Fundamentals of Electrical Circuits and Systems

Chapter 1: Basic Concepts

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EPFL

07.11 @1:15-4:00	Lecture	Introduction, Basic concepts - circuit elements
07.11 @4:15-5:00	LTSpice	LTSpice Exercise 1: Introduction to LTSpice
14.11 @1:15-3:00	Lecture	Kirchhoff's laws, Fundamental theorems
14.11 @3:15-5:00	Exercises	Problem Sets 1 and 2
21.11 @1:15-3:00		Exam Part I (Prof. Thiran)
21.11 @3:15-4:00	Lecture	Nodal analysis, Mesh analysis
21.11 @4:15-5:00	LTSpice	LTSpice Exercise 2 : DC Analysis
28.11 @1:15-3:00	Lecture	Sinusuoidal regime: phasors, impedance, admittance
28.11 @3:15-5:00	Exercise, LTSpice	Problem Set 3, LTSpice Exercise 3: Impedance
05.12 @1:15-3:00	Lab Session	Circuits in sinusoidal regime
05.12 @3:15-4:00	Lecture	Sinusuoidal regime: Thévenin/Nortpon equivalents, Active and reactive power
05.12 @4:15-5:00	Exercise, LTSpice	Problem Set 4, LTSpice Exercise 4: Cos phi
12.12 @1:15-4:00	Lecture	Three-phase circuits 1
12.12 @4:15-5:00	Exercise	Problem Set 5
19.12 @1:15-3:00	Lecture	Three-phase circuits 2
19.12 @3:15-5:00	Exercise, LTSpice	Problem Sets 6, 7, 8, LTSpice Exercise 5: 3-phase

Search

Simulation Models

Reference Designs

Evaluation Hardware & Software

Packaging, Quality, Symbols & Footprints

Circuit Design Tools & Calculators
- Amplifier & Linear
- Clock & Timing
- Data Converter
- **LTspice**
- Power Management
- RF & Synthesis

Processors & DSP

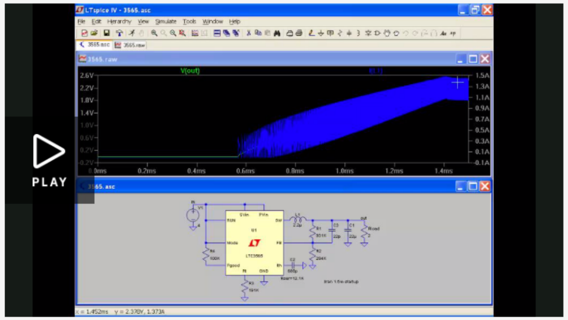
LTspice

LTspice® is a high performance SPICE simulation software, schematic capture and waveform viewer with enhancements and models for easing the simulation of analog circuits. Included in the download of LTspice are macromodels for a majority of Analog Devices switching regulators, amplifiers, as well as a library of devices for general circuit simulation.

Contact Technical Support for assistance

Benefits of using LTspice

Our enhancements to SPICE have made simulating switching regulators extremely fast compared to normal SPICE simulators, allowing the user to view waveforms for most switching regulators in just a few minutes. This video provides an overview of the advantages of using LTspice in an analog circuit design and how easy it is to get started.



Download LTspice

Download our LTspice simulation software for the following operating systems:

Download for Windows 7, 8 and 10 Updated on Oct 15 2020 *

Download for Mac 10.9+ Updated on Oct 15 2020 *

Feedback

Evaluation

- Final exams during the winter session
- EE-407: 60% exam, 30% LTSpice simulations, 10% lab

Electrical circuits: Fundamentals

- What is an electrical circuit ?

- Unit systems

- Usual quantities
  - Charge, current, voltage
  - Power and energy

- Basic circuit elements
  - Resistance
  - Capacitance
  - Inductance
  - Voltage and current sources

- Kirchhoff's law
- Combination of elements
- Voltage / current dividers

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  - to transport energy from one point to another

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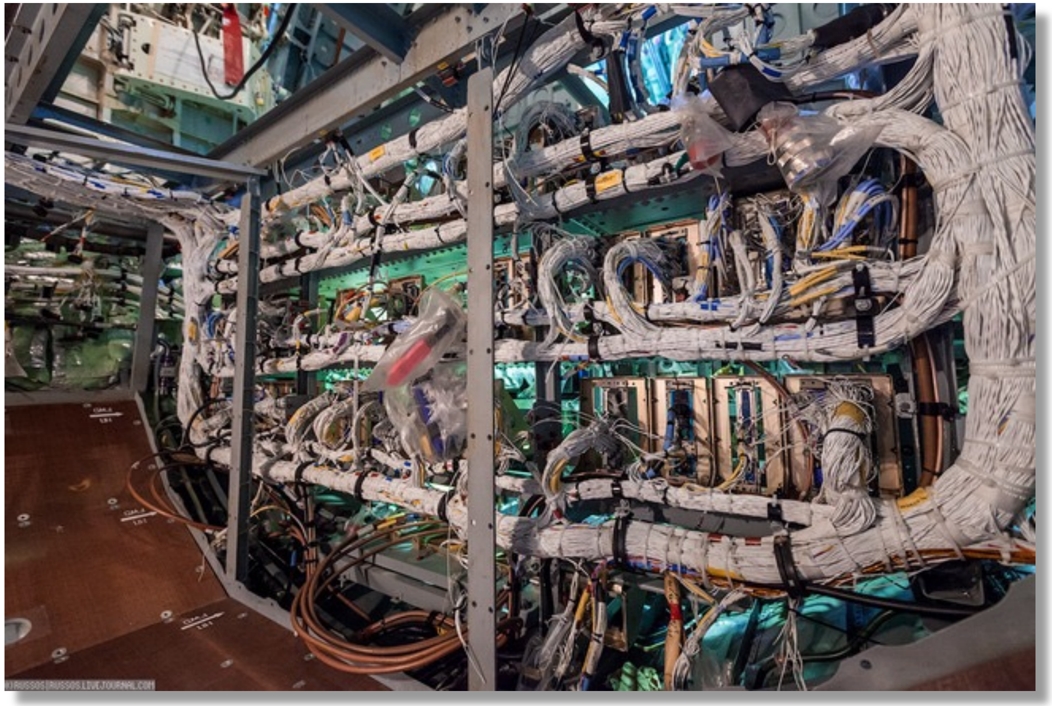
- An electrical circuit allows
  - to transport energy from one point to another
  - to transmit information



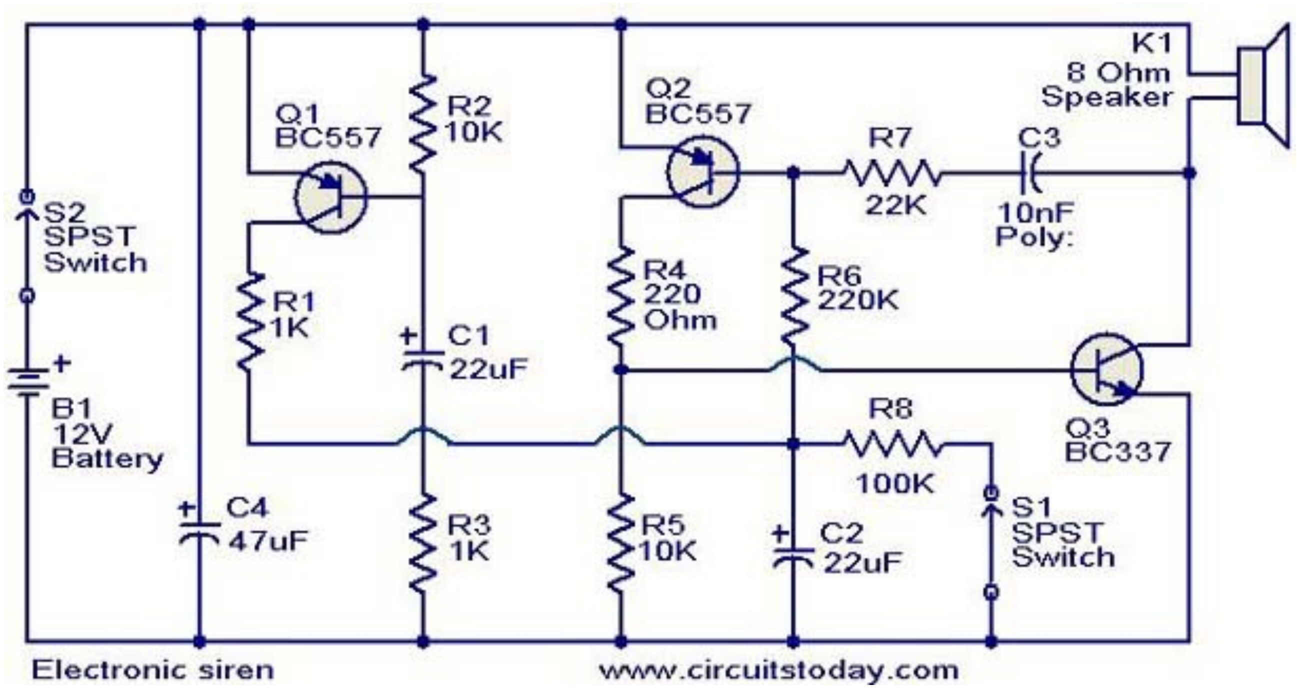
Wiring in an airplane

What is an electrical circuit ?

- An electrical circuit allows
  - to transport energy from one point to another
  - to transmit information
  - to process information

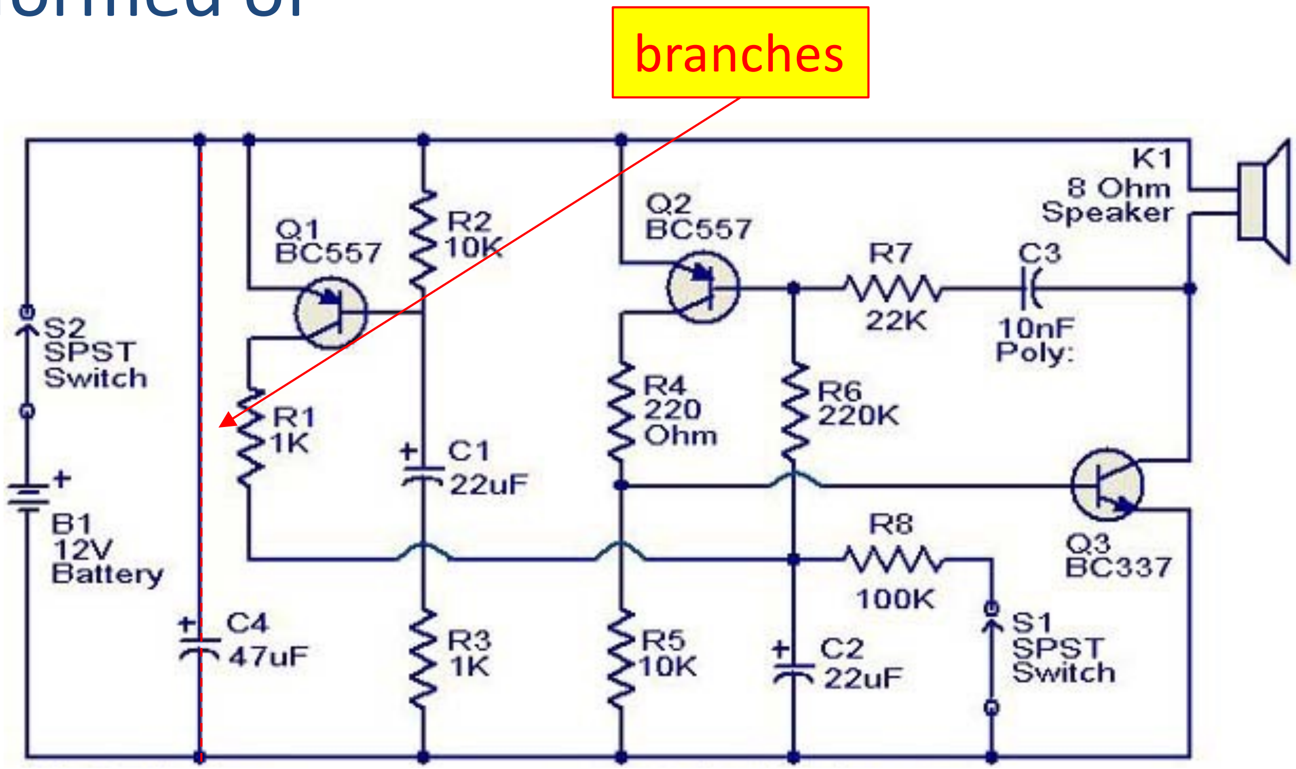
What is an eletrical circuit ?

- An electrical circuit is an interconnection between electrical elements (components).



What is an electrical circuit ?

- An electrical circuit is an interconnection between electrical elements (components).
- It is formed of



