

EE-334

Digital System Design

Custom Digital Circuits
From Algorithms to Architectures

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Starting from Specifications as Sequential Code

- **Algorithms are** usually **specified** as **mathematical expressions** or **as sequential programs**
- Mathematical representation must be written with care to avoid ambiguities
- Representation as sequential programs
 - usually remove already significant amounts of overhead (re-use)
 - often imply a step by step execution on few shared computing resources
 - Longer programs generally take only more time on a sequential processor
 - Sequential execution may resolve ambiguities (imply priority)
- Example algorithm

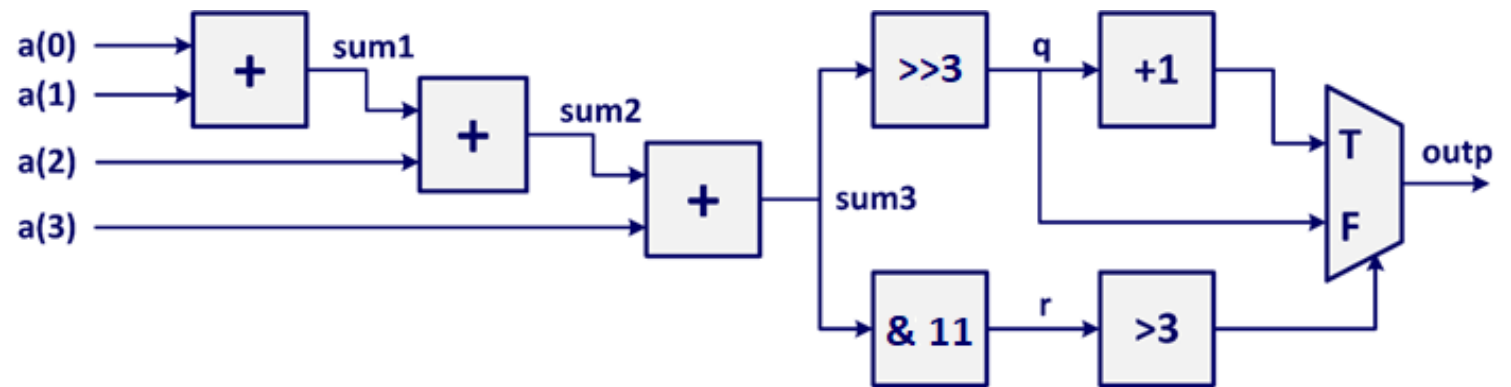
$$q = \begin{cases} \frac{1}{8} \sum_{i=0}^3 a[i] + 1 & \text{mod}_8 \left(\sum_{i=0}^3 a[i] \right) > 3 \\ \frac{1}{8} \sum_{i=0}^3 a[i] & \text{else} \end{cases}$$

```
int sigproc(int a[]) {  
    int q, r;  
    int sum = a[0];  
    for (int i=1; i<=3;i++) {  
        sum += a[i];  
    }  
    q = sum/8;  
    r = sum % 8;  
    if (r > 3) q += 1;  
    return q;  
}
```

Isomorphic Architecture

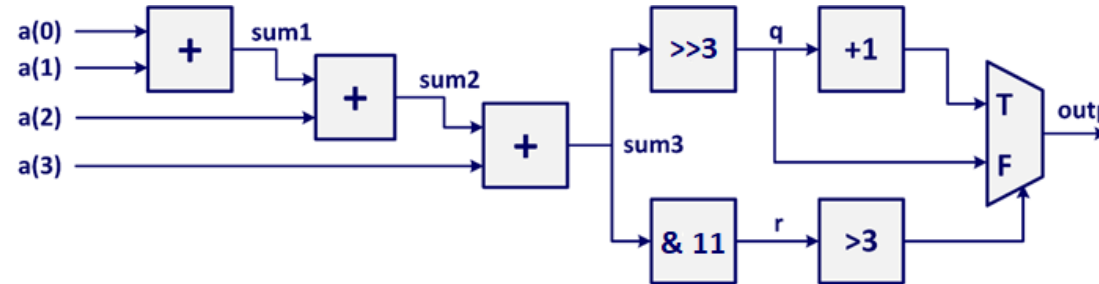
- **Isomorphic architecture:** straightforward mapping of an algorithm to hardware
 - Corresponds to the data flow graph of the algorithm
 - Vertices correspond to operations, edges route intermediate results to other vertices
 - **Every operation is mapped to a dedicated combinational hardware unit**
 - **Loops are unrolled:** loop counter must be computable (known) at design time
 - **Conditional statements become multiplexers**

```
int sigproc(int a[]) {  
    int q, r;  
    int sum = a[0];  
    for (int i=1; i<=3;i++) {  
        sum += a[i];  
    }  
    q = sum/8;  
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    if (r > 3) q += 1;  
    return q;  
}
```



