Course Exercises

Power Systems Analysis

Fall 2024

2.1 Fundamental aspects for the study of AC circuits

Problem 2.1.1: Voltage and Current Phasors

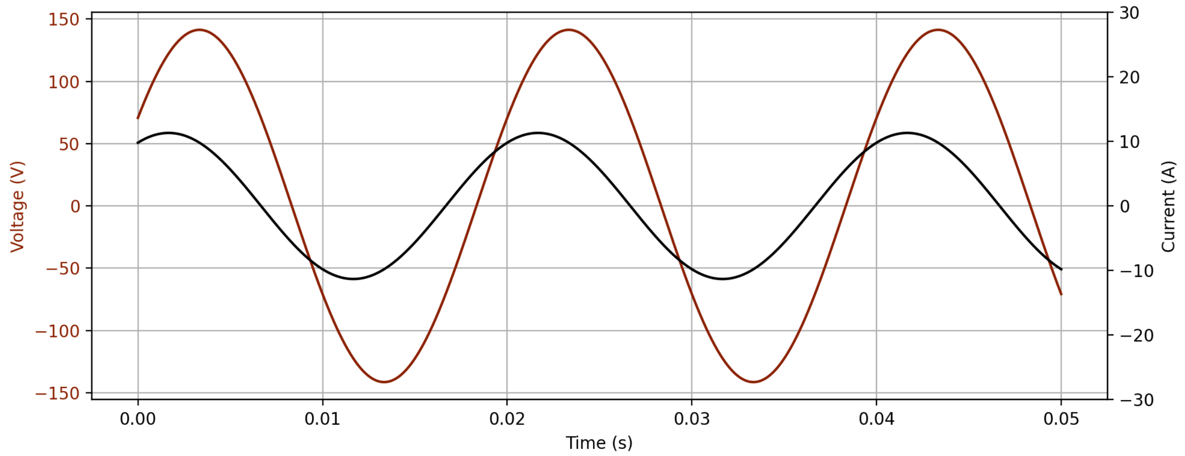
If $v(t) = 141.4 \sin(\omega t + 30°)$ V and $i(t) = 11.31 \cos(\omega t - 30°)$ A, find for each:

(a) the maximum value,

(b) the RMS value, and

(c) the phasor expression in polar and rectangular form if the voltage is the reference.

(d) Is the circuit inductive or capacitive?



Problem 2.1.2: Impedance Calculation

If the circuit of Problem 2.1.1 consists of a purely resistive and a purely reactive element, find their parameters:

(a) if the elements are in series, and

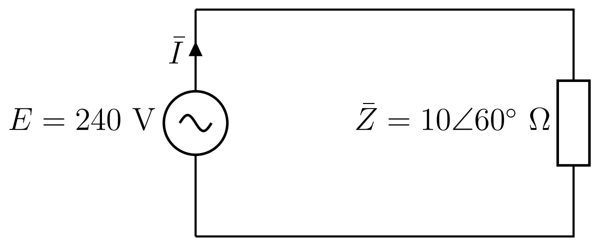
(b) if the elements are in parallel.

Problem 2.1.3: Voltage Difference

In a single-phase circuit $\bar{E}_a = 120\angle 45°$ (V) and $\bar{E}_b = 100\angle(-15°)$ V with respect to a reference node O. Find $\bar{V}_{ba}$ in polar form.

Problem 2.1.4: AC Circuit Power Calculation

A single-phase AC voltage of 240 V is applied to a series circuit whose impedance is $10\angle 60°$ (Ω). Find R, X, P, Q, and the power factor of the circuit.



Problem 2.1.5: Effect of Parallel Capacitor

If a capacitor is connected in parallel with the circuit of Problem 2.1.4, and if this capacitor supplies 1250 VAr, find the P and Q supplied by the 240-V source, and find the resultant power factor.

Problem 2.1.6: Power Factor Correction

A single-phase inductive load draws 10 MW at 0.6 power-factor lagging. Draw the power triangle and determine the reactive power of a capacitor to be connected in parallel with the load to raise the power factor to 0.85.

Problem 2.1.7: Three Phase Power

A balanced three-phase load is connected to a 415 V (line to line), 50 Hz, three-phase power supply. The load draws a line current of 20 A with a power factor of 0.8 lagging. Find the values of P and Q drawn by the load.

Solutions

Solution to Problem 2.1.1 (Voltage and Current Phasors):

(a) The maximum value of the voltage $v(t)$ is $141.4\,\text{V}$ and the maximum value of the current $i(t)$ is $11.31\,\text{A}$.