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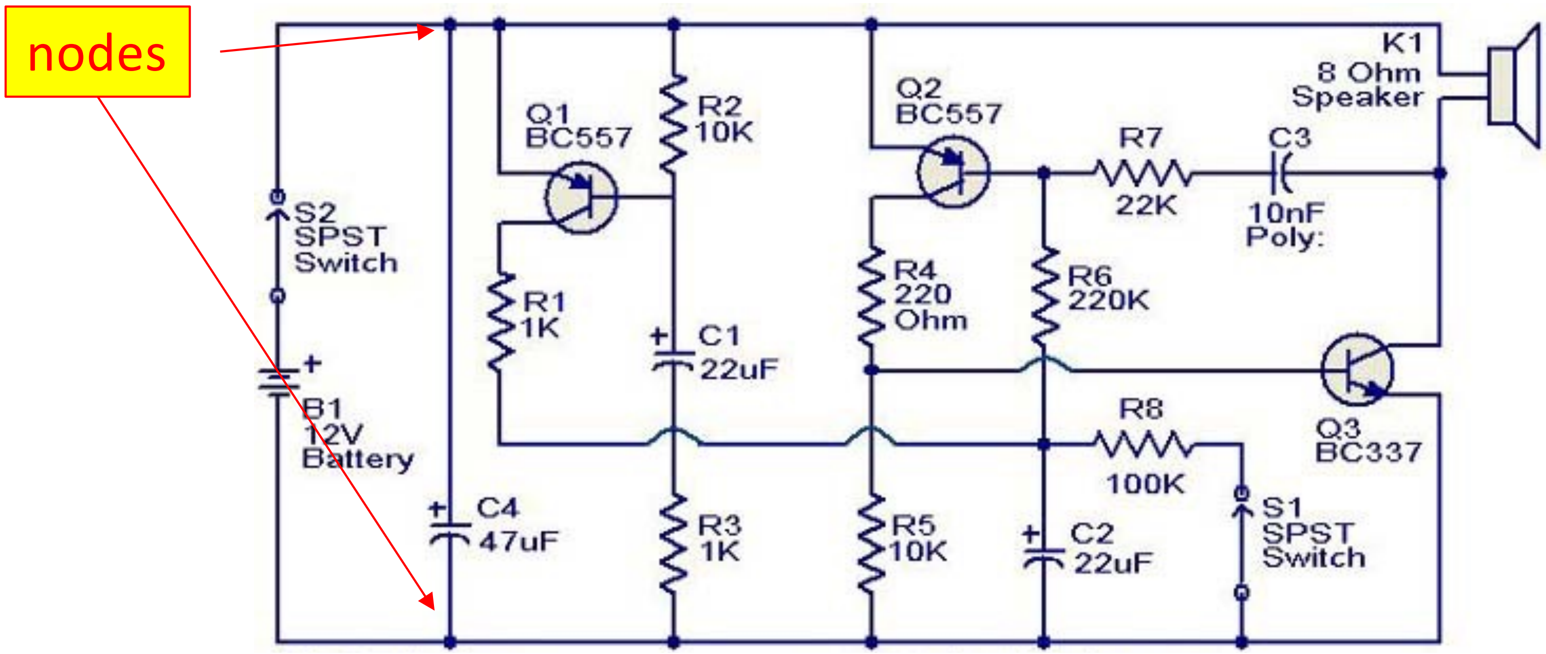
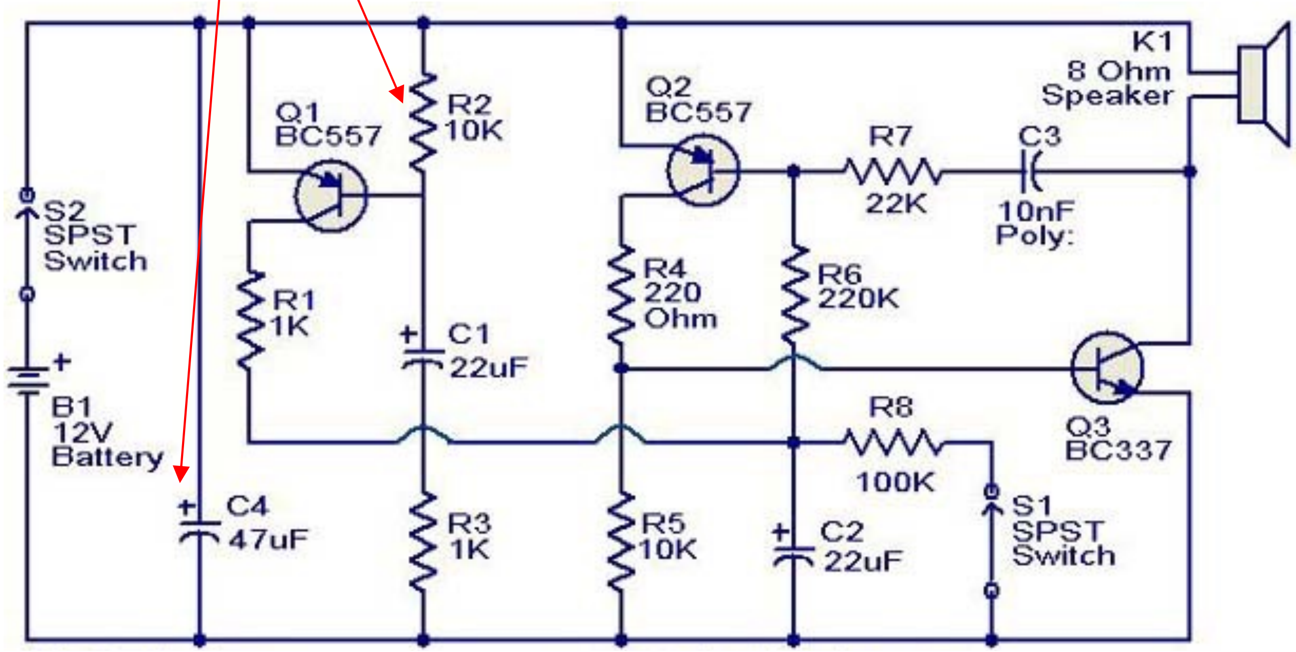


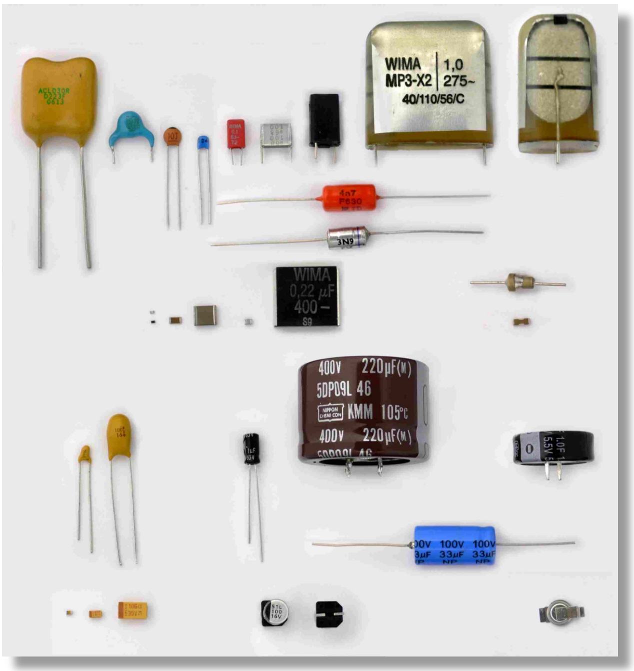
What is an electrical circuit ?

Passive linear elements



What is an electrical circuit ?

- An electrical circuit is an interconnection between electrical elements (components).
- It is formed of branches and nodes
- Passive linear elements are resistors, inductors and capacitors.



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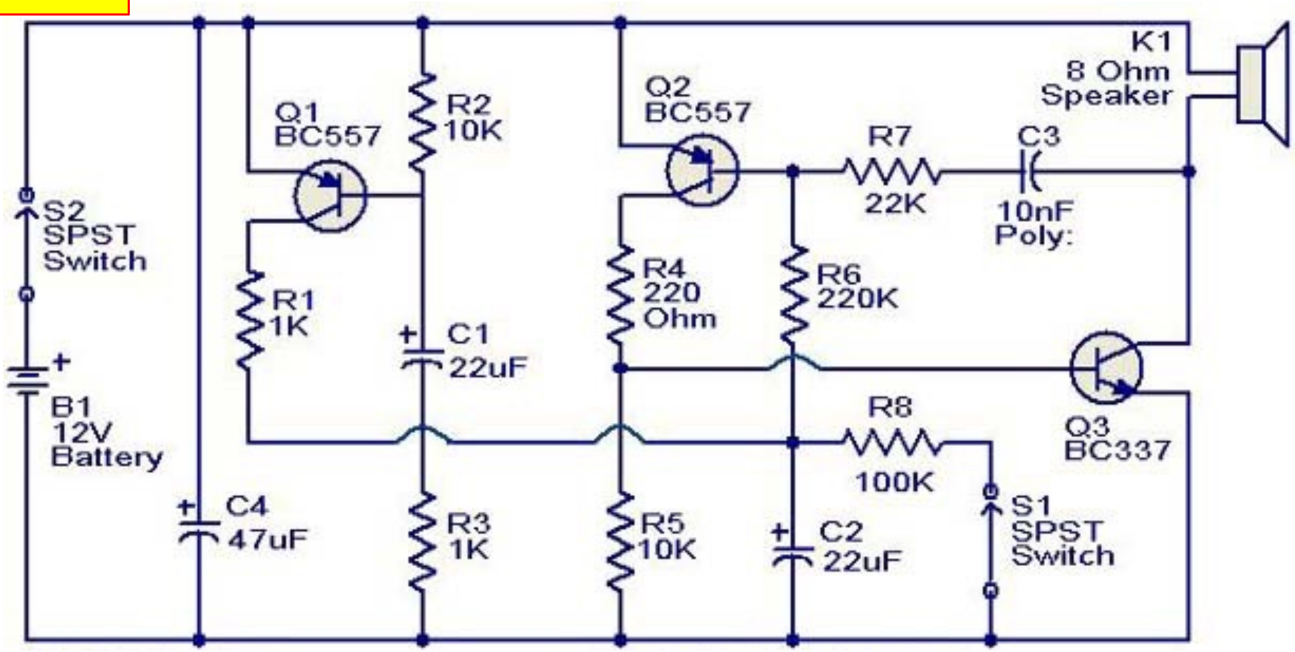
- Active elements (sources) are sources of voltage and current.

What is an electrical circuit ?

Active elements
(sources)



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- An electrical circuit is an interconnection between electrical elements (components).
- It is formed of branches and nodes
- Passive linear elements are resistors, inductors and capacitors.
- Active elements are sources of voltage and current.
- The circuits are expressed in terms of two basic quantities: current and voltage.

Electrical circuits: Fundamentals

- What is an electrical circuit ?
- Unit systems
- Usual quantities
  - Charge, current, voltage
  - Power and energy
- Basic circuit elements
  - Resistance
  - Capacitance
  - Inductance
  - Voltage and current sources
- Kirchhoff's law
- Combination of elements
- Voltage / current dividers

Electric charge

- The electric charge is an electrical property of the atomic particles that make up the matter, it is measured in Coulomb (C).

- $1\ C = 1/1.602\text{x}10^{-19} = 6.24\text{x}10^{18}$ electrons

- The only charges that exist in nature at our scale are integer multiples of the elementary charge
  $e = -1.602\text{x}10^{-19}\ C$.

Charles de **Coulomb** (1736-1806),
French physicist.

Charge and electric current

- The electric current is the time rate of change of the charge, measured in Ampere (A).

André-Marie **Ampère** (1775-1836), French physicist, founder of electrodynamics. We owe him the terms of voltage and current.

Charge and electric current

- The electric current is the time rate of change of the charge, measured in Ampere (A).

- The relationship between the charge q and the current i is given by

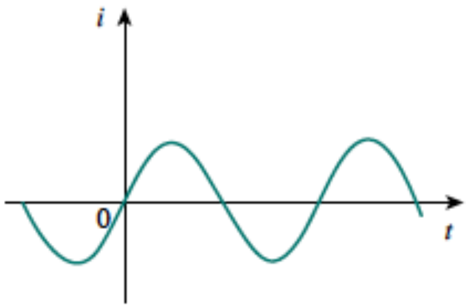
$$i(t) = \frac{dq}{dt}$$

$$q(t) = \int_0^t i(\tau)\, d\tau$$

Charge and electric current

- A direct current I is a current independent of time.

- An alternating current is a current that periodically changes direction.