Isomorphic Architecture in VHDL

- VHDL offers multiple **options to describe an isomorphic architecture**:



Sequential code in process statements

```
process (a) is
    variable acc, q, r : unsigned(7 downto 0);
begin
    acc := unsigned(a(0));
    for i in 1 to 3 loop
        acc := acc + unsigned(a(i));
    end loop;
    q := "000" & acc(7 downto 3); -- /8
    r := "00000" & acc(2 downto 0); -- rem 8
    if r > 3 then
        q := q + 1;
    end if;
    outp <= std_logic_vector(q);
end process;
```

Explicit modelling with concurrent statements

```
architecture dfl of sigproc is
    signal sum0, sum1 : unsigned(7 downto 0);
    signal sum2, sum3, q, r : unsigned(7 downto 0);
begin
    sum0 <= unsigned(a(0));
    sum1 <= sum0 + unsigned(a(1));
    sum2 <= sum1 + unsigned(a(2));
    sum3 <= sum2 + unsigned(a(3));
    q <= "000" & sum3(7 downto 3); -- /8
    r <= "00000" & sum3(2 downto 0); -- rem 8
    outp <= std_logic_vector(q + 1) when r > 3 else
            std_logic_vector(q);
end architecture dfl;
```

Isomorphic Architecture in VHDL

- VHDL offers multiple **options to describe an isomorphic architecture:**

- **Sequential code in process statements**

- **Compact code** is interpreted and **expanded automatically** in the elaboration phase of the hardware synthesis
 - Loops are unrolled automatically
 - Hardware resources are automatically allocated and connected
- Variables are used instead of signals to pass results between sequential statements
 - **Variables** are similar to signals, but are assigned immediately and **can be re-used**. They only exist until the end of the process.

```
process (a) is
    variable acc, q, r : unsigned(7 downto 0);
begin
    acc := unsigned(a(0));
    for i in 1 to 3 loop
        acc := acc + unsigned(a(i));
    end loop;
    q := "000" & acc(7 downto 3); -- /8
    r := "00000" & acc(2 downto 0); -- rem 8
    if r > 3 then
        q := q + 1;
    end if;
    outp <= std_logic_vector(q);
end process;
```

- **Convenient, but dangerous approach : IDEALLY DO NOT USE!**

- Many pitfalls for code that can not be synthesized (e.g., non-constant loop counters)
- Often hides complexity: loops do not imply reuse of resources!!!

