_{out}}{2S_{out}(0)}$$

- If the input noise is a white noise of PSD S_0, the output noise is then given by $S_{out}(f) = |H(f)|^2 \cdot S_0$ and the **equivalent bandwidth only depends on the transfer function**

$$B_n = \frac{1}{2} \cdot \frac{1}{|H(0)|^2 S_0} \cdot \int_{-\infty}^{+\infty} |H(f)|^2 \cdot S_0 \cdot df = \frac{1}{|H(0)|^2} \cdot \int_{0}^{+\infty} |H(f)|^2 \cdot df$$

Equivalent Noise Bandwidth of a 1st-order LP Filtered White Noise



- The equivalent noise bandwidth of a **1st-order low-pass filtered white noise** with a cut-off frequency f_c is given by

$$B_n = \int_0^{+\infty} \frac{df}{1 + (f/f_c)^2} = f_c \cdot \underbrace{\int_0^{+\infty} \frac{dx}{1 + x^2}}_{=\frac{\pi}{2}} = \frac{\pi}{2} f_c$$

Noise Bandwidth of Various Filters

Type	Noise Transfer Function	Noise Bandwidth
1st-order LP	$\dfrac{1}{1+\dfrac{s}{\omega_c}}$	$\dfrac{\omega_c}{4}$
2nd-order LP	$\dfrac{1}{1+\dfrac{s}{\omega_0 \cdot Q}+\left(\dfrac{s}{\omega_0}\right)^2}$	$\dfrac{\omega_0 \cdot Q}{4}$
2nd-order BP[1]	$\dfrac{\dfrac{s}{\omega_0 \cdot Q}}{1+\dfrac{s}{\omega_0 \cdot Q}+\left(\dfrac{s}{\omega_0}\right)^2}$	$\dfrac{\omega_0}{4Q}$
2nd-order LP (with zero)	$\dfrac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_0 \cdot Q}+\left(\dfrac{s}{\omega_0}\right)^2}$	$\dfrac{\omega_0 \cdot Q}{4} \cdot \left[1+\left(\dfrac{\omega_0}{\omega_z}\right)^2\right]$

1. In the case of the bandpass filter the dc gain $H_0(0)$ is zero and should be replaced in the above definition by the gain at the resonance frequency which in the above case has been set to one.

Higher Order Transfer Functions

3rd-order LP

- Noise transfer function:

$$H_{n3}(s) = \frac{n_2 s^2 + n_1 s + n_0}{d_3 s^3 + d_2 s^2 + d_1 s + d_0}$$

- Noise bandwidth:

$$B_{n3} = \frac{1}{4} \frac{n_2^2 d_1 d_0 + n_1^2 d_3 d_0 + n_0^2 d_3 d_2 - 2 n_2 n_0 d_3 d_0}{d_3 (d_2 d_1 - d_3 d_0) d_0}$$

4th-order LP

- Noise transfer function:

$$H_{n4}(s) = \frac{n_3 s^3 + n_2 s^2 + n_1 s + n_0}{d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0}$$

- Noise bandwidth:

$$B_{n4} = \frac{1}{4} \frac{n_3^2 d_3 d_0^2 + 2 n_3 n_1 d_4 d_1 d_0 - n_3^2 d_2 d_1 d_0 - n_2^2 d_4 d_1 d_0 - n_1^2 d_4 d_3 d_0 - n_0^2 d_4 d_3 d_2 + 2 n_2 n_0 d_4 d_3 d_0 + n_0^2 d_4^2 d_1}{d_4 \left(d_4 d_1^2 - d_3 d_2 d_1 + d_3^2 d_0\right) d_0}$$

A. Dastgheib and B. Murmann, “Calculation of Total Integrated Noise in Analog Circuits,” TCAS I, vol. 55, no. 10, Nov. 2008.