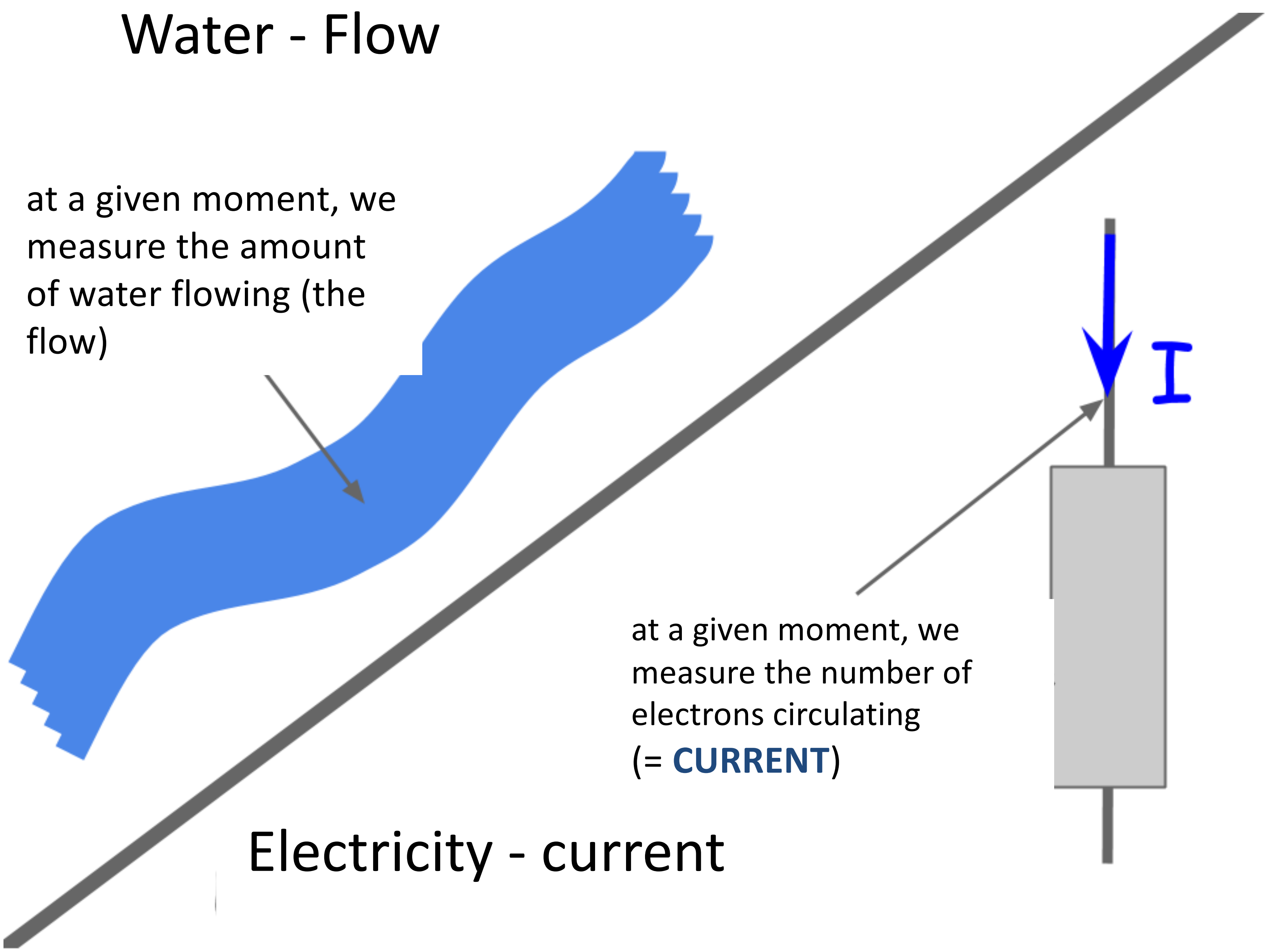
Sinusoidal
alternating current

Voltage, potential difference, electromotive force

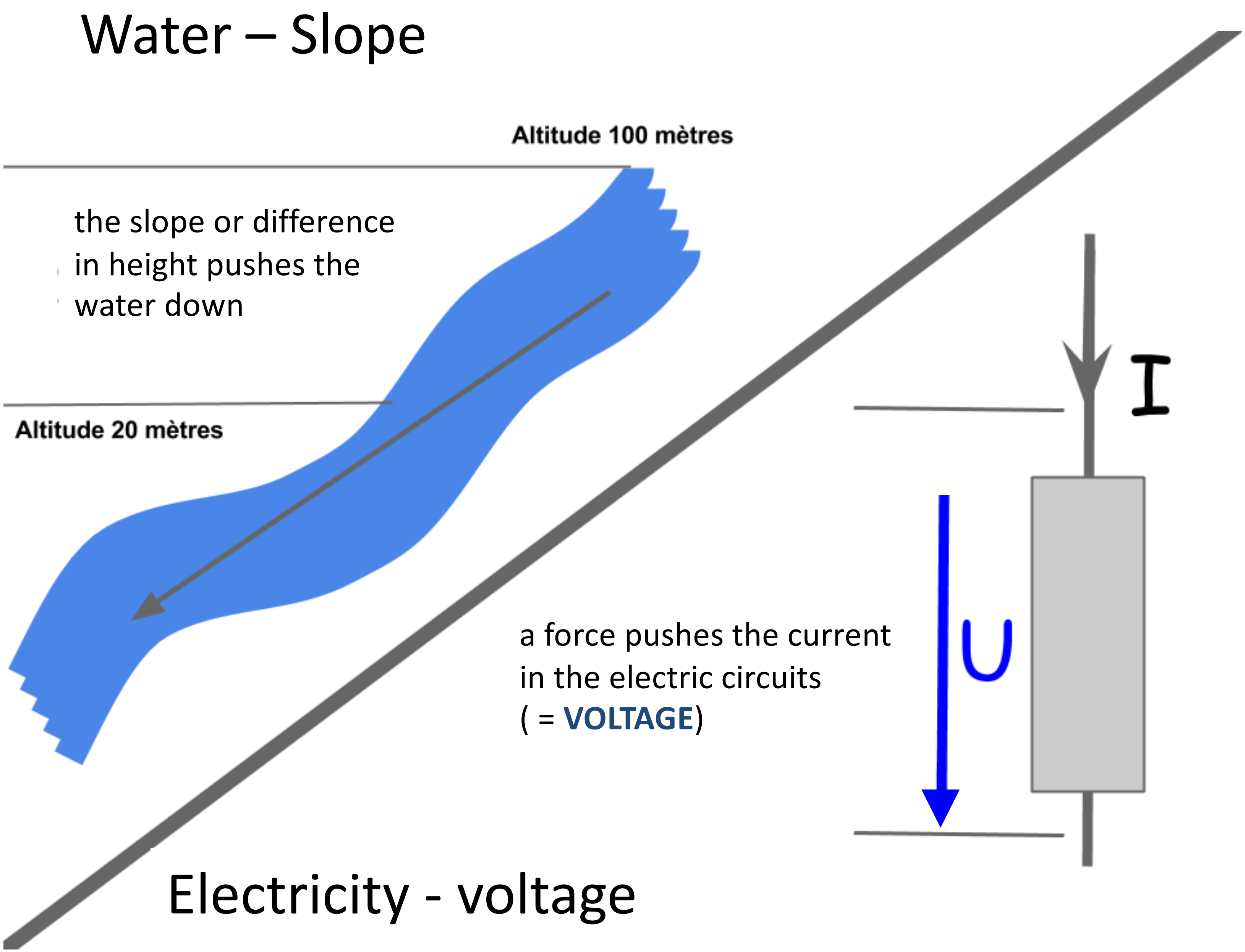
- Moving electrons in a conductor in a particular direction requires some work or energy transfer.
- This work is performed by an external force to the charge provided by a power source and called electromotive force, voltage or potential difference.
- The voltage or potential difference v_{ab} between two points of a circuit is the energy required to move a unit of electric charge from point a to point b.

Comte Alessandro **Volta** (1745-1827), italian physicist, inventor of the electric battery (1800).

Analogy


Water - Flow
at a given moment, we measure the amount of water flowing (the flow)
I
at a given moment, we measure the number of electrons circulating (= **CURRENT**)
Electricity - current

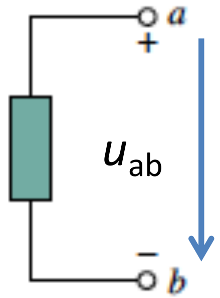
Analogy


Water – Slope
Altitude 100 mètres
the slope or difference in height pushes the water down
Altitude 20 mètres
I
a force pushes the current in the electric circuits (= VOLTAGE)
U
Electricity - voltage

Voltage, potential difference, electromotive force

- Mathematically :

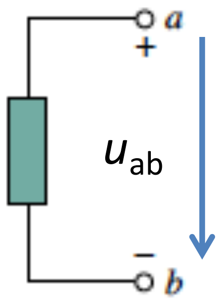
$$u_{ab} = dw / dq$$

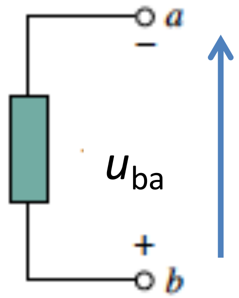


- w is the energy (J) and q is the charge(C).

- The voltage u_{ab} is defined between two points of the circuit a and b.
- if $u_{ab}>0$: the potential of point a is *greater* than the potential of point b
- if $u_{ab}<0$: the potential of point a is *smaller* than the potential of point b

Voltage, potential difference, electromotive force





u_{ab}= 9 V

u_{ba}= -9 V

Voltage and electric field

- The voltage between two points a and b can also be defined as the integral of the electric field between these two points

$$u_{ab} = \int_a^b \vec{E} \cdot \overrightarrow{dl}$$

Power and Energy

- Power is the rate at which you consume energy.

- The watt (W), unit of power, corresponding to the transfer of 1 joule of energy over one second.

- Mathematically :

$$p = \frac{dw}{dt}$$

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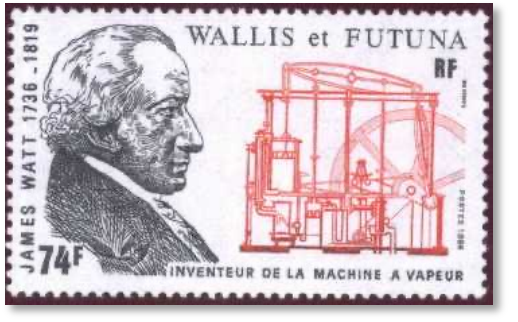
James **Watt** (1736-1819), mechanical engineer and British inventor. We owe him the steam engine.



Power and Energy

- In an electrical circuit, the instantaneous power p(t) is equal to the product of the instantaneous voltage and the instantaneous current, and the average power P is equal to the mean value of the instantaneous power

$$p(t) = u(t) \cdot i(t)$$

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u(t) i(t) dt \quad \text{ou} \quad P = \frac{1}{T} \int_{0}^{T} u(t) i(t) dt$$

Power and Energy

- The energy absorbed or supplied by an element during the time interval from 0 to t is :

$$w(t) = \int_0^t p(\tau)d\tau$$

- The law of energy conservation implies that the algebraic sum of the power in a circuit, at any instant, must be zero :

$$\sum P = 0$$

Joule

- The Joule (J) is equivalent to the work produced by a force of 1 N whose point of application moves one meter in the direction of the force.

$$w(t) = \int_0^t p(\tau) d\tau$$

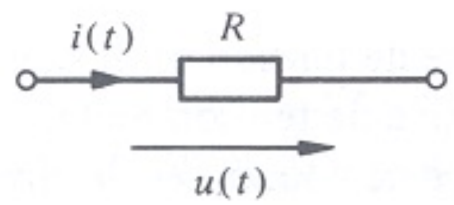
James prescott **Joule** (1818-1890), British physicist. He studied the heat released by the electric currents (Joule heating).

Passive elements

Resistance/Resistor

- Circuit element that satisfies Ohm's law

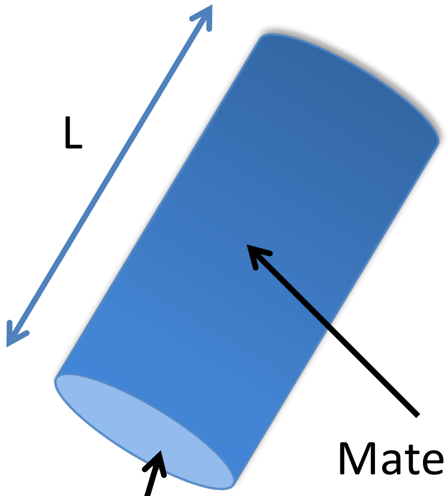
$$u(t) = Ri(t)$$

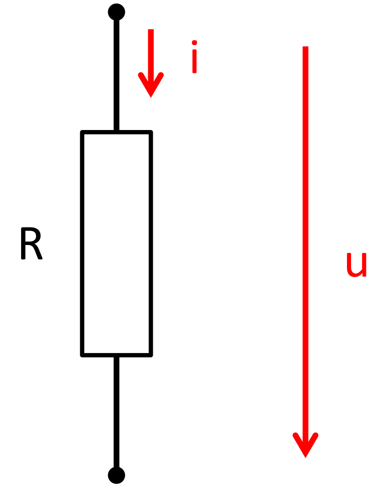


- The unit of resistance is the ohm (Ω)

Georg Simon **Ohm** (1789-1854), German physicist. He discovered in 1827 the fundamental laws of electric currents.

Ohm's Law and Resistance

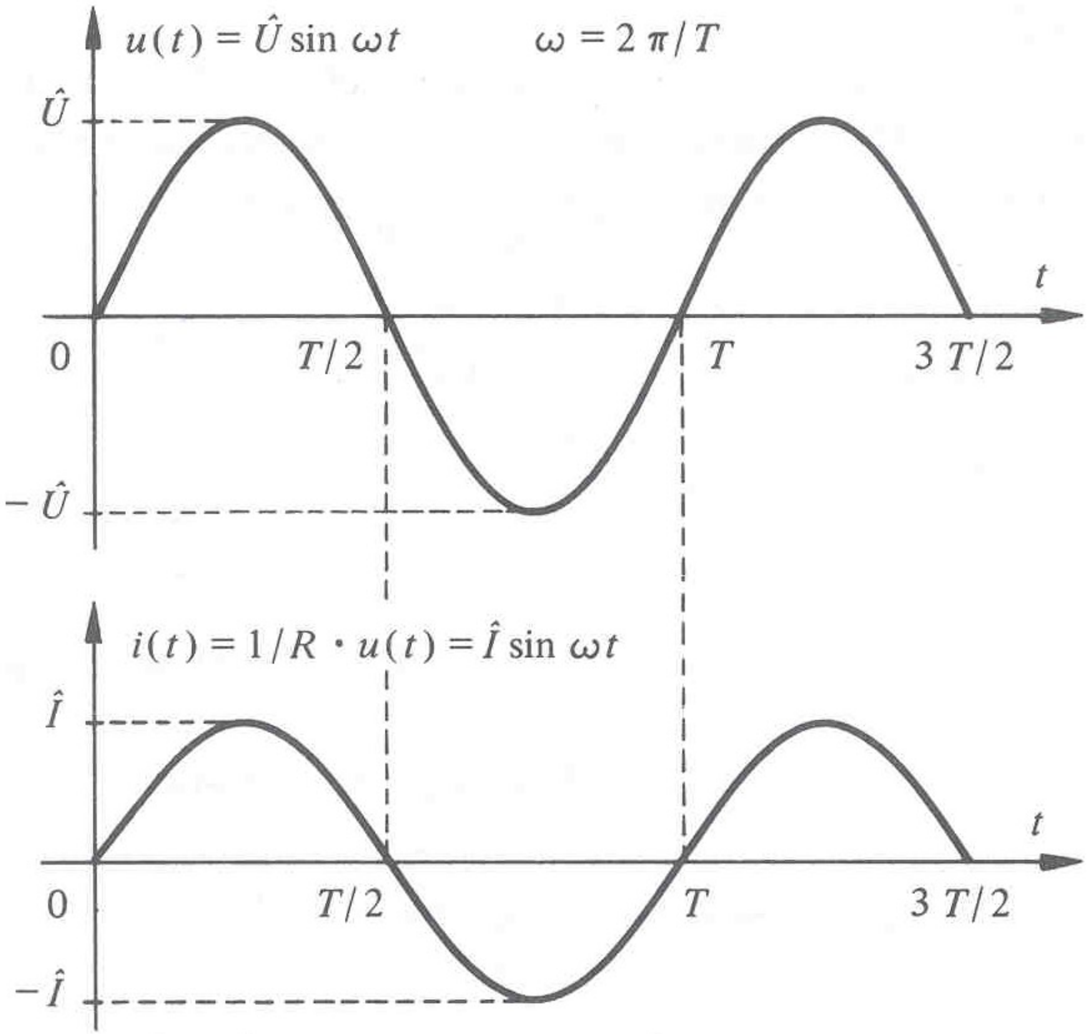




Ohm's law : $E = \rho J$ J: Current density A/m²

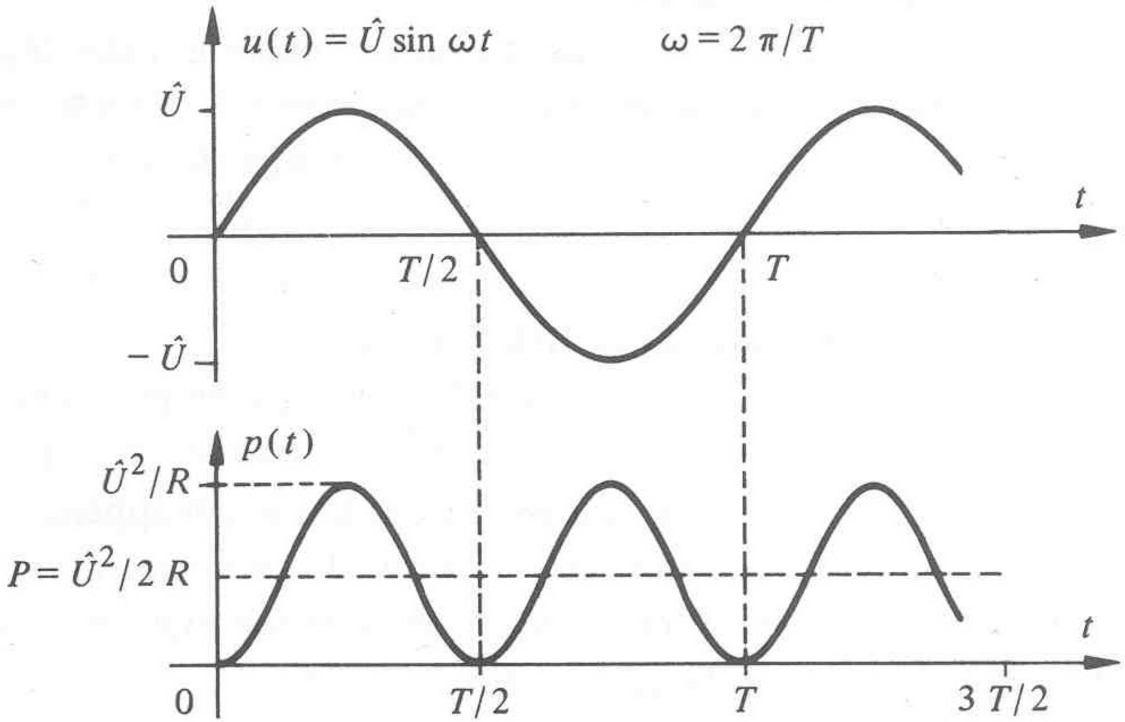
$\Rightarrow$ $u = R \cdot i$ $R = \rho \dfrac{L}{S}$