Isomorphic Architecture in VHDL

- VHDL offers multiple **options to describe an isomorphic architecture:**

- **Explicit modelling with concurrent statements**

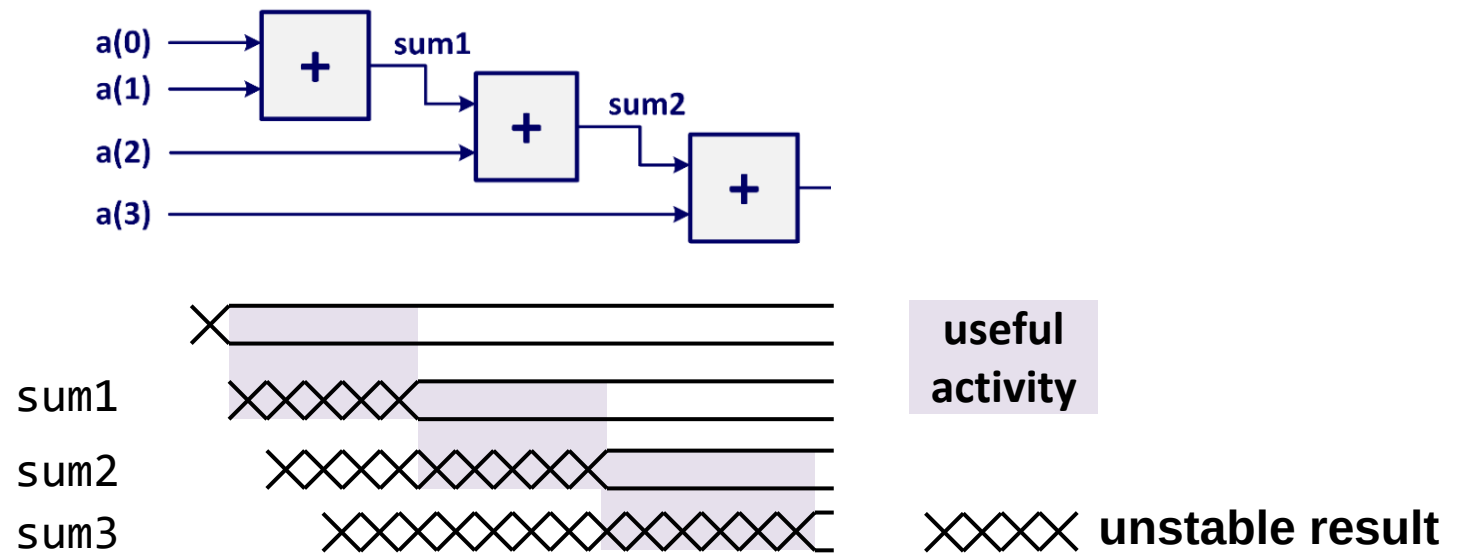
- Sequential code must be expanded manually
- Components connected with signals
 - Need for many signals that must be defined explicitly
- Does not use many of the features that render sequential code compact

```
architecture dfl of sigproc is
    signal sum0, sum1 : unsigned(7 downto 0);
    signal sum2, sum3, q, r : unsigned(7 downto 0);
begin
    sum0 <= unsigned(a(0));
    sum1 <= sum0 + unsigned(a(1));
    sum2 <= sum1 + unsigned(a(2));
    sum3 <= sum2 + unsigned(a(3));
    q <= "000" & sum3(7 downto 3); -- /8
    r <= "00000" & sum3(2 downto 0); -- rem 8
    outp <= std_logic_vector(q + 1) when r > 3 else
            std_logic_vector(q);
end architecture dfl;
```

- **A bit more tedious to write, but much more close to the architecture**
 - All statements map directly to corresponding hardware