Contribution of 1/f Noise to the Total Noise Power

- The PSD (single-sided) of an amplifier including the 1/f noise component can be written as

$$S_n(f) = S_0 \cdot \left(1 + \frac{f_k}{|f|}\right)$$

- where S_0 is the white noise component and f_k the corner frequency (frequency at which the 1/f noise becomes equal to the white noise)
- The noise power assuming the noise is filtered by a 1st-order low-pass filter having a cut-off frequency f_c is the given by

$$V_n^2 = \int_0^{+\infty} \frac{S_n(f)}{1 + (f/f_c)^2} \cdot df$$

- We then can define an equivalent noise bandwidth B_n including 1/f noise defined as

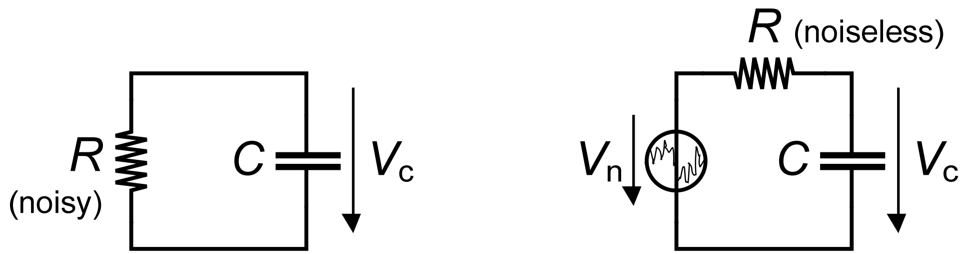
$$B_n \triangleq \frac{V_n^2}{S_0} = \int_0^{+\infty} \frac{1 + f_k/f}{1 + (f/f_c)^2} \cdot df = f_c \cdot \int_0^{+\infty} \frac{dx}{1 + x^2} + f_k \cdot \int_{x_\ell}^{+\infty} \frac{dx}{x \cdot (1 + x^2)}$$

- where $x \triangleq f/f_c$ and $x_\ell \triangleq f_\ell/f_c \ll 1$. It is convenient to choose $f_\ell = 1\,Hz$

$$B_n = \frac{\pi}{2} f_c + \frac{f_k}{2} \ln\left[1 + \left(\frac{f_c}{f_\ell}\right)^2\right] \cong \frac{\pi}{2} f_c + f_k \ln\left(\frac{f_c}{f_\ell}\right) = \frac{\pi}{2} f_c + f_k \ln\left(\frac{f_c}{1Hz}\right)$$

- We see that the contribution of 1/f noise to the total noise power is scaling only with the $\ln f_c$ whereas the white noise scales proportionally to f_c

Thermal Noise – kT/C Noise



- The **variance** of the thermal noise voltage V_c across C is given by

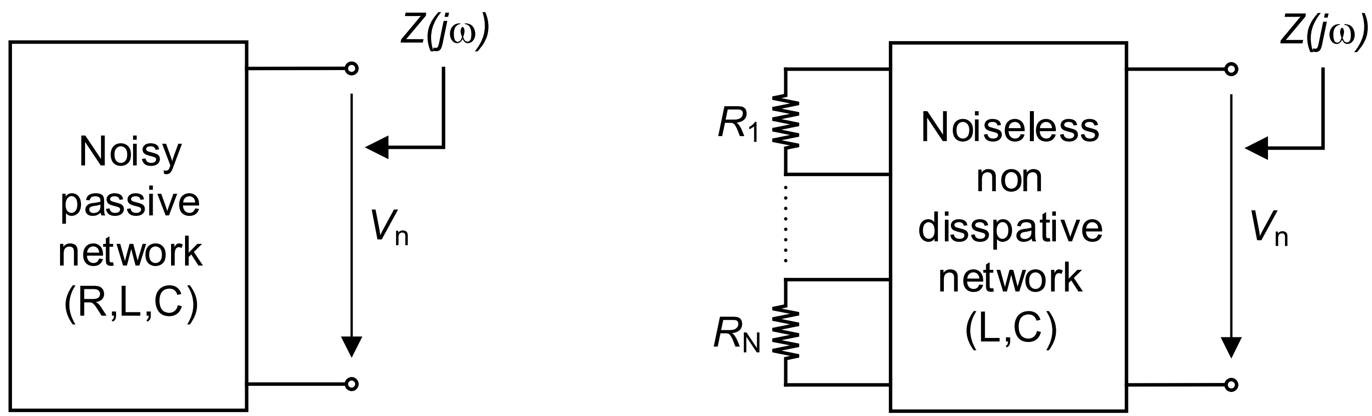
$$\overline{V_c^2} = 4kTR \cdot \int_0^{+\infty} \frac{df}{1+(2\pi f \tau)^2} = 4kTR \cdot B_n = 4kTR \cdot \frac{\pi}{2}\frac{1}{2\pi RC} = \frac{kT}{C}$$