Resistance: current and voltage



Resistance: dissipated power

$$p(t) = u(t)i(t) = Ri^2(t) = \frac{u^2(t)}{R}$$



Resistance: dissipated energy

- The electrical energy supplied to a resistor for a duration t is dissipated in the form of heat

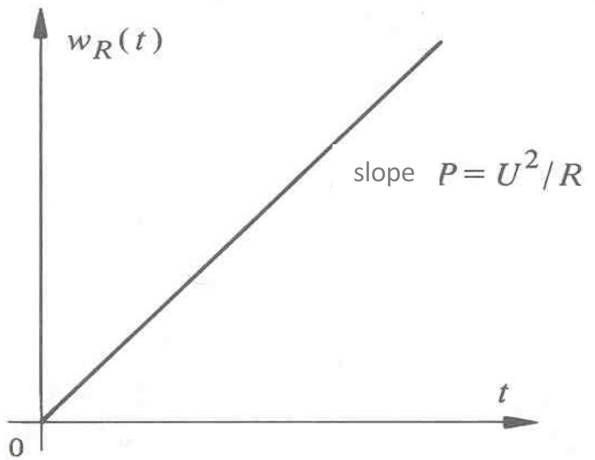
$$w_R(t) = R\int_0^t i^2(\tau)d\tau = \frac{1}{R}\int_0^t u^2(\tau)d\tau$$

- Continuous regime:

$$i(t) = I \quad u(t) = U \Rightarrow \ w_R(t) = RI^2t = \frac{U^2t}{R}$$

Resistance: dissipated energy

- Continuous regime :

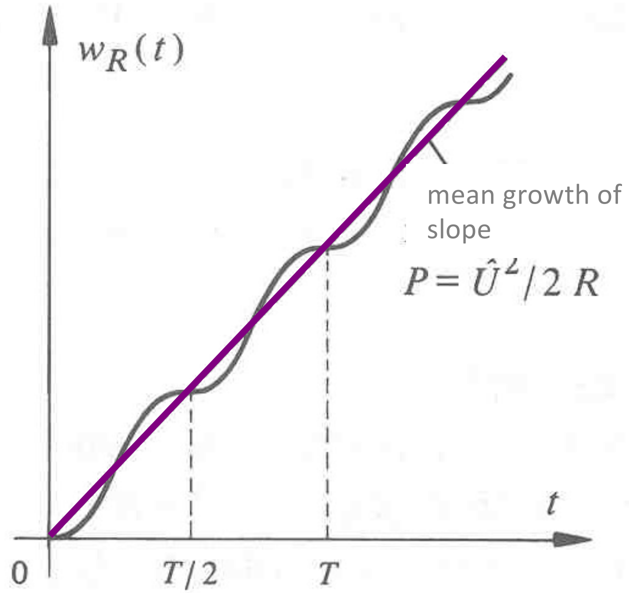


$$w_R(t) = RI^2 t = \frac{U^2 t}{R}$$

- Sinusoidal regime:

$$u(t) = \hat{U} \sin \omega t$$

$$w_R(t) = \frac{\hat{U}^2}{2R}\left[t - \frac{1}{2\omega} \sin(2\omega t) \right]$$



Short circuit

- Let a circuit whose two points are connected by a resistor. If the value of this resistance tends to zero, these two points become short circuited.

- In practice, a piece of wire is often (not always!) a short circuit.

Open circuit

- Let a circuit whose two points are connected by a resistor. If one makes the value of this resistance tend towards infinity, these two points become in open circuit.

- In practice, an open circuit is realized by removing all the elements that connect the two points of the circuit.

Conductance

- Inverse of resistance G=1/R

$$i(t) = Gu(t)$$

- The unit of conductance is the Siemens (S)

Werner von **Siemens** (1816-1892),
German engineer and industrialist.

Capacitance/Capacitor

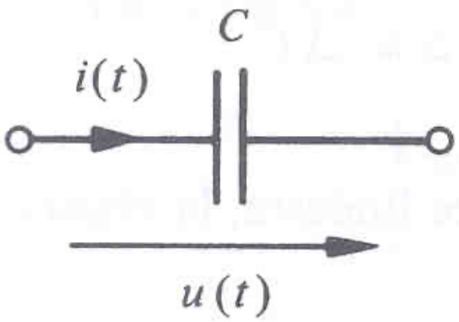
- Circuit element that obeys the following relationship : $$q(t) = Cu(t)$$
- Knowing that the current is the flow of charges :

$$i(t) = \frac{dq}{dt}$$

$$\boxed{i(t) = C\frac{du}{dt} \qquad u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = u(0) + \frac{1}{C}\int_{0}^{t} i(\tau)d\tau}$$

Capacitance

The value C of the capacitance is expressed in farad (F).

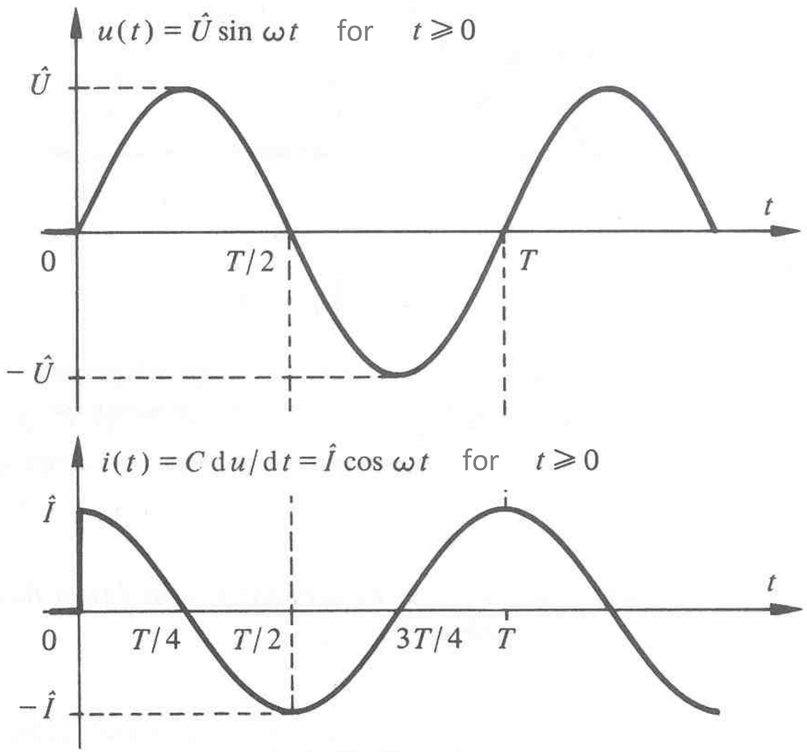


Michael **Faraday** (1791-1867),
British physicist.

Capacitance: current and voltage

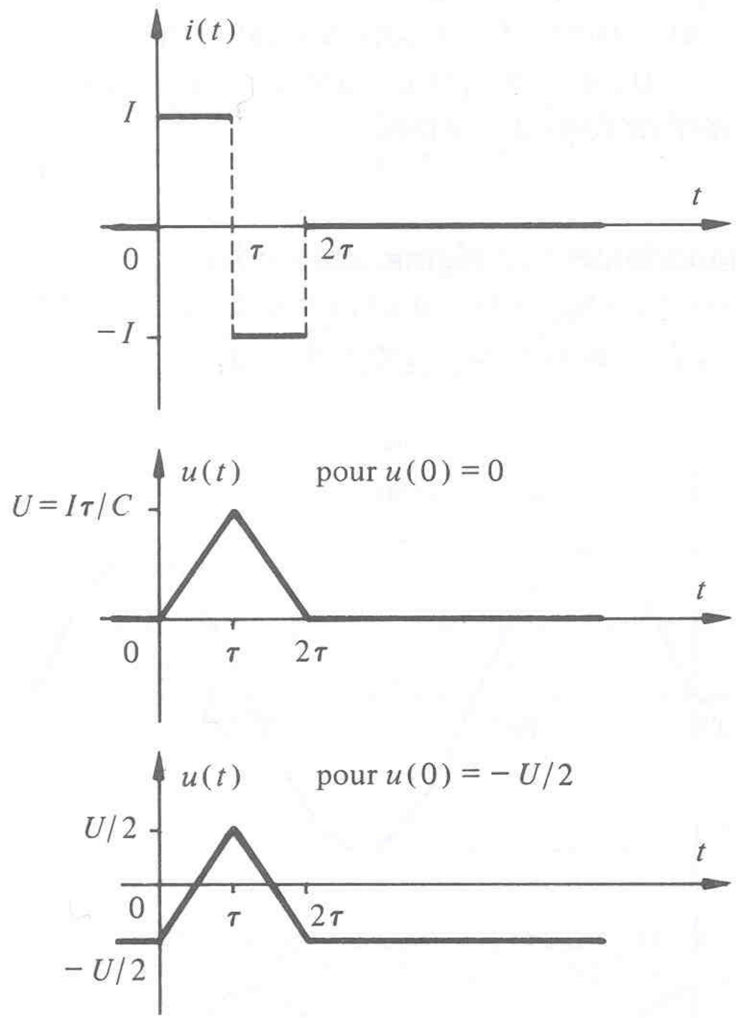
- In continuous regime : open circuit

- In sinusoidal regime :



$$\hat{I} = \omega C \hat{U}$$

Capacitance: no voltage step!



$$u(t) = u(0) + \frac{1}{C}\int_0^t i(\tau)d\tau$$

Capacitance: electrostatic energy

$$w_c(t) = \frac{1}{2} C u^2(t)$$

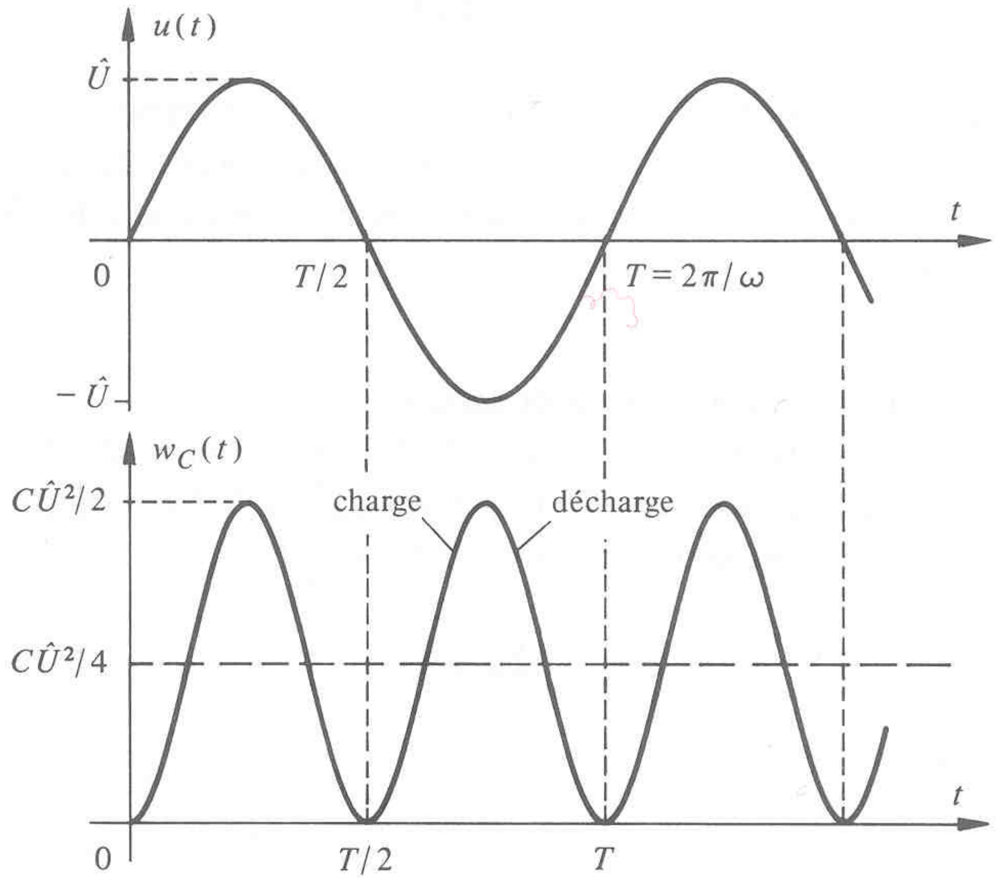
- Sinusoidal regime: $u(t) = \hat{U} \sin \omega t$

$$w_c(t) = \frac{C\hat{U}^2}{2} \sin^2 \omega t$$

$$w_c(t) = \frac{C\hat{U}^2}{4} \left[1 - \cos 2\omega t\right]$$

Capacitance: electrostatic energy

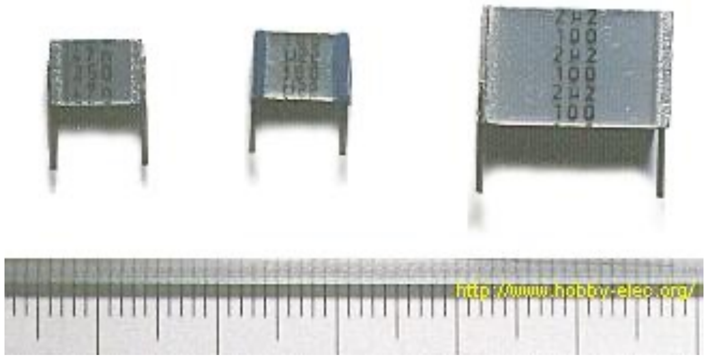
- Sinusoidal regime:



$$w_c(t) = \frac{C\hat{U}^2}{4}\left[1 - \cos 2\omega t\right]$$

Capacitor: physical element

Ceramic (100 pF, 10 nF)

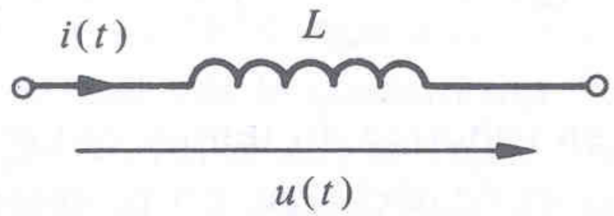


Metallic polyester film
(1 nF, 220 nF, 2.2 mF)

Inductance/Inductor

- Circuit element that obeys the following relationship :

$$u(t) = L\frac{di}{dt} \qquad i(t) = \frac{1}{L}\int_{-\infty}^{t} u(\tau)d\tau = i(0) + \frac{1}{L}\int_{0}^{t} u(\tau)d\tau$$



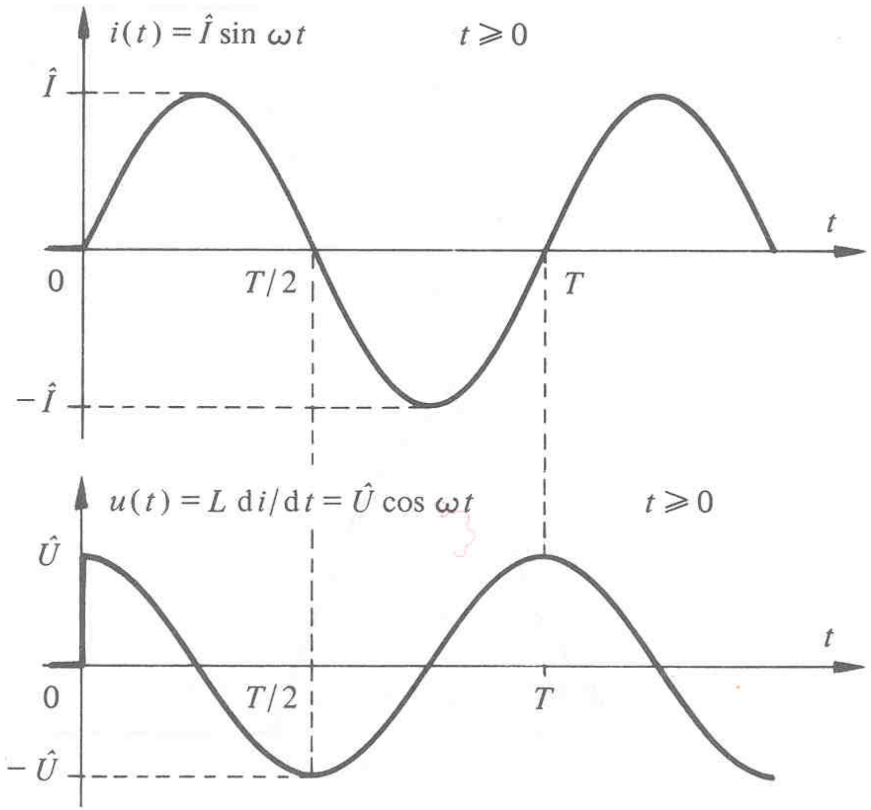
- The unit of inductance is henry (H).