(b) The RMS value of the voltage $v(t)$ is:

$$V_{\text{RMS}} = \frac{V_{\max}}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100\,\text{V}$$

The RMS value of the current $i(t)$ is:

$$I_{\text{RMS}} = \frac{I_{\max}}{\sqrt{2}} = \frac{11.31}{\sqrt{2}} = 8\,\text{A}$$

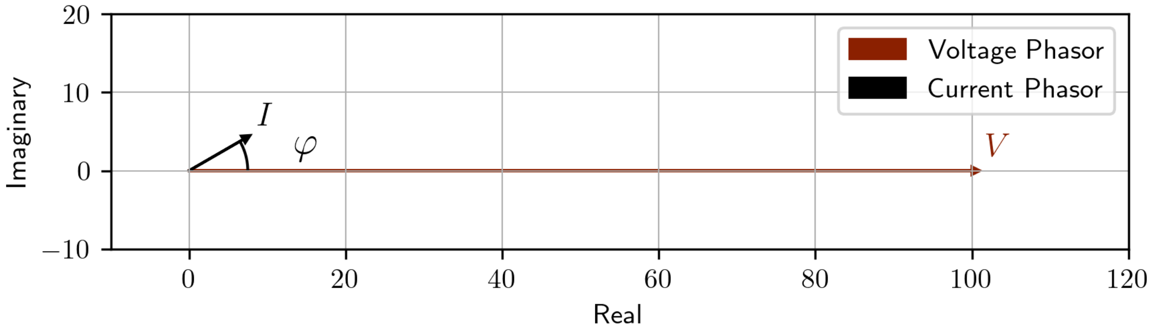
(c) Phasor expressions:

- Voltage phasor: $\bar{V} = 100\angle 0°$ where the angle is $0°$ because it is taken as a reference. The Cartesian form is $\bar{V} = 100 + j0$ V.
- Current phasor: Convert cos to sin:

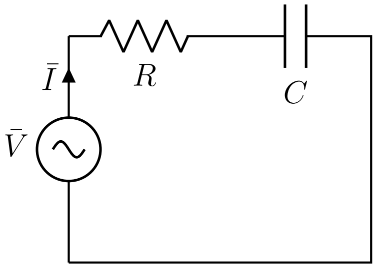
  $$i(t) = 11.31\cos(\omega t - 60°) = 11.31\sin(\omega t - 60° + 90°) = 11.31\sin(\omega t + 30°)\text{ A}$$

  So, the current phasor is $\bar{I} = 8\angle 30°$ A and, in its Cartesian form: $6.93 + j4$ A.

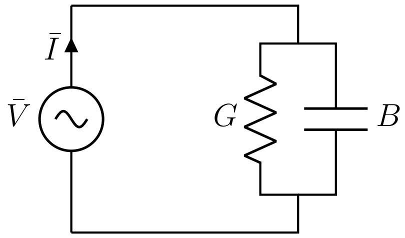
(d) The circuit is capacitive. The current is leading the voltage.



Solution to Problem 2.1.2 (Impedance Calculation): Since the circuit is capacitive (2.1.1d), it consists of a resistive and capacitive element, R and C, respectively.



(a) Series RC Circuit



(b) Parallel RC Circuit

(a) **Series Connection:** The equivalent impedance can be expressed as:

$$\bar{Z} = R + jX$$

In the case of connection in series, the equivalent impedance is the sum of the impedances of both elements:

$$\bar{Z} = \bar{Z}_R + \bar{Z}_C$$

Therefore

$$\bar{Z} = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C}$$

The impedance can be calculated directly from the voltage and current phasors as:

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{Ve^{j\theta_V}}{Ie^{j\theta_I}} = \frac{V}{I}e^{j(\theta_V - \theta_I)} = Ze^{j\varphi}$$

- The magnitude of $\bar{Z}$ is:

$$|\bar{Z}| = \frac{V}{I} = \frac{100}{8} = 12.5\,\Omega$$

- The phase angle φ of $\bar{Z}$ is:

$$\varphi = \theta_V - \theta_I = -30^\circ$$

- Solving for R and X:

$$R = |\bar{Z}|\cos(-30^\circ) = 12.5 \cdot \frac{\sqrt{3}}{2} = 10.83\,\Omega$$
$$X = |\bar{Z}|\sin(-30^\circ) = 12.5 \cdot \left(-\frac{1}{2}\right) = -6.25\,\Omega$$