- This result can be obtained directly by applying the **equipartition theorem**
- Average stored energy in C is given by $\overline{W} = \frac{1}{2} C \cdot \overline{V_c^2}$
- Since resistor R and capacitor C are in thermal equilibrium and there is only one degree of freedom, we have

$$\overline{W} = kT/2$$

- We then get $$C \cdot \overline{V_c^2} = kT \Longrightarrow \overline{V_c^2} = \frac{kT}{C}$$

Thermal Noise in Passive Networks – The Nyquist Theorem



- The power spectral density (PSD) of noise voltage V_n is given by

$$S_{v_n}(f) = 4kT \cdot \Re\{Z(j2\pi f)\}$$

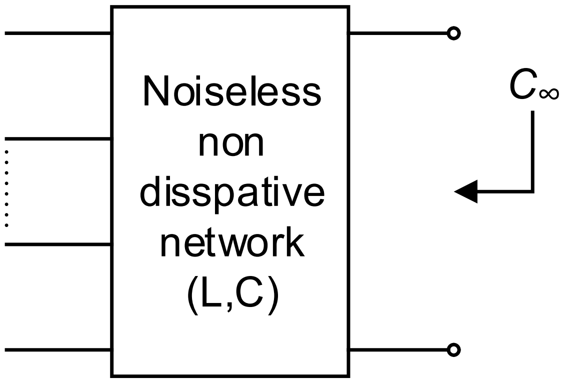
- The variance of voltage V_n is then given by

$$\overline{V_n^2} = \int_0^{+\infty} S_{v_n}(f) \cdot df = 4kT \cdot \int_0^{+\infty} \Re\{Z(j2\pi f)\} \cdot df$$

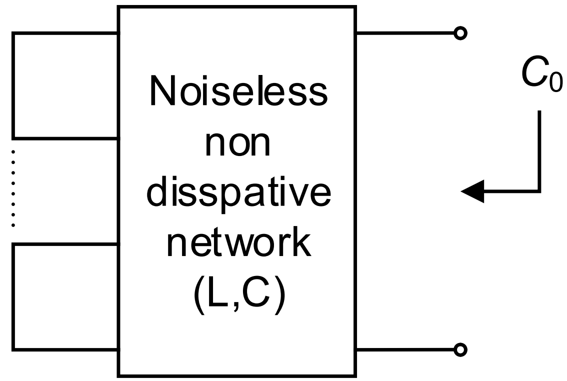
- Or we can use the Bode theorem given in the next slide

H. Nyquist, "Thermal Agitation of Electric Charge in Conductors," Physical Review B, vol. 32, pp. 110-113, July 1928.
A. Papoulis, *Probability, Random Variables and Stochastic Processes*, 1st ed., pp. 362-363, 1981.

Thermal Noise in Passive Networks – The Bode Theorem



$$\frac{1}{C_\infty} = \lim_{s \to \infty}[s \cdot Z(s)]$$



$$\frac{1}{C_0} = \lim_{s \to 0}[s \cdot Z(s)]$$

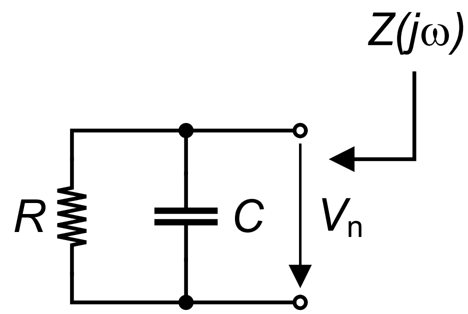
- The variance of noise voltage V_n can be obtained without computing the integral by using the Bode theorem stating

$$\overline{V_n^2} = kT \cdot \left[\frac{1}{C_\infty} - \frac{1}{C_0}\right]$$

- Where C_∞ and C_0 are define as follows

H. W. Bode, *Network Analysis and Feedback Amplifier Design*, van Nostrand Company, New York, pp. 281-282, 1945.

Nyquist and Bode Theorems – Example (1/2)



- The impedance is given by

$$Z(j\omega) = \frac{1}{1/R + j\omega C} = \frac{R}{1 + j\omega RC} = \frac{R}{1 + (\omega RC)^2} - j\frac{\omega R^2 C}{1 + (\omega RC)^2}$$