Joseph **Henry** (1797-1878),
american physicist.

Inductance: current and voltage

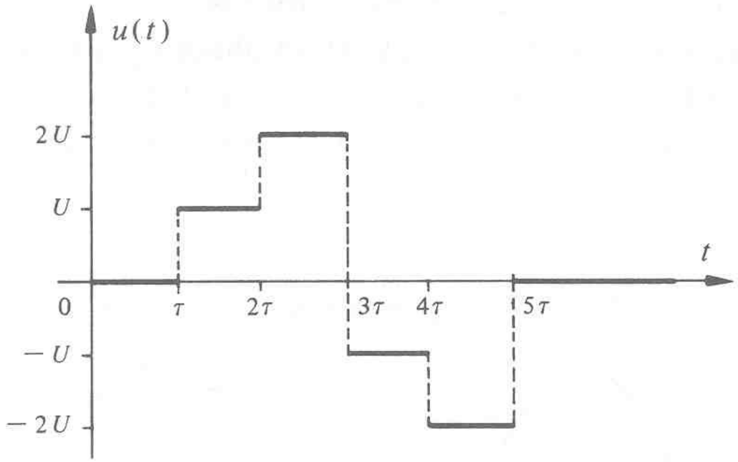
- In continuous regime: short circuit

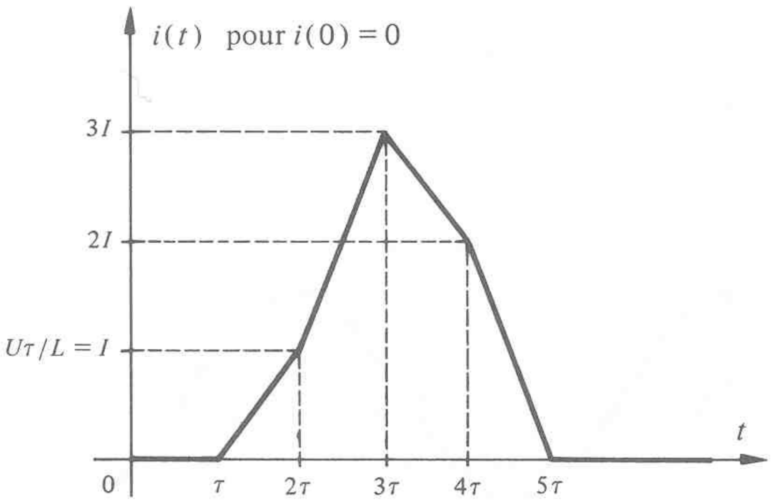
- In sinusoidal regime:



$$\hat{U} = \omega L \hat{I}$$

Inductance: no current step!





Inductance: magnetic energy

$$w_L(t) = \frac{1}{2} L i^2(t)$$

- Sinusoidal regime: $i(t) = \hat{I} \sin \omega t$

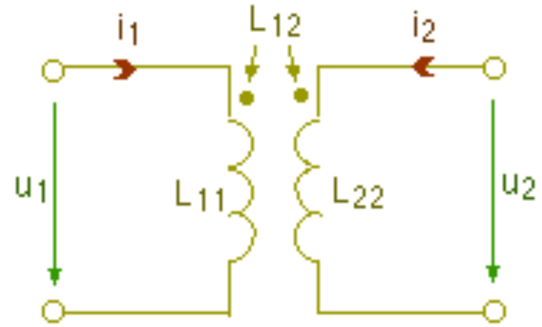
$$w_L(t) = \frac{L\hat{I}^2}{2} \sin^2 \omega t = \frac{L\hat{I}^2}{4} \left[1 - \cos 2\omega t\right]$$

Coupled (Mutual) inductors

- Coupled inductors constitute a biporte defined by the constitutive relations :

$$u_1 = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$u_2 = L_{12}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$



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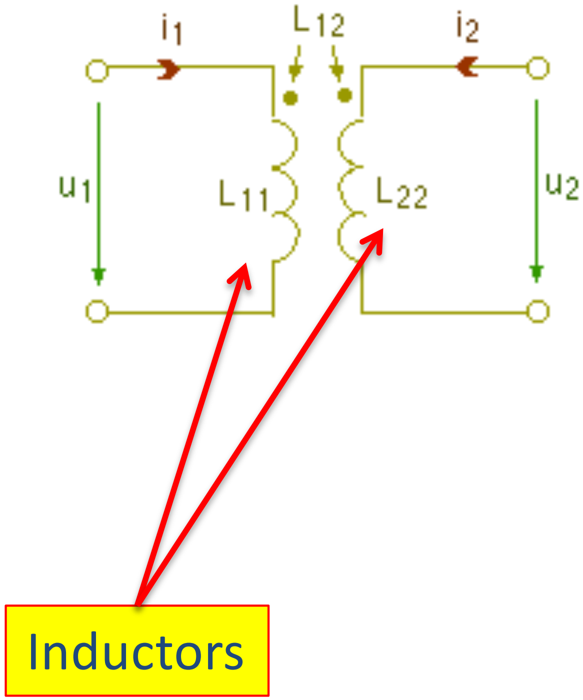

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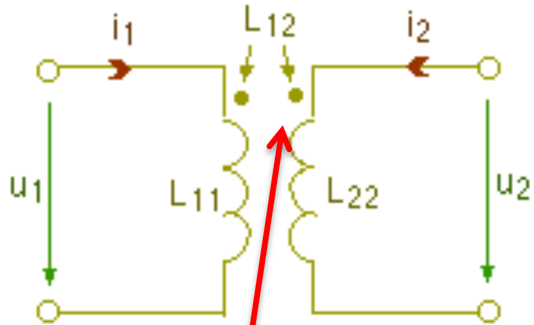
$$u_1 = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$u_2 = L_{12}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$



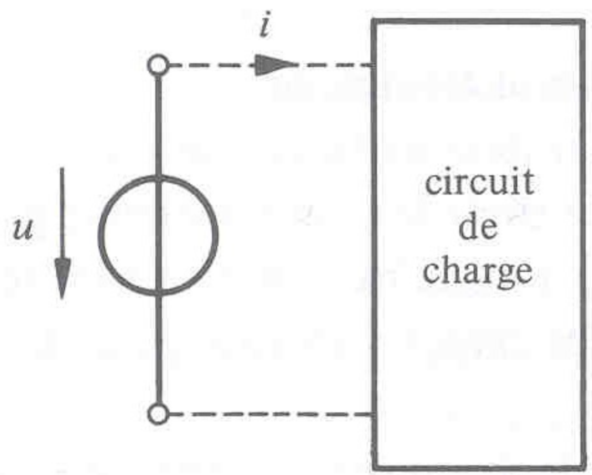
Mutual inductances

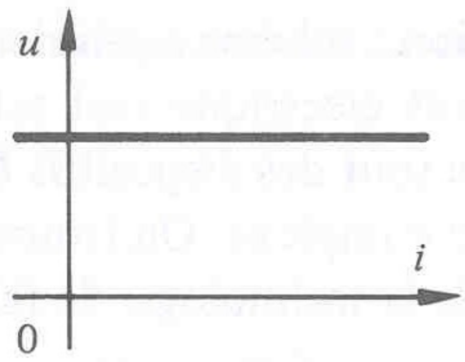
Active circuit elements

Ideal voltage source

- Circuit element capable of supplying or absorbing electrical energy and whose voltage across its terminals is constant and independent of the charge connected to it.
- A voltage source provides no energy when it is in open circuit.

- Symbol :



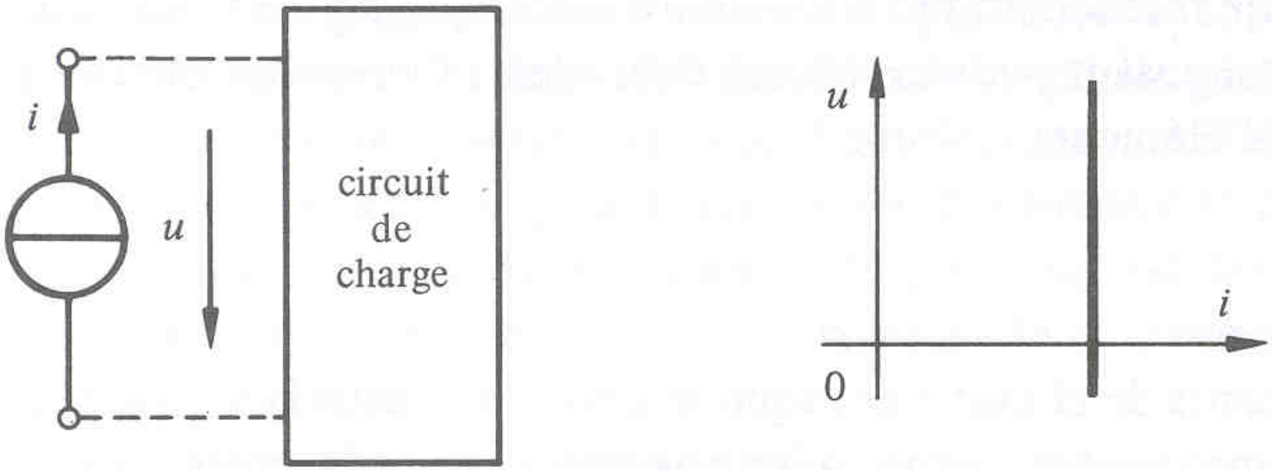


Examples of sources

Ideal current source

- Circuit element capable of supplying or absorbing electrical energy and whose current flowing through it is constant and independent of the charge connected to it.
- A current source provides no energy when it is short-circuited.

- Symbol :

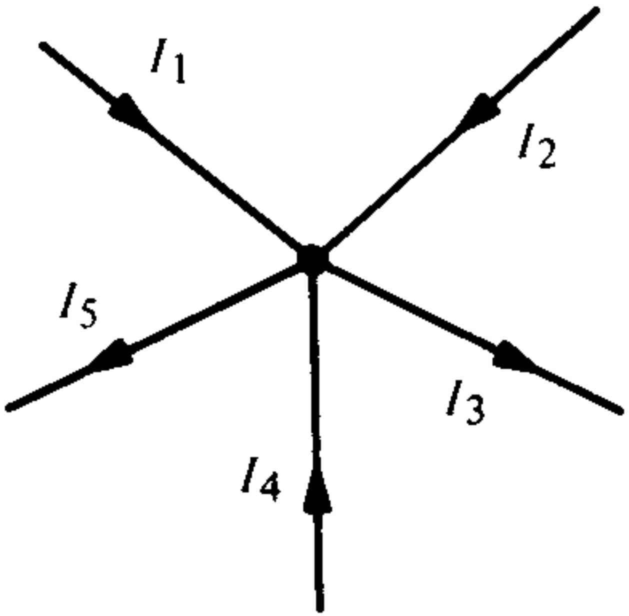


Kirchhoff’s Laws

- Fundamental laws of electrical circuits

Gustav Robert **Kirchhoff** (1824-1897),
German physicist.