Since in this case $X_C = \dfrac{1}{\omega C} = -X$, then

$$C = \frac{1}{\omega \cdot (-X)} = \frac{1}{2\pi \cdot 50 \cdot 6.25} = 509.3\ \mu\text{F}.$$

(b) **Parallel Connection:** The equivalent admittance can be expressed as

$$\bar{Y} = G + jB$$

In the case of connection in parallel, the equivalent admittance is the sum of the admittances of both elements:

$$\bar{Y} = \bar{Y}_R + \bar{Y}_C$$

Therefore:

$$\bar{Y} = \frac{1}{R} + j\omega C$$

The equivalent admittance is:

$$\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{12.5e^{j(-30°)}} = 0.0693 + j0.0400 \text{ S}$$

Therefore, $G = 0.0693$ S and $B = 0.04$ S. In this case $R = \dfrac{1}{G} = 14.43\ \Omega$ and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{B} = \dfrac{1}{0.04} = 25\ \Omega$. Therefore,

$$C = \frac{1}{\omega X_C} = \frac{1}{2 \cdot \pi \cdot 50 \cdot 25} = 127.3\ \mu\text{F}.$$

Solution to Problem 2.1.3 (Voltage Difference):

- Given $\bar{E}_a = 120\angle 45°$ and $\bar{E}_b = 100\angle - 15°$, the voltage $\bar{V}_{ba}$ is found using:

$$\bar{V}_{ba} = \bar{E}_b - \bar{E}_a$$

- Converting to rectangular form:

$$\bar{E}_a = 120(\cos 45° + j\sin 45°) = 84.85 + j84.85, \mathrm{V}$$

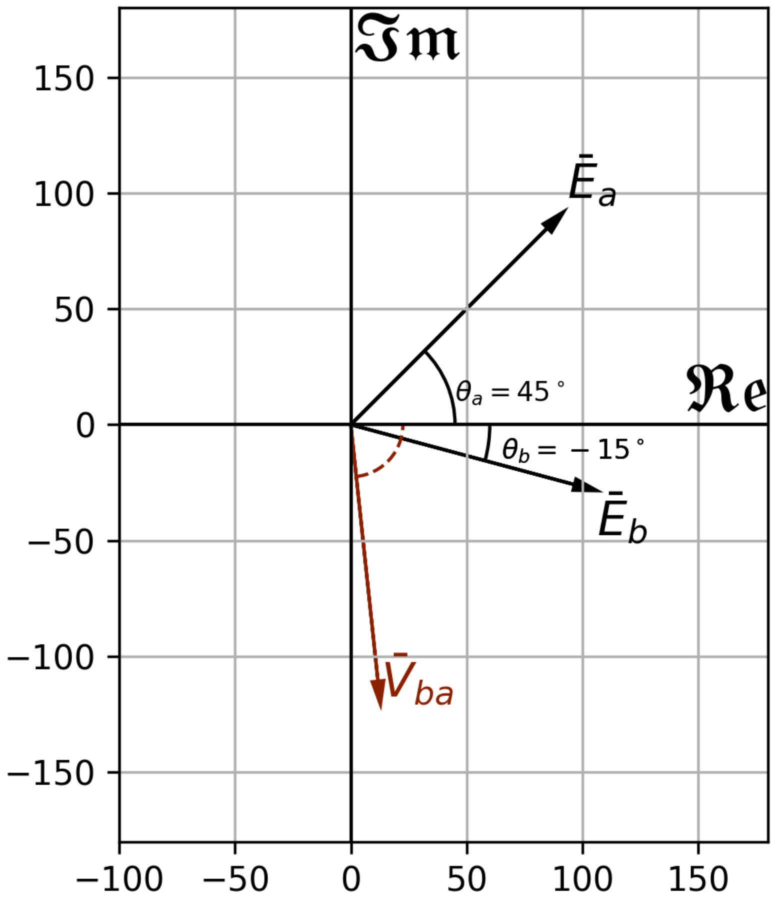
$$E_b = 100(\cos(-15°) + j\sin(-15°)) = 96.59 - j25.88\,\mathrm{V}$$

- Subtracting:

$$\bar{V}_{ba} = (96.59 - j25.88) - (84.85 + j84.85) = 11.74 - j110.73\,\mathrm{V}$$

- Converting back to polar form:

$$\bar{V}_{ba} = \sqrt{11.74^2 + (-110.73)^2}\angle\tan^{-1}\left(\frac{-110.73}{11.74}\right) = 111.35\angle - 84°\,\mathrm{V}$$



Solution to Problem 2.1.4 (AC Circuit Power Calculation):