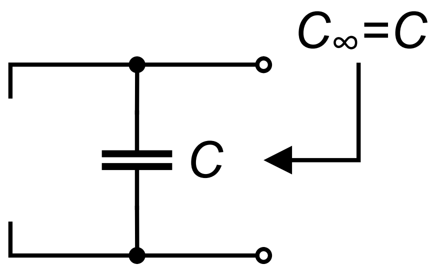
- And hence $S_{v_n}(f) = 4kT \cdot \Re\{Z(j2\pi f)\} = \frac{4kTR}{1+(\omega RC)^2}$

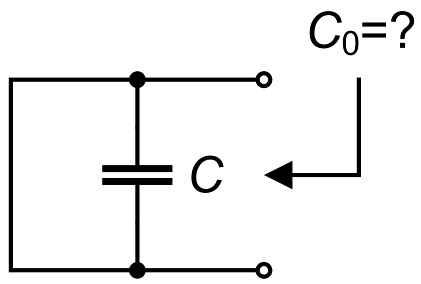
- The noise variance is then given by

$$\overline{V_n^2} = \int_0^{+\infty} S_{v_n}(f) \cdot df = 4kTR \cdot \int_0^{+\infty} \frac{df}{1 + (2\pi f\tau)^2} = 4kTR \cdot B_n$$

$$= 4kTR \cdot \frac{\pi}{2} \frac{1}{2\pi RC} = \frac{kT}{C}$$

Nyquist and Bode Theorems – Example (2/2)





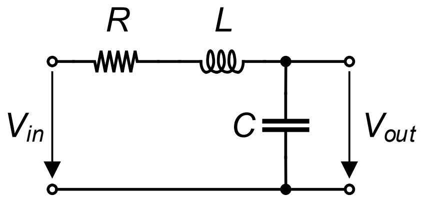
$$\frac{1}{C_\infty} = \lim_{s\to\infty}[s \cdot Z(s)] = \lim_{s\to\infty} \frac{sR}{1+sRC} = \frac{1}{C} \qquad \frac{1}{C_0} = \lim_{s\to 0}[s \cdot Z(s)] = \lim_{s\to 0} \frac{sR}{1+sRC} = 0$$

- Noise variance of a 1st-order RC circuit using Bode theorem results in

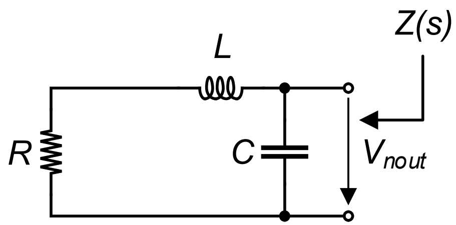
$$\overline{V_n^2} = kT \cdot \left[\frac{1}{C_\infty} - \frac{1}{C_0}\right] = kT \cdot \left[\frac{1}{C} - 0\right] = \frac{kT}{C}$$

- Just by circuit inspection without any integration!
- However, unfortunately only applies to passive circuits!

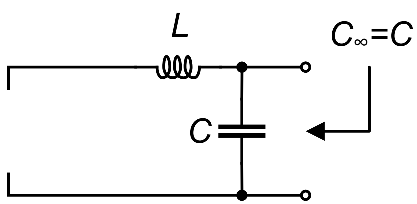
Passive RLC Low-pass Filter



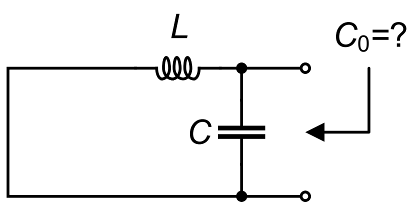
$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{1}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2} \text{ with } \omega_0 = \frac{1}{\sqrt{LC}} \text{ and } Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$



$$Z(s) = \frac{R + sL}{1 + sRC + s^2 LC}$$



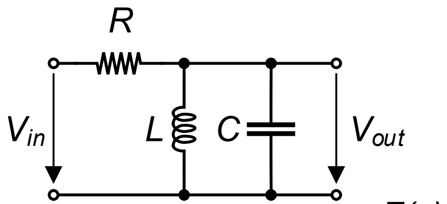
$$\frac{1}{C_\infty} = \lim_{s\to\infty}[s \cdot Z(s)] = \frac{1}{C}$$