Kirchhoff’s current Law

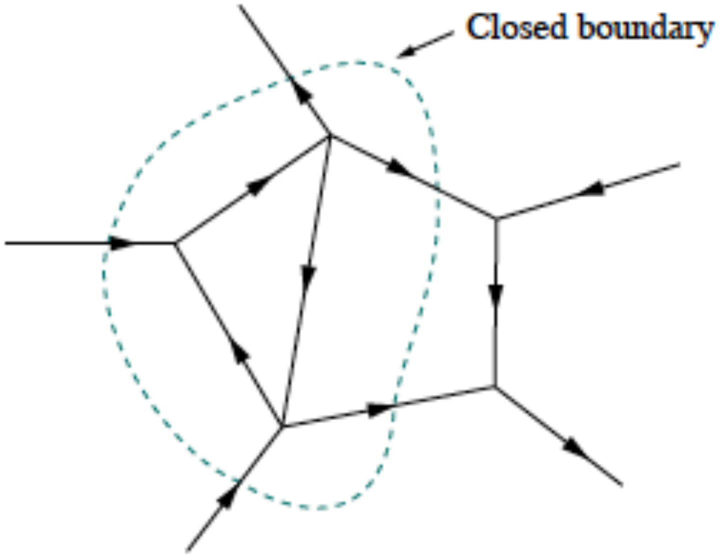


$$\sum_{j=1}^{N} i_j = 0$$

$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

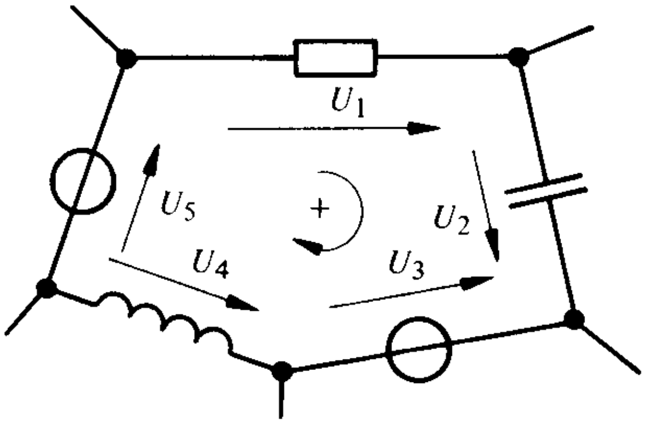
Kirchhoff’s current Law

- The sum of currents in a closed boundary is zero



$$\sum_{j=1}^{N} i_j = 0$$

Kirchhoff’s voltage Law



$$\sum_{j=1}^{N} u_j = 0$$

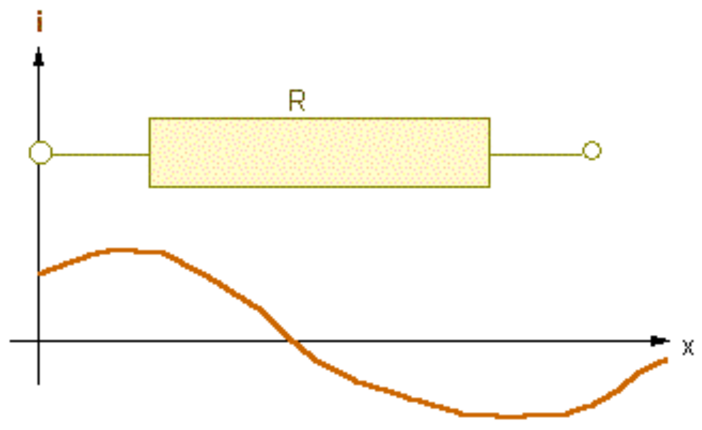
$$U_1 + U_2 - U_3 - U_4 + U_5 = 0$$

Limitations of the Kirchhoff model

- Kirchhoff's model is a special case of a more general model, the Maxwell model.
- Maxwell's equations describe the phenomenon of wave propagation. A basic assumption of Kirchhoff's model is the equality of the current entering and leaving a node.

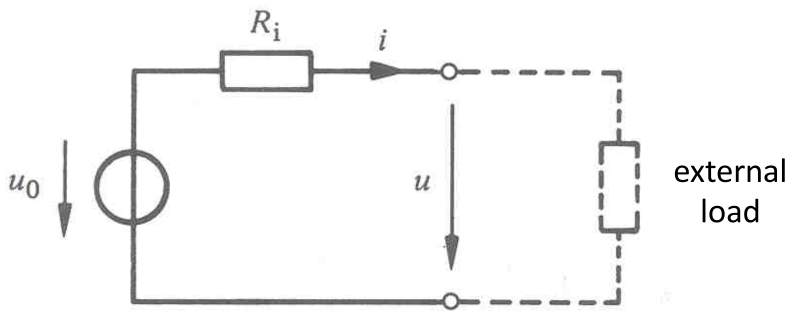
- For example, the current at both ends of a resistor is the same at all times. This is no longer the case if a wave propagates along the resistance.

- However, if the wavelength is much greater than the length of the resistance, the Kirchhoff model remains valid.



'Real' voltage source

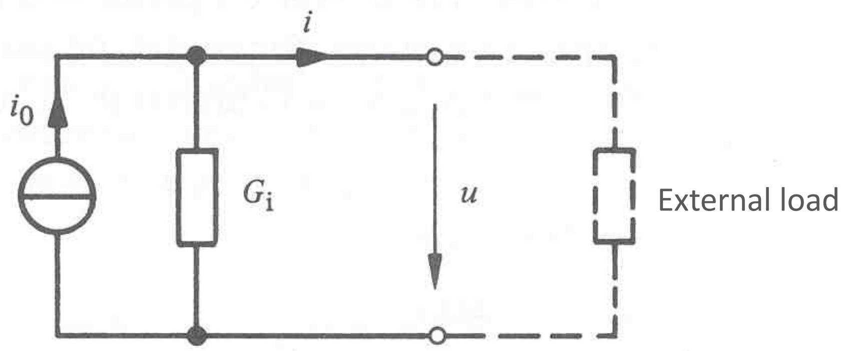
- The voltage of a real voltage source such as a battery decreases with the current it provides. We can model such a source by an ideal source in series with a resistance :



$$u = u_o - R_i i$$

'Real' current source

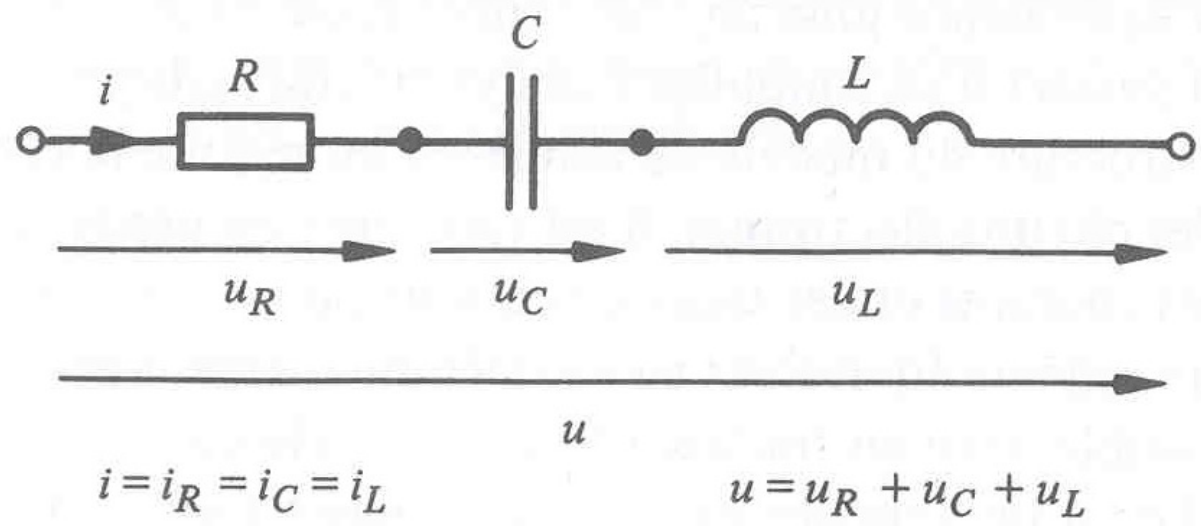
- Current-voltage characteristic and equivalent circuit



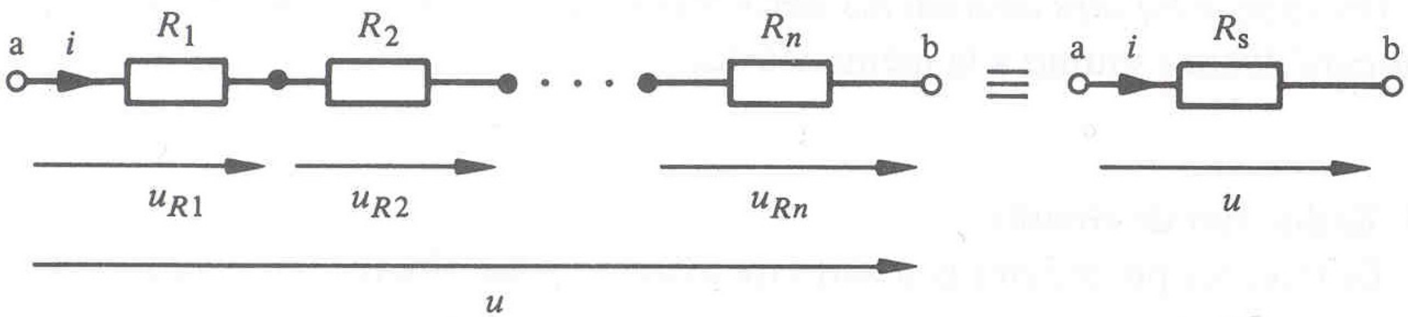
$$i = i_o - G_i u$$

Elements in series

- Elements connected in series are traversed by the same current. The voltage across the circuit is equal to the sum of the voltages relative to each element.

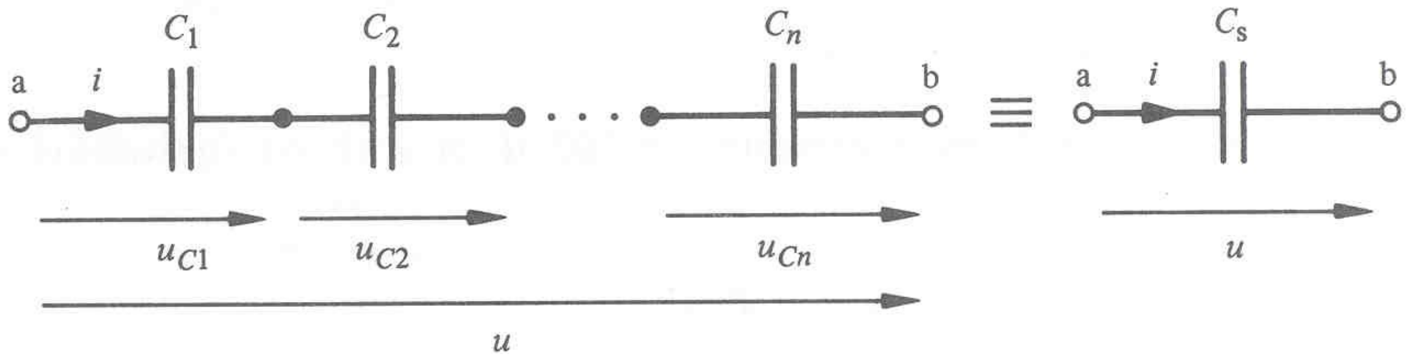


Resistors in series



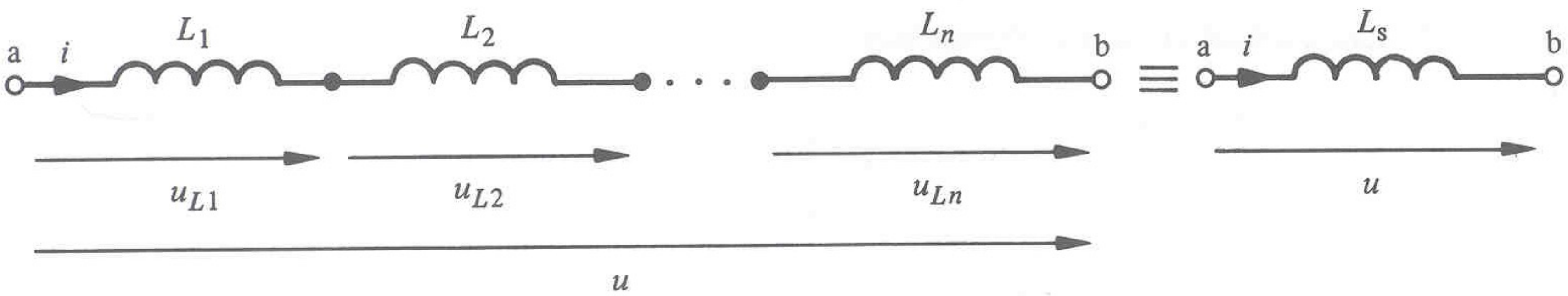
$$R_s = \sum_{k=1}^{n} R_k$$

Capacitors in series



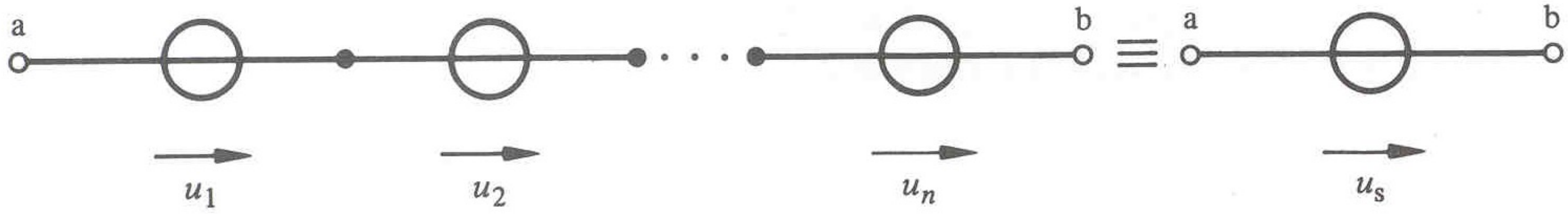
$$\frac{1}{C_s} = \sum_{k=1}^{n} \frac{1}{C_k} \quad \text{and} \quad u(0) = \sum_{k=1}^{n} u_{Ck}(0)$$