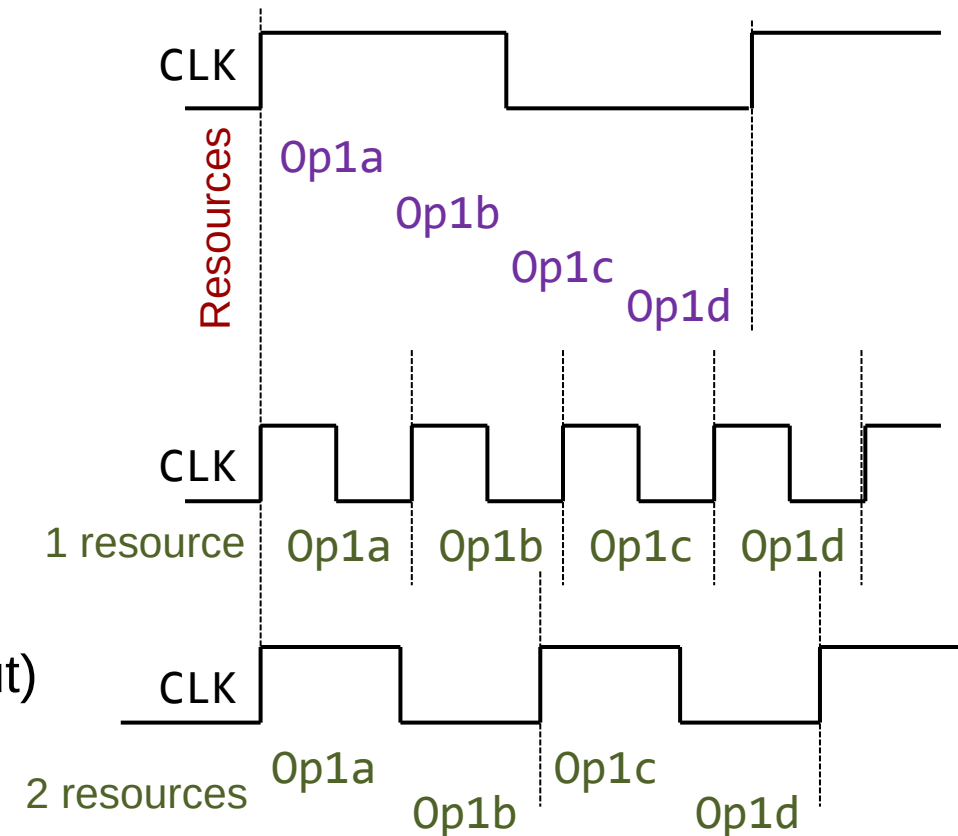
Efficiency of Combinational Circuits

- **Combinational circuits are actually very inefficient:**
 - **Every element has** only **a very small delay** (**time in which it performs useful computations**)
 - Synchronous paradigm requires **every gate** to **wait almost 1 complete cycle** for the next input
- Useful activity propagates like a wave through the combinational circuit



Sequential Processing in Custom Hardware

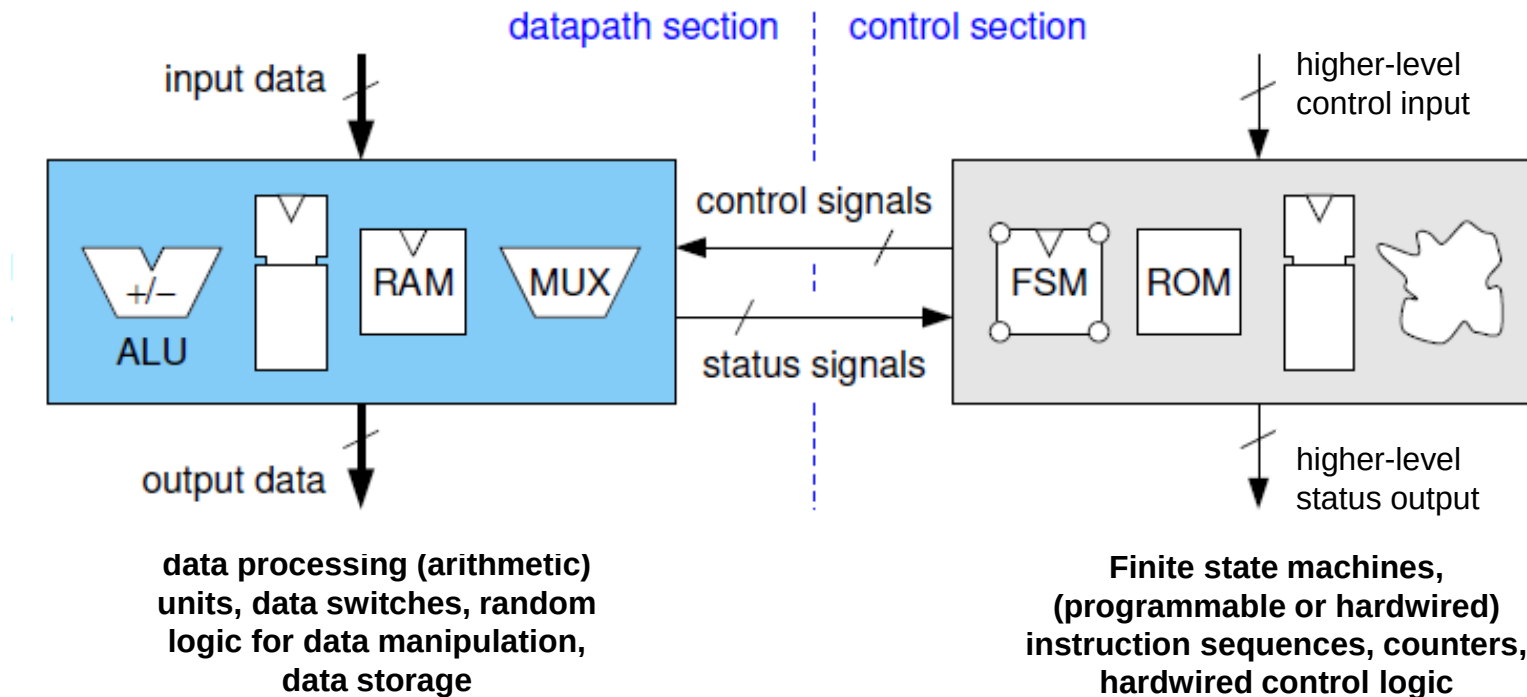
- Single-cycle (**combinational**) **processing** is **not efficient** and **not flexible**
 - Algorithm defines both circuit complexity and computation time (throughput)
- **Multi-cycle** computation **architecture** enables
 - Decomposition of computations into multiple steps
 - Re-use of computation resources between the steps
 - Flexibility in scheduling operations in time and binding them to different resources
- **Distribution of computations** over multiple cycles (time steps) **provides more flexibility** to trade time for complexity (area)
 - Ability to design for given requirements (area or throughput)
 - Often better overall efficiency (AT-product)