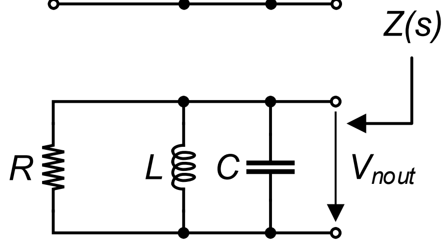
$$\frac{1}{C_0} = \lim_{s\to 0}[s \cdot Z(s)] = 0$$

$$\overline{V_{nout}^2} = kT \cdot \left[\frac{1}{C_\infty} - \frac{1}{C_0}\right] = kT \cdot \left[\frac{1}{C} - 0\right] = \frac{kT}{C}$$

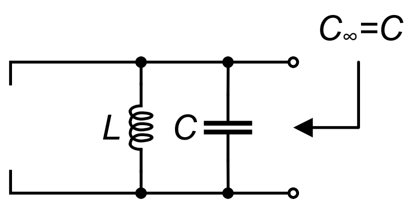
Passive RLC Bandpass Filter



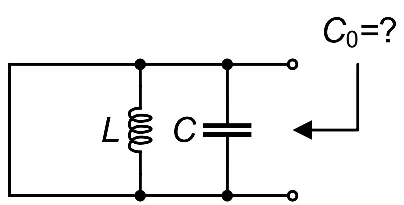
$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{\frac{s}{\omega_0 Q}}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2} \text{ with } \omega_0 = \frac{1}{\sqrt{LC}} \text{ and } Q = R\sqrt{\frac{C}{L}}$$



$$Z(s) = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$



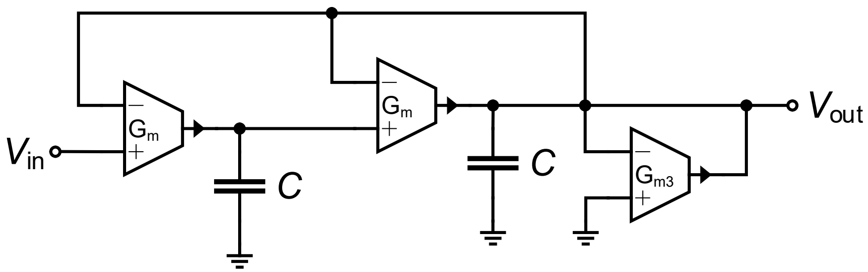
$$\frac{1}{C_\infty} = \lim_{s \to \infty}[s \cdot Z(s)] = \frac{1}{C}$$



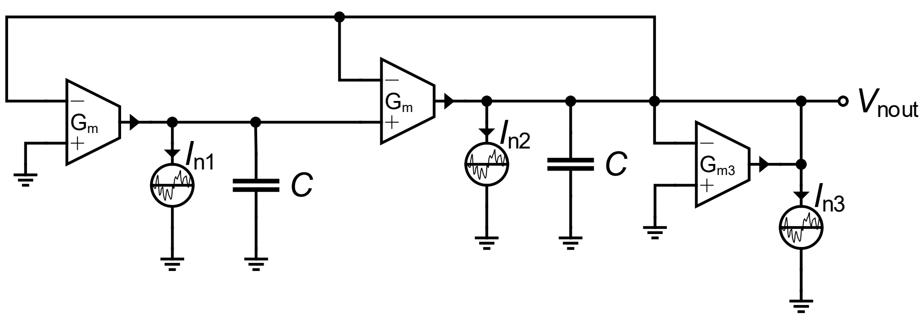
$$\frac{1}{C_0} = \lim_{s \to 0}[s \cdot Z(s)] = 0$$

$$\overline{V_{nout}^2} = kT \cdot \left[\frac{1}{C_\infty} - \frac{1}{C_0}\right] = kT \cdot \left[\frac{1}{C} - 0\right] = \frac{kT}{C}$$

Noise in G_m-C Tow-Thomas Biquad



$$G_{m1} = G_{m2} = G_m \text{ and } C_1 = C_2 = C$$



$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1}$$

with $\omega_0 = \frac{G_m}{C}$ and $Q = \frac{G_m}{G_m + G_{m3}}$

$$R_{m1}(s) \triangleq \frac{V_{nout}}{I_{n1}} = -\frac{1}{G_m} \cdot \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1}$$

$$R_{m2}(s) \triangleq \frac{V_{nout}}{I_{n2}} = -\frac{1}{G_m} \cdot \frac{\frac{s}{\omega_0}}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1}$$

$$R_{m3}(s) \triangleq \frac{V_{nout}}{I_{n3}} = R_{m2}(s)$$

$$S_{V_{out}^2} = |R_{m1}(f)|^2 \cdot S_{I_{n1}} + |R_{m2}(f)|^2 \cdot \left(S_{I_{n2}} + S_{I_{n3}}\right)$$

$$S_{I_{ni}} = 4k_B T \cdot \gamma_{ni} \cdot G_{mi} \text{ for } i = 1,2,3 \text{ (ignoring flicker noise)}$$