Inductors in series

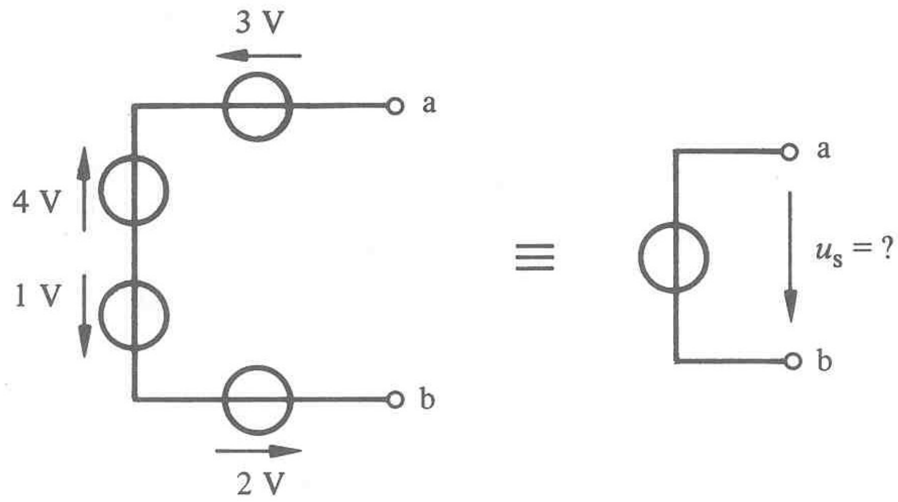


$$L_s = \sum_{k=1}^{n} L_k \qquad \text{and} \qquad i(0) = i_{Lk}(0)$$

Voltage sources in series



$$u_s = \sum_{k=1}^{n} u_k$$

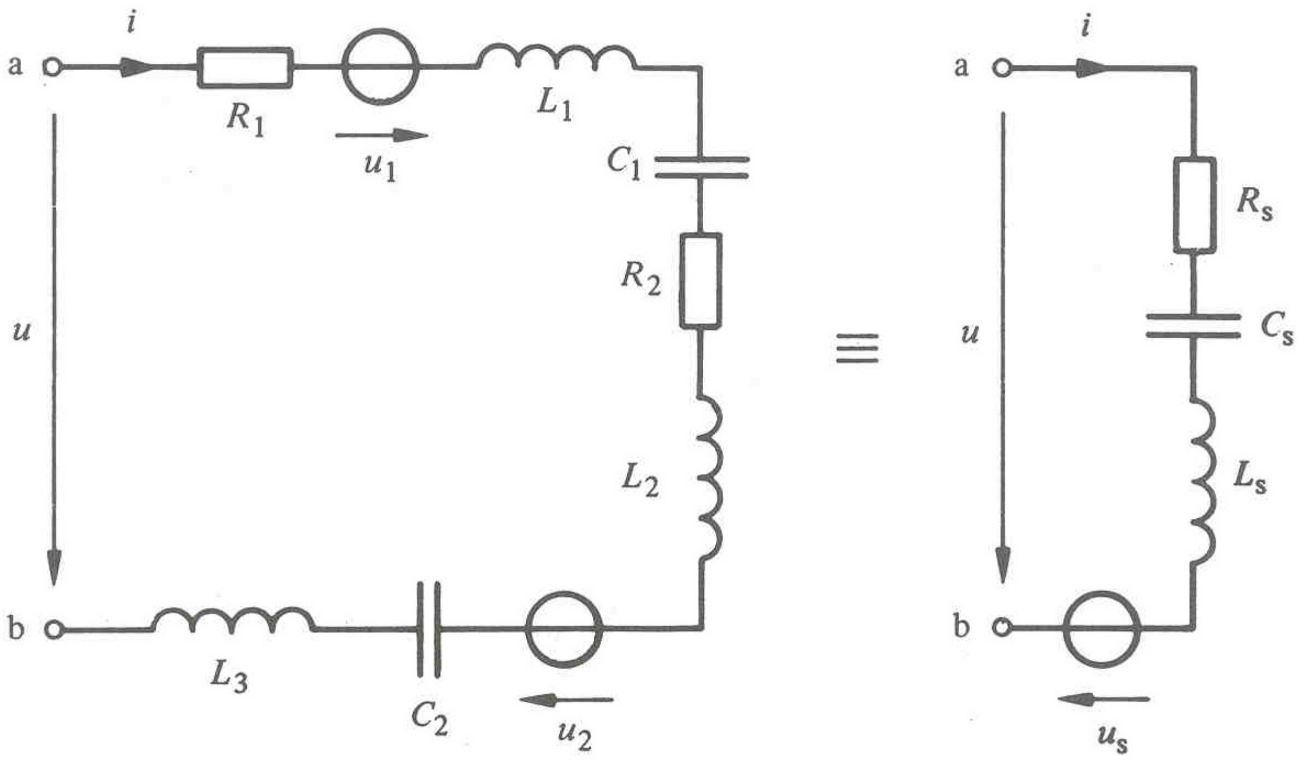


$$u_s = 3 - 4 + 1 + 2 = 2\ V$$

Current sources in series

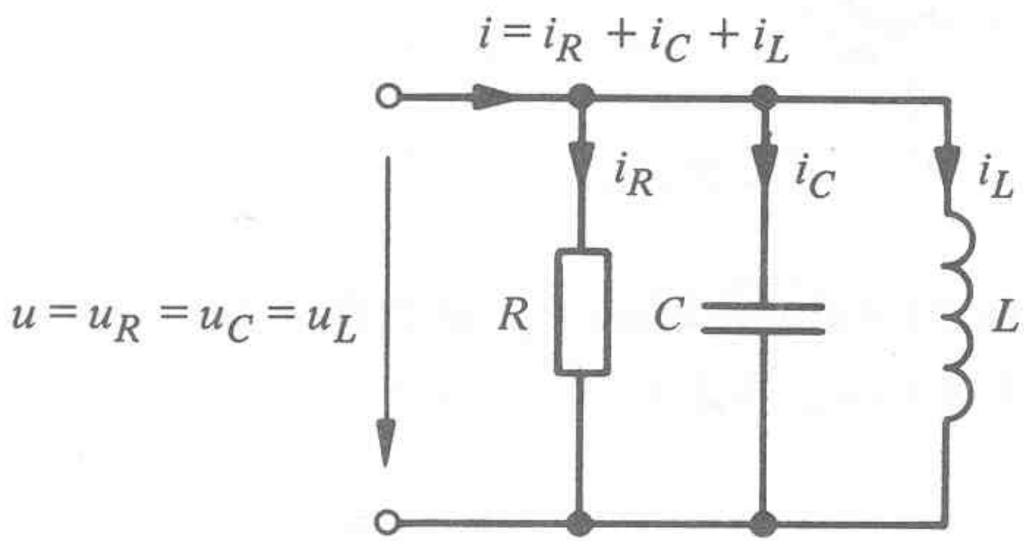
- Impossible ***unless all individual sources produce the same current***

Series circuit: example

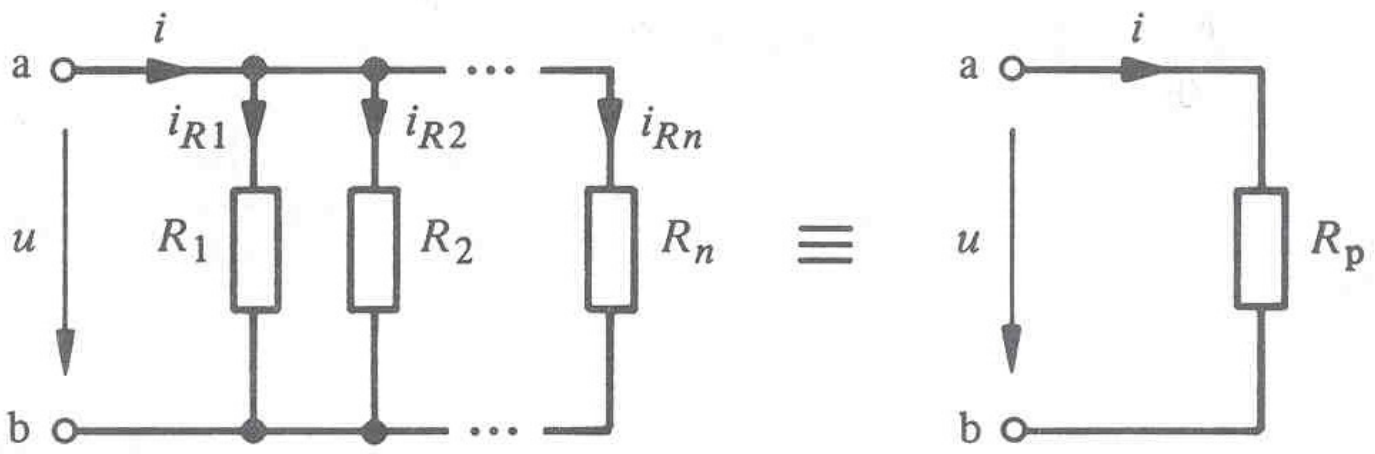


Elements in parallel

- Elements connected in parallel are subjected to the same voltage. The total current is the sum of the individual currents.

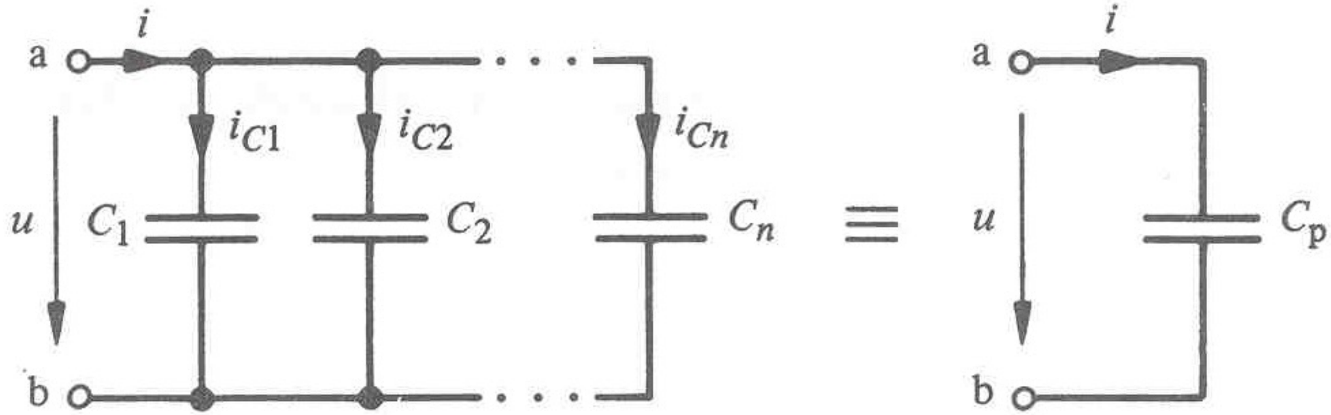


Resistors in parallel



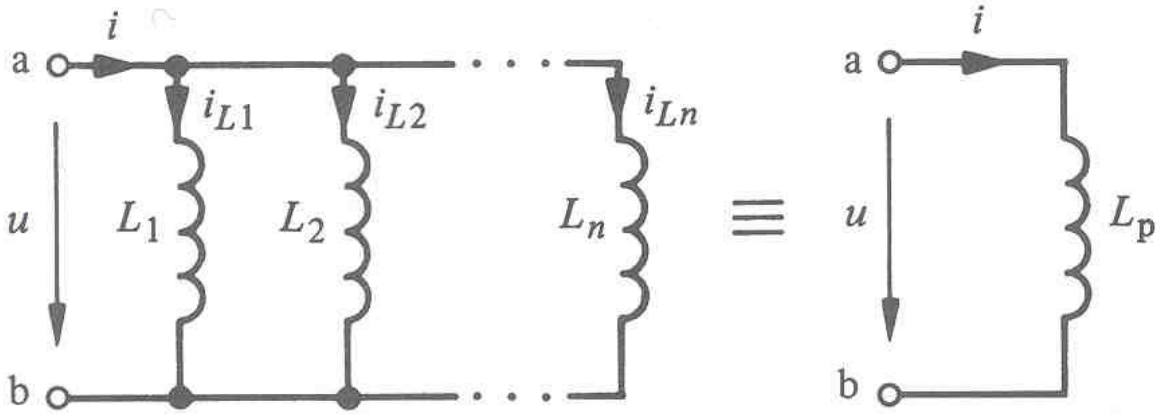
$$\frac{1}{R_p} = \sum_{k=1}^{n} \frac{1}{R_k}$$

Capacitors in parallel



$$C_p = \sum_{k=1}^{n} C_k \quad \text{and} \quad u(0) = u_k(0)$$

Inductors in parallel

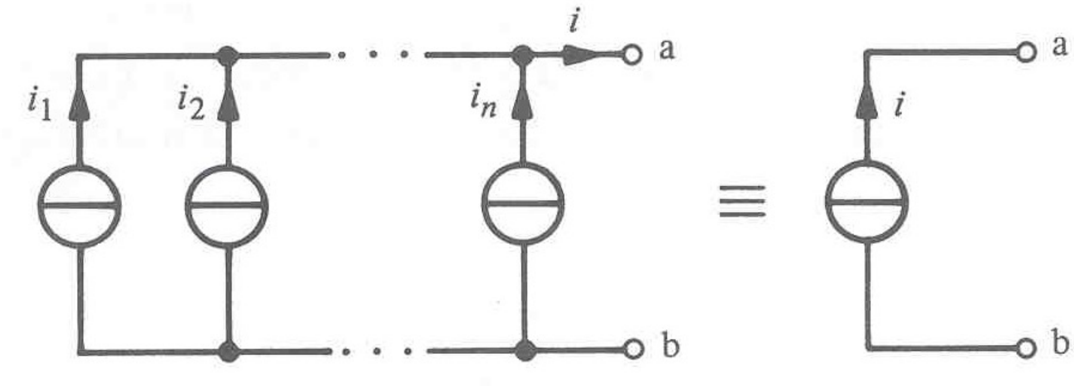


$$\frac{1}{L_p} = \sum_{k=1}^{n} \frac{1}{L_k} \qquad \text{and} \qquad i(0) = \sum_{k=1}^{n} i_{Lk}(0)$$