Sequential Processing in Custom Hardware

- **Separation between computations and control**

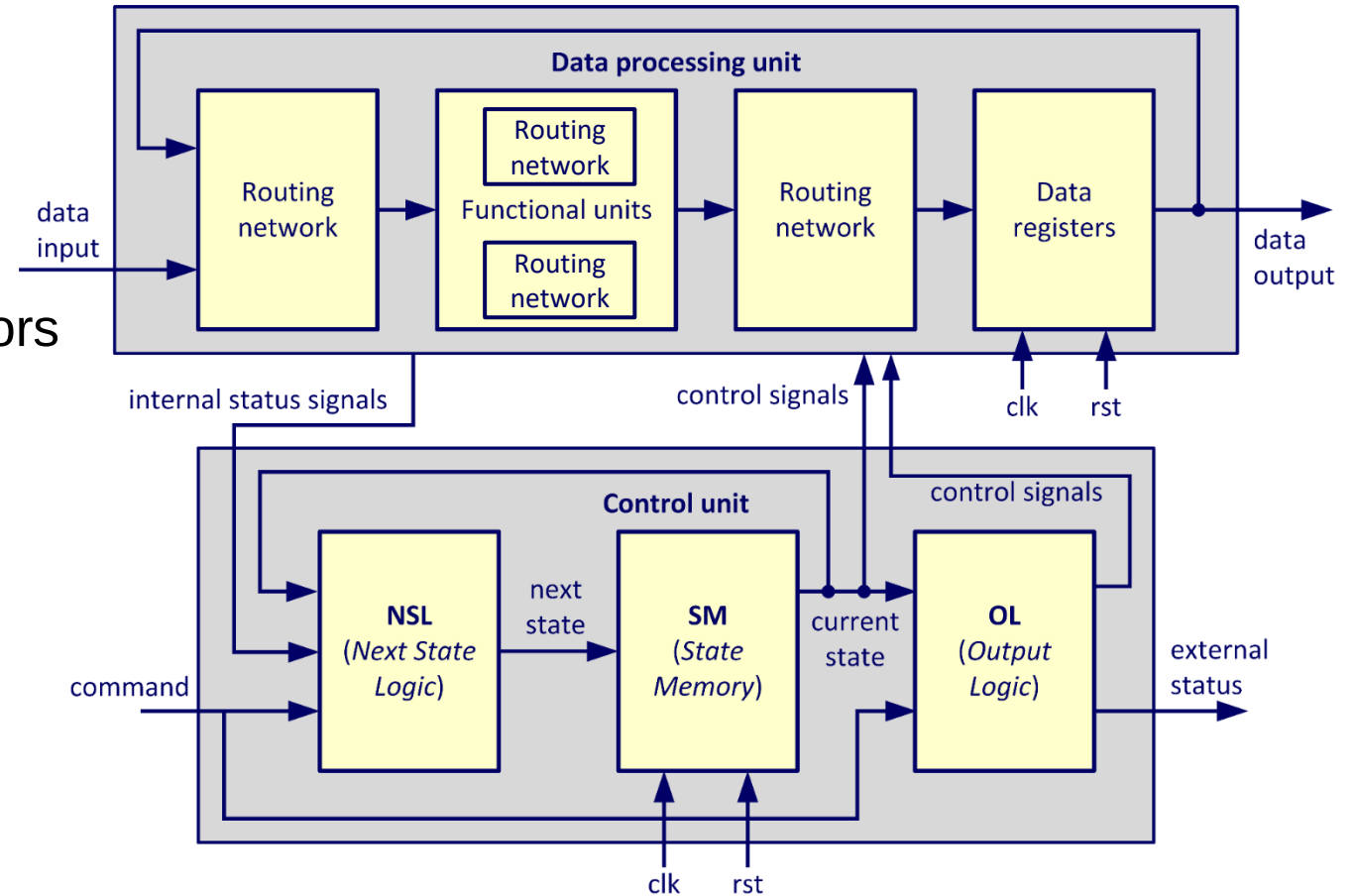
- **Datapath** : performs (arithmetic or other) operations on data and keeps intermediate results in memory elements. Control inputs (from control logic) define what should be done in each cycle
- **Control** : controls datapath and manages the sequence of operations (potentially based on feedback from the datapath), but does not manipulate data itself