Noise in G_m-C Tow-Thomas Biquad

- Considering only thermal noise, the equivalent noise bandwidth for noise sources I_{n1}, I_{n2} and I_{n3} are given by

$$B_{n1} = Q \cdot \frac{\omega_0}{4} \text{ and } B_{n2} = B_{n3} = \frac{\omega_0}{4}$$

- The corresponding output thermal noise voltage is given by

$$V_{nout}^2\Big|_{I_{n1}} = 4k_B T \cdot \gamma_{n1} G_m \cdot \frac{1}{G_m^2} \cdot B_{n1} = \frac{\gamma_{n1} k_B T}{C} \cdot Q$$

$$V_{nout}^2\Big|_{I_{n2},I_{n3}} = 4k_B T \cdot \gamma_{n2} G_m \cdot \frac{1}{G_m^2} \cdot B_{n2} = \frac{\gamma_{n2} k_B T}{C}$$

- The total output thermal noise voltage

$$V_{nout}^2 = \frac{k_B T}{C} \cdot (\gamma_{n1} Q + \gamma_{n2} + \gamma_{n3})$$

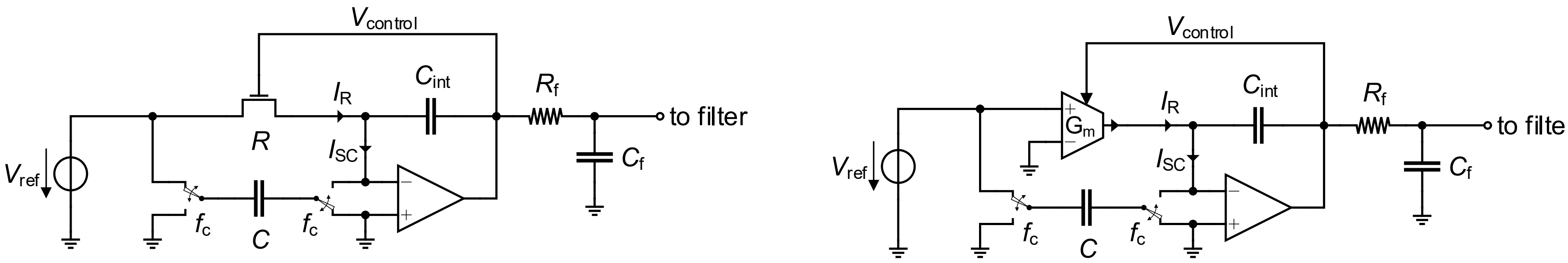
Outline

- Introduction
- RC-active filters
- MOSFET-C filters
- G_m-C filters
- Source-follower CTFs
- Noise in CTFs
- **Automatic tuning**

Component Accuracy and Automatic Tuning Control

- Component mismatch on-chip remains small
  - $\frac{\Delta C}{C} \sim 0.05\ \%$
  - $\frac{\Delta R}{R} \sim 2\ \%$
  - $\frac{\Delta G_m}{G_m} \sim 10\ \%$ (often due to bias current mismatch)
- However, absolute errors can be very large
  - RC time constants only accurate to 20-50 %
  - C/G_m time constants even less accurate
- Tuning control circuit is needed to obtain 1-5% accuracy in filter time constants
- Tuning control circuits rely on tight matching between a reference circuit in the control loop and the filter to be tuned
- An external clock signal of precisely controlled frequency is commonly used as reference signal

Simple SC Control Circuit



- Assuming an ideal OPAMP we then have

$$I_R = \frac{V_{ref}}{R} \text{ and } \overline{I_{SC}} = f_c \cdot C \cdot V_{ref}$$