- Given impedance $\bar{Z} = 10\angle 60°, \Omega$:

$$\bar{Z} = R + jX = 10(\cos 60° + j \sin 60°) = 5 + j8.66\,\Omega$$

- The resistive component $R = 5\ \Omega$, and the reactive component $X = 8.66\ \Omega$.
- The current $\bar{I}$ is:

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{240}{10\angle 60°} = 24\angle(-60°)\ \mathrm{A}$$

- Real power P:

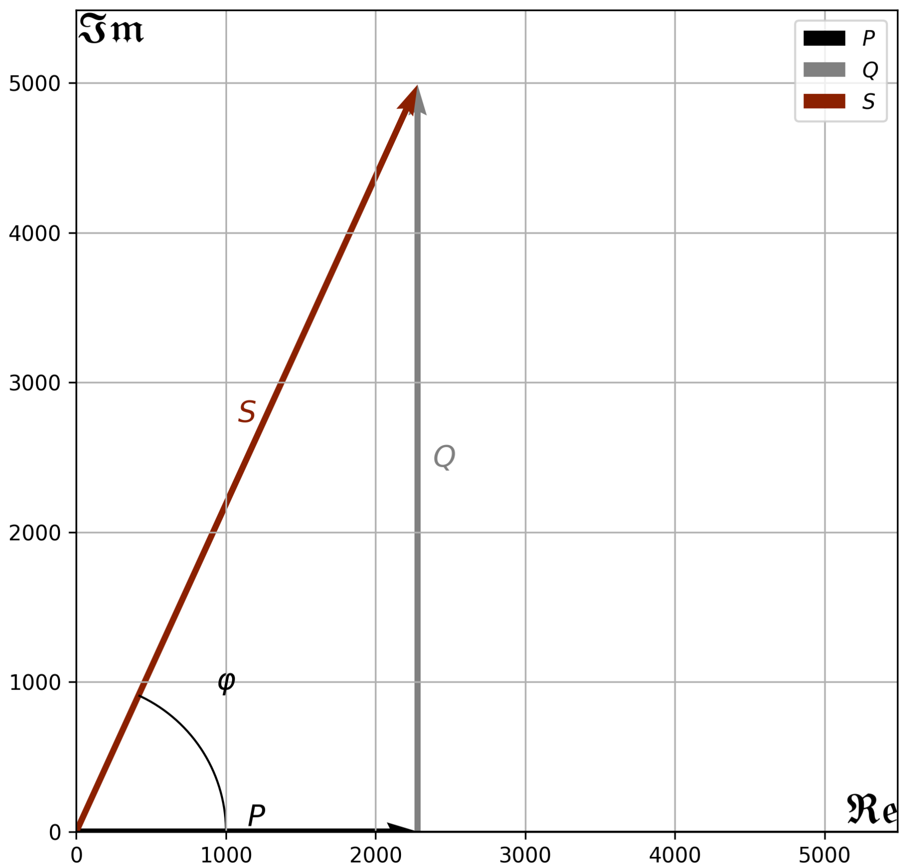
$$P = VI\cos\varphi = 240 \cdot 24 \cdot \cos 60° = 2880\,\mathrm{W}$$

- Reactive power Q:

$$Q = VI\sin\varphi = 240 \cdot 24 \cdot \sin 60° = 4988\,\mathrm{VAr}$$

- Power factor:

$$PF = \cos\varphi = \cos 60° = 0.5$$



Solution to Problem 2.1.5 (Effect of Parallel Capacitor): Given that the capacitor supplies $Q_c = 1250\text{VAr}$.

- The total reactive power Q_t supplied by the source is:

$$Q_t = Q - Q_c = 4988 - 1250 = 3738\,\text{VAr}$$

- The apparent power S is:

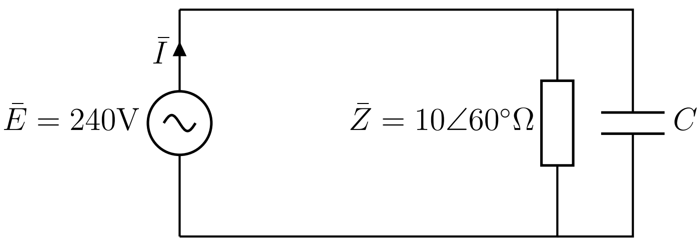
$$S = \sqrt{P^2 + Q_t^2} = \sqrt{2880^2 + 3738^2} = 4718.8\,\text{VA}$$