Sequential Processing in Custom Hardware

- Architectural template follows the RTL principle

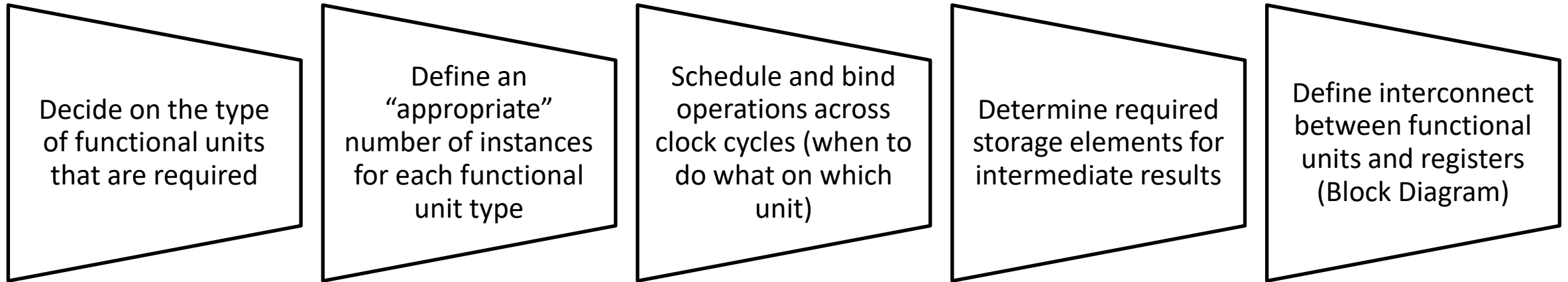
- **FSM** for control with connections to and from the datapath
- **Datapath**: organized similar to processors
 - Storage elements keep partial results for subsequent clock cycle
 - Functional units perform computations
 - Reconfigurable with control signals
 - Routing networks connect
 - Functional units and storage elements
 - Functional units with other functional units



Datapath Design Procedure

- **Datapath determines complexity and throughput**
 - **Many degrees of freedom** render finding optimal the design difficult

- **Datapath design in five steps**



- **Leads to an RTL diagram** of the datapath with various control inputs
- **Control path:** follows from the design of the datapath