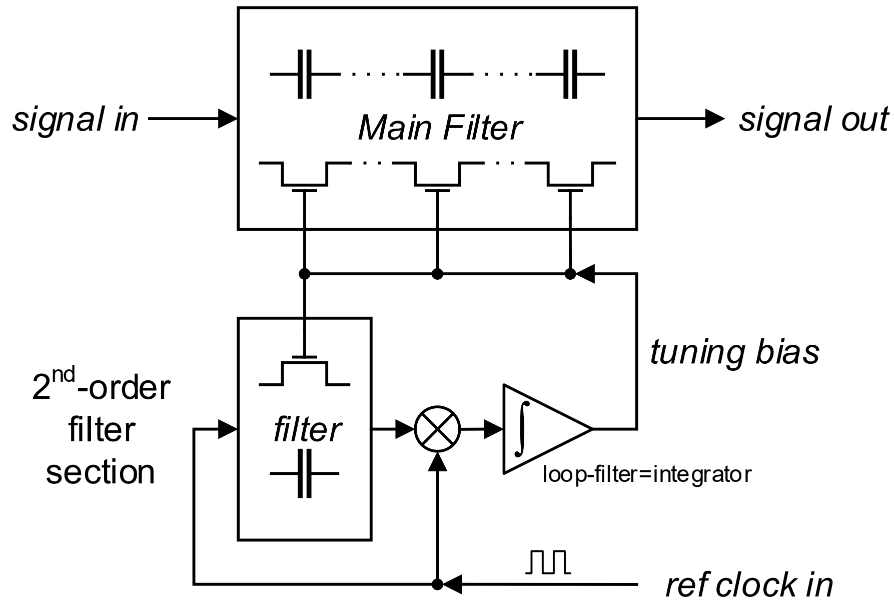
- The circuit imposes that $I_R = \overline{I_{SC}}$ and hence

$$R \cdot C = \frac{1}{f_c}$$

- Accuracy is limited to a few percent by
  - clock feedthrough in the MOS switches
  - OPAMP or OTA offset and speed
  - Imperfect matching between R, C in the control circuit and R,C's in the filter
- For G_m-C filters, the MOSFET resistor is simply replaced by a transconductor

Courtesy F. Krummenacher.

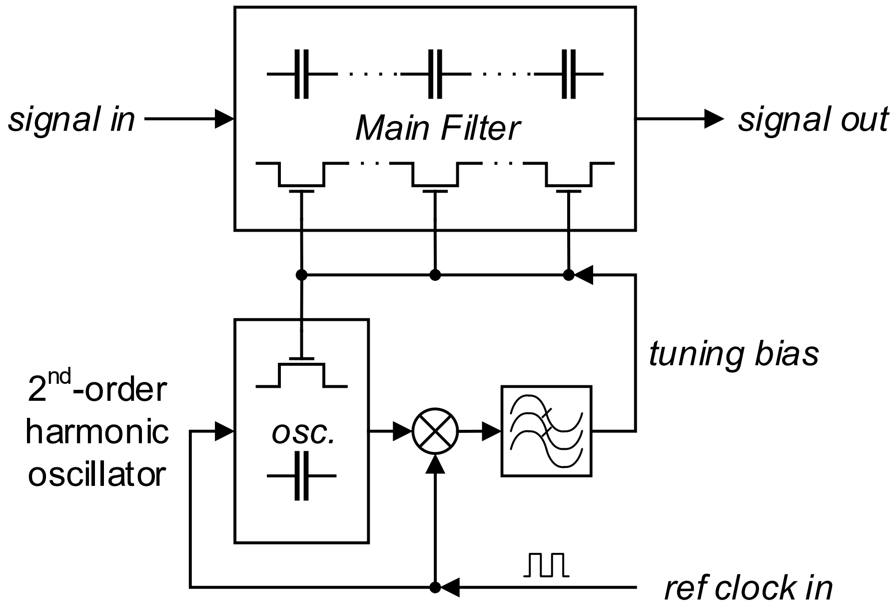
PLL with Voltage-controlled Filter (VCF)



- The phase of the auxiliary filter output signal is compared to that of the reference clock signal and the tuning bias is adjusted until the two differ by a predetermined value (i.e. $\pi/2$)

Courtesy F. Krummenacher.