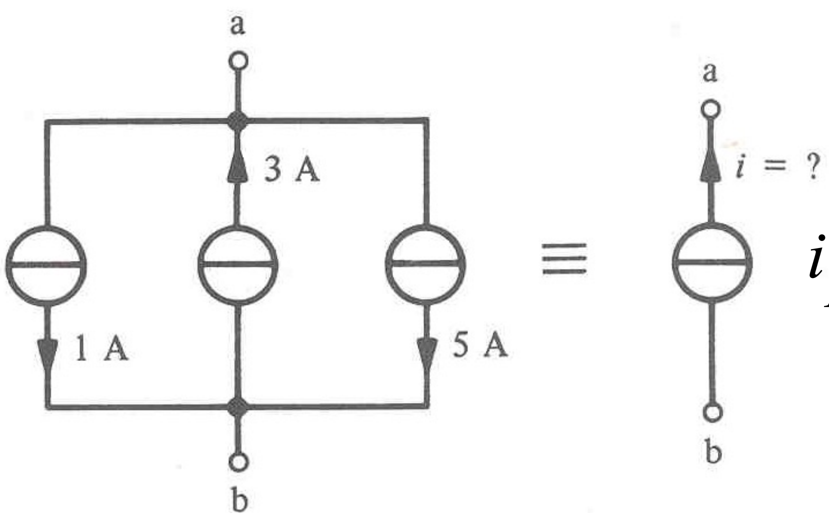
Voltage sources in parallel

- Impossible ***unless all individual sources produce the same voltage***

Current sources in parallel

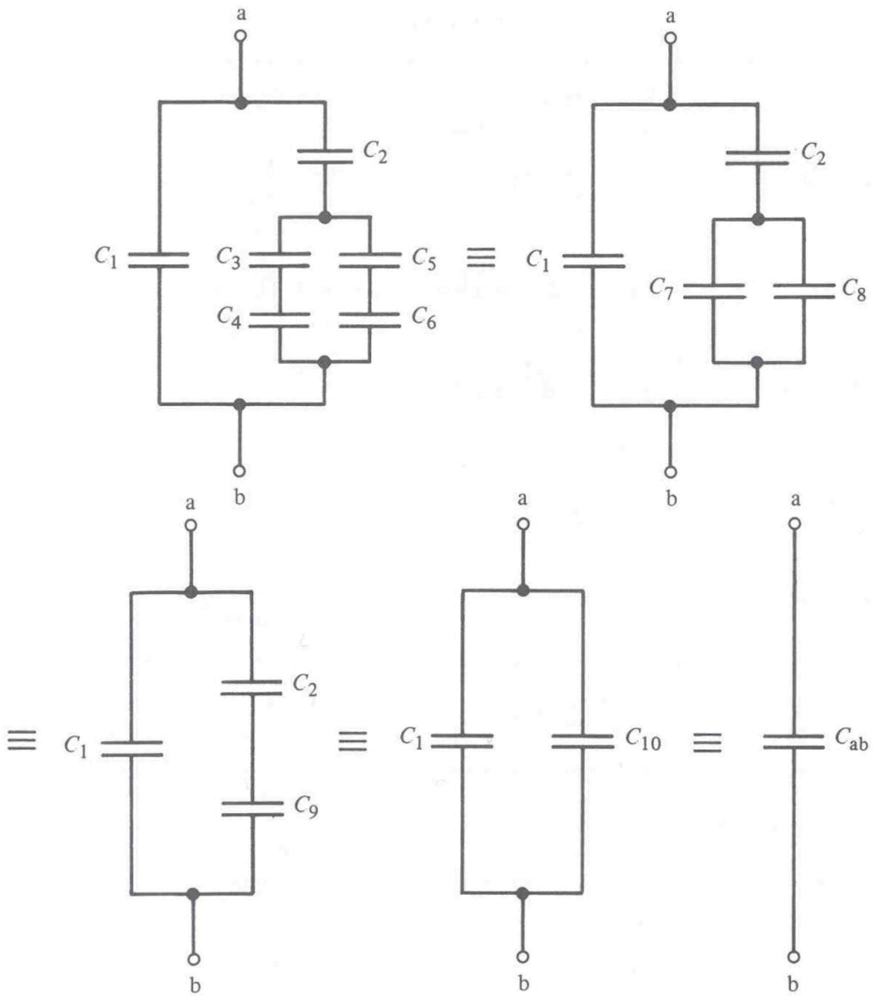


$$i_p = \sum_{k=1}^{n} i_k$$

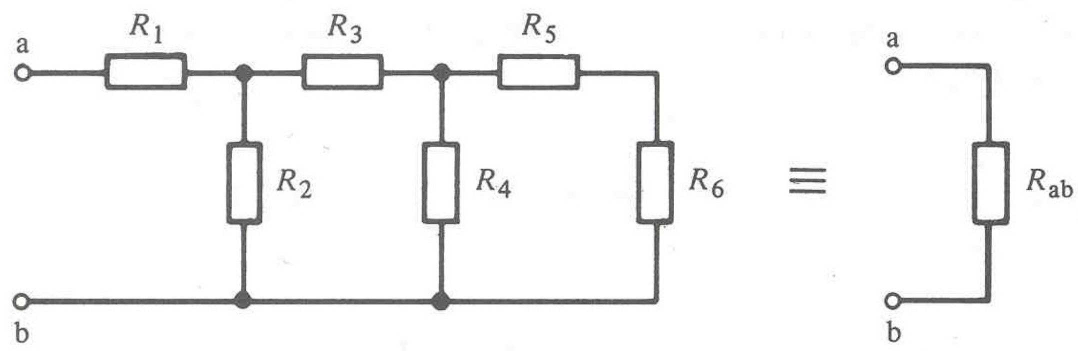


$$i_p = -1 + 3 - 5 = -3\ A$$

Series-parallel circuits

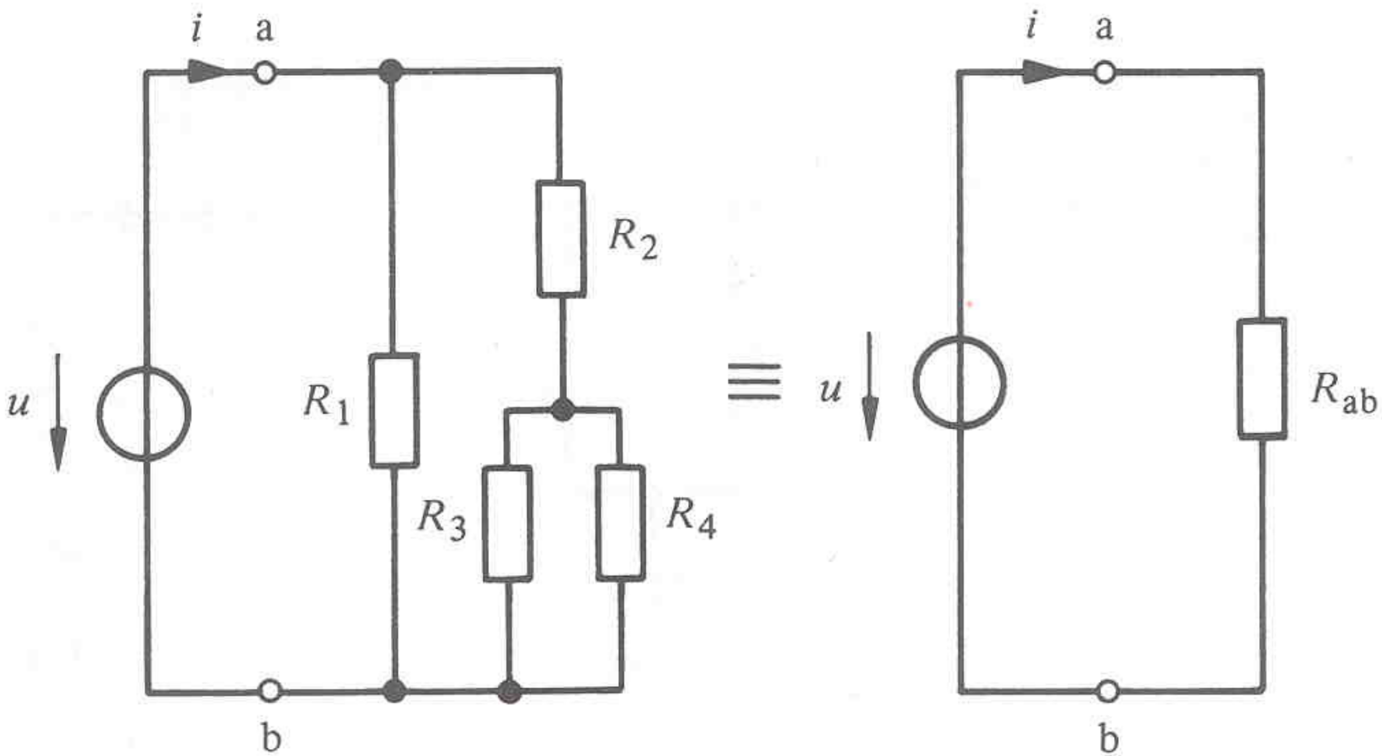


Series-parallel circuits



$$R_{ab} = R_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{R_3 + \cfrac{1}{\cfrac{1}{R_4} + \cfrac{1}{R_5 + R_6}}}}$$

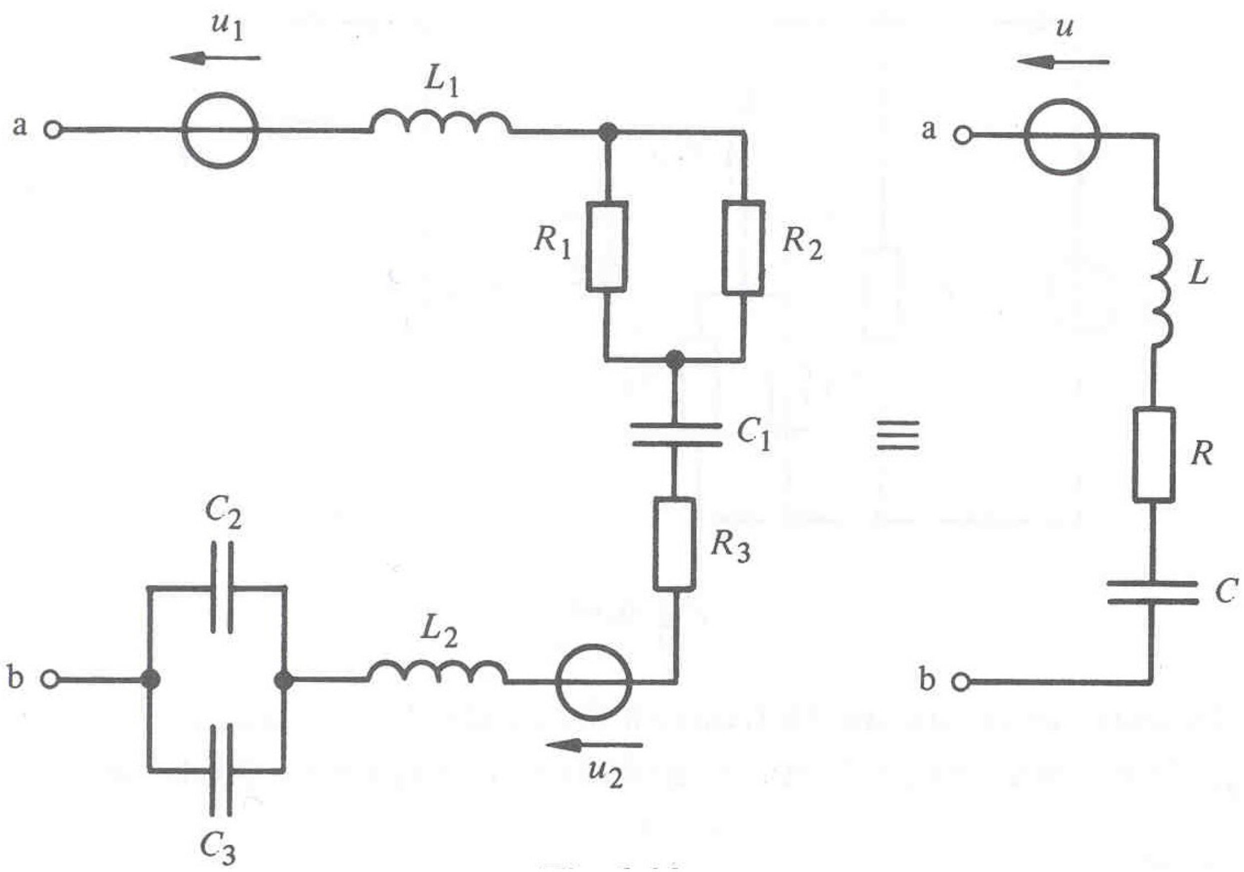
Series-parallel circuits



$u = 73{,}45\ \mathrm{V}$
$R_1 = 22\ \Omega$
$R_2 = 36\ \Omega$
$R_3 = 18\ \Omega$
$R_4 = 15\ \Omega$

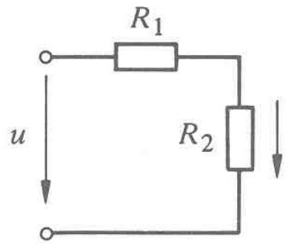
$$R_{ab} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2 + \dfrac{R_3 R_4}{R_3 + R_4}}}$$

Series-parallel circuits

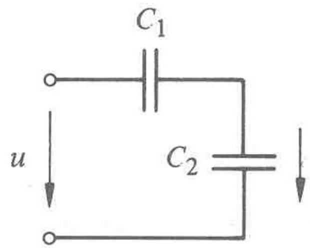


$$u = u_1 - u_2 \qquad R = R_3 + \frac{R_1 R_2}{R_1 + R_2} \qquad L = L_1 + L_2 \qquad C = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

Voltage divider

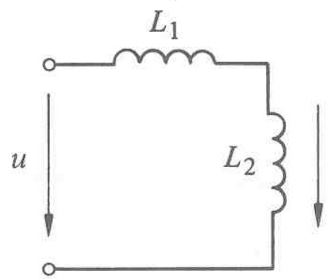


$$u_2 = u\frac{R_2}{R_1 + R_2}$$



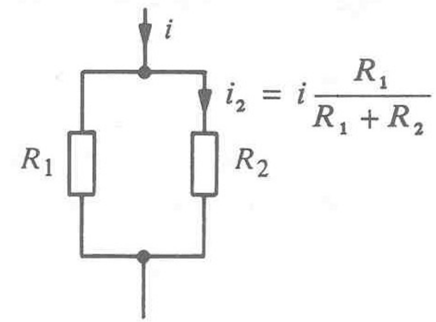
$$u_2 = u\frac{C_1}{C_1 + C_2}$$

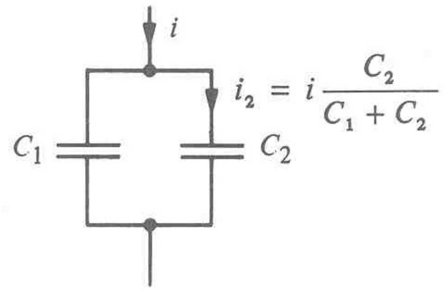
(No initial voltages)

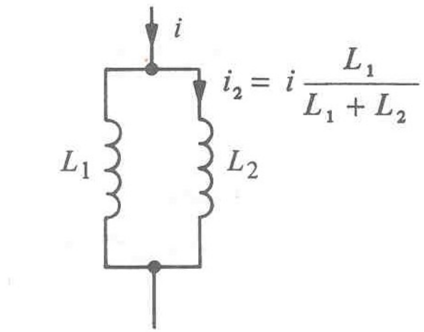


$$u_2 = u\frac{L_2}{L_1 + L_2}$$

Current divider

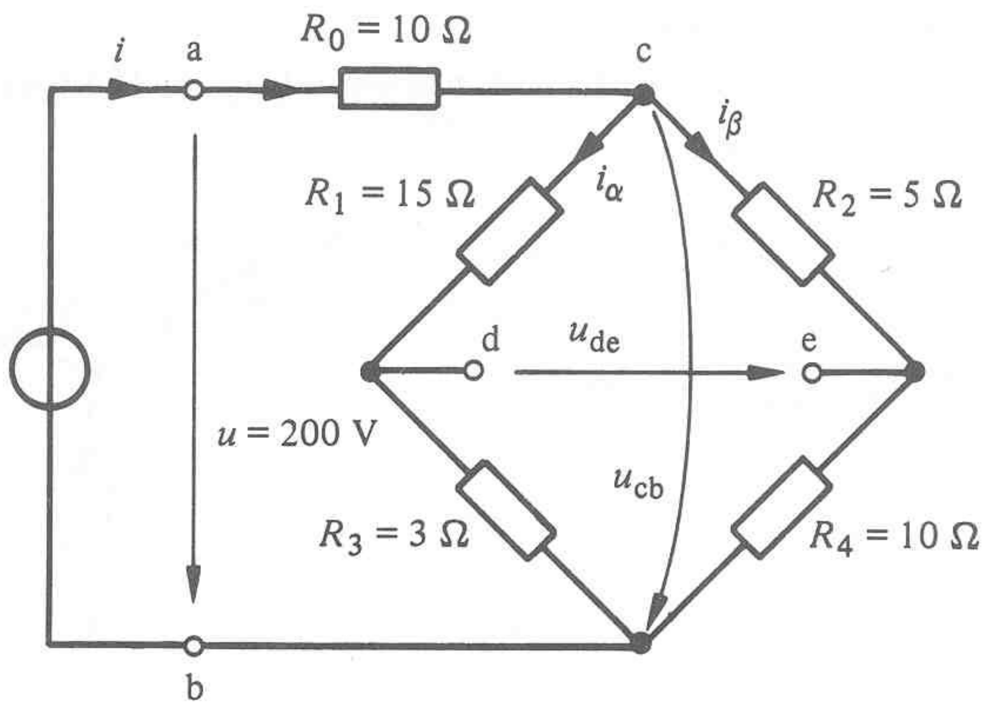






(No initial currents)

Voltage / current divider : example



Solution on the board