- The resultant power factor is:

$$\text{Power Factor} = \frac{P}{S} = \frac{2880}{4718.8} = 0.61$$

  or alternatively

$$\text{Power Factor} = \cos\left(\tan^{-1}\left(\frac{3738}{2880}\right)\right) = 0.61$$



Solution to Problem 2.1.6 (Power Factor Correction):

- Real power $P = 10$ MW and initial power factor $PF_{\text{initial}} = 0.6$.
- Initial reactive power Q_{initial}:

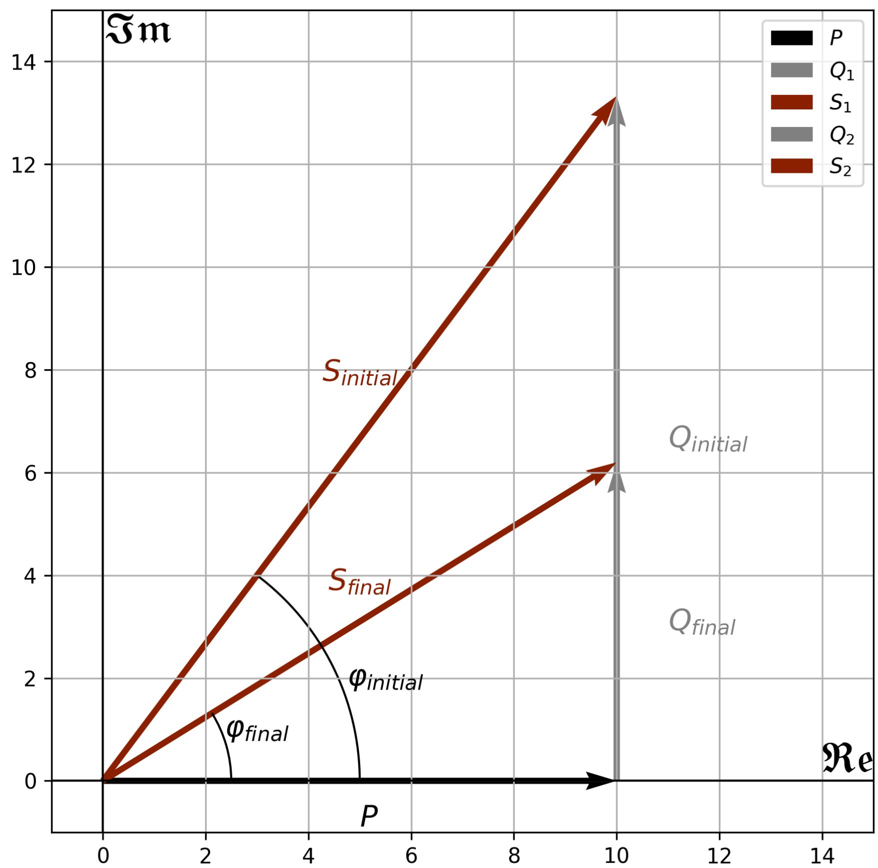
$$Q_{\text{initial}} = P \tan\cos^{-1}(0.6) = 10 \times \tan\cos^{-1}(0.6) = 13.33\,\text{MVAr}$$

- Final reactive power Q_{final} with power factor $PF_{\text{final}} = 0.85$:

$$Q_{\text{final}} = P \tan\cos^{-1}(0.85) = 10 \cdot \tan\cos^{-1}(0.85) = 6.20\,\text{MVAr}$$

- Required capacitor reactive power Q_c:

$$Q_c = Q_{\text{initial}} - Q_{\text{final}} = 6.20 - 13.33 = -7.13\,\text{MVAr}$$



Solution to Problem 2.1.7 (Three Phase Power): The real power P drawn from the line can be calculated using:

$$P = \sqrt{3} V I \cos\varphi$$

Substituting the values:

$$P = \sqrt{3} \cdot 415 \cdot 20 \cdot 0.8 = 11.5\,\mathrm{kW}$$

The reactive power Q can be calculated similarly by using:

$$Q = \sqrt{3} V I \sin(\cos^{-1}(\text{Power Factor}))$$

$$Q = \sqrt{3} \cdot 415 \cdot 20 \cdot 0.6 = 8.63\,\mathrm{kVAr}$$

The power factor $\cos\varphi$ is also given by:

$$\cos\varphi = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$

Substitute the values of P and Q calculated:

$$\varphi = \tan^{-1}\left(\frac{8.63}{11.5}\right) = \tan^{-1}(0.75) = 36.87^\circ$$

$$\cos\varphi = \cos(36.87^\circ) = 0.8$$

This confirms that the power factor is indeed 0.8.