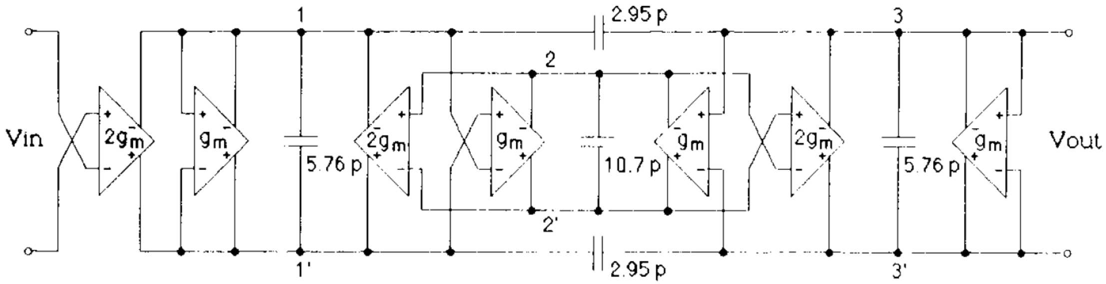
PLL with Voltage-controlled Oscillator (VCO)

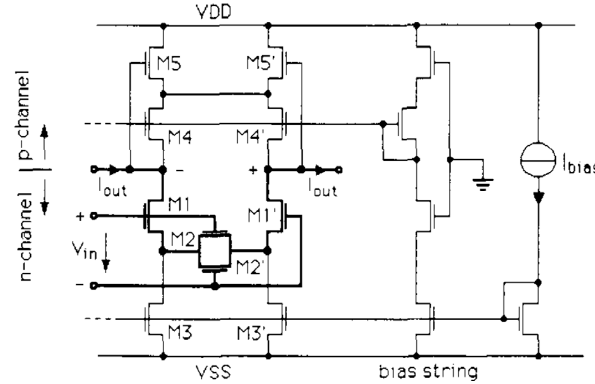


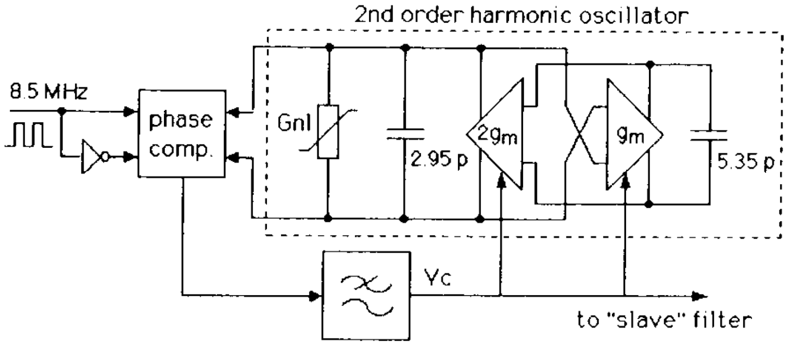
- The frequency (i.e. phase variations) of the oscillator's output signal is compared to that of the clock reference signal

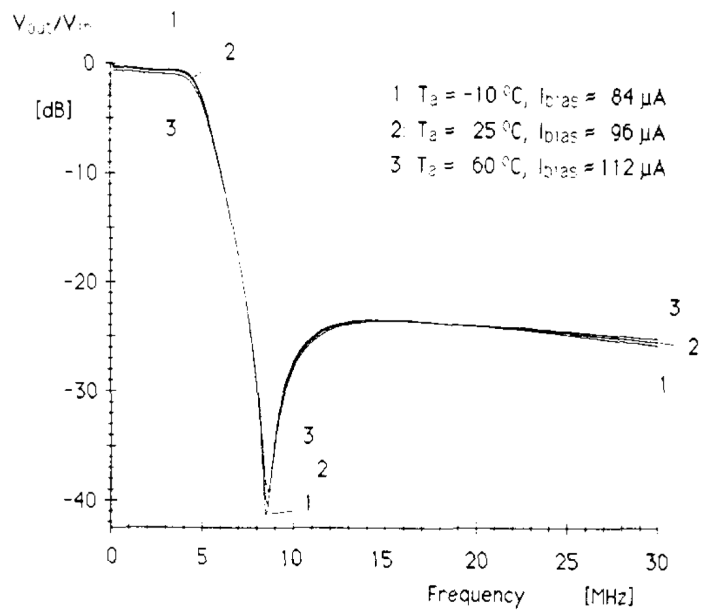
Courtesy F. Krummenacher.

Example of G_m-C Filter with Automatic Tuning









F. Krummenacher and N. Joel, JSSCC, June 1988.