Design Procedure Example

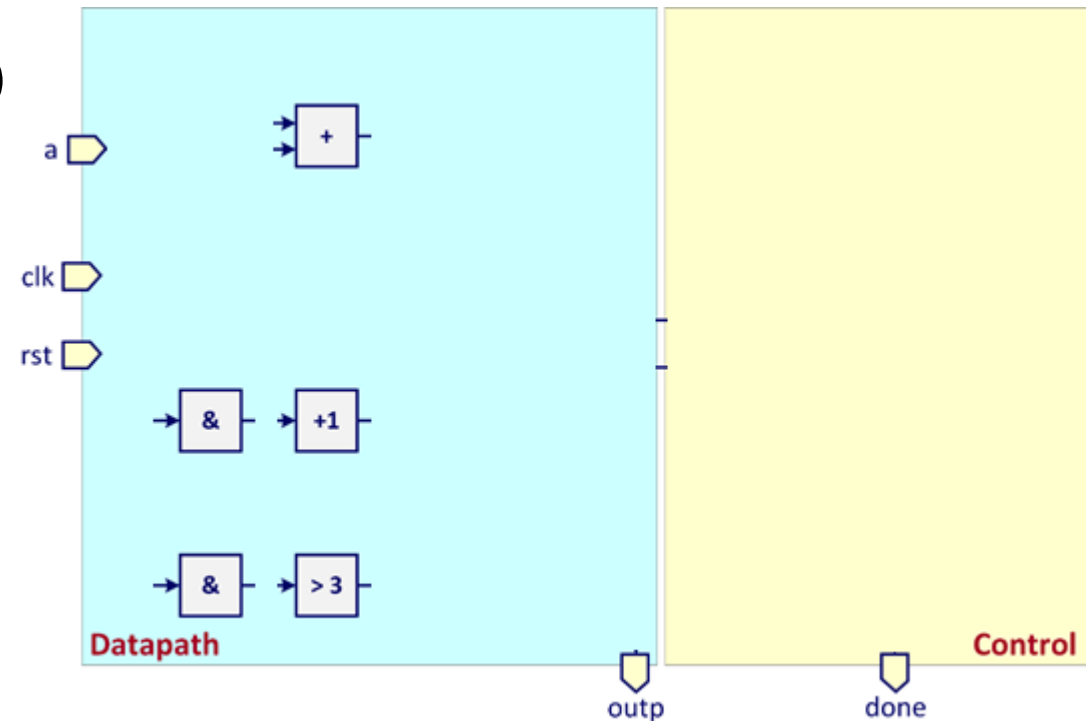
- **Step-1:** identify **functional units**
 - Adder, Div-by-8 (shift-right-3), Modulo-8 (extract 3 LSBs)
Compare to 3, Add-1

```
int sigproc(int a[]) {  
    int q, r;  
    int sum = a[0];  
    for (int i=1; i<=3;i++) {  
        sum += a[i];  
    }  
    q = sum/8;  
    r = sum % 8;  
    if (r > 3) q += 1;  
    return q;  
}
```

Design Procedure Example

- **Step-1:** identify **functional units**
 - Adder, Div-by-8 (shift-right-3), Modulo-8 (extract 3 LSBs)
Compare to 3, Add-1
- **Step-2:** define **hardware resources**
 - **One instance for each type** of resource (max reuse)

```
int sigproc(int a[]) {  
    int q, r;  
    int sum = a[0];  
    for (int i=1; i<=3;i++) {  
        sum += a[i];  
    }  
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}
```



Design Procedure Example

- **Step-1:** identify **functional units**
 - Adder, Div-by-8 (shift-right-3), Modulo-8 (extract 3 LSBs) Compare to 3, Add-1
- **Step-2:** define **hardware resources**
 - **One instance for each type** of resource (max reuse)
 - **Can be used only once in each cycle**
- **Step-3:** schedule operations across cycles and bind to hardware resources
 - For-loop in cycles 0-3, update of q in cycle 4
 - Binding with 1 operation
- **Step-4:** determine storage elements
 - Registers (2) for intermediate sum and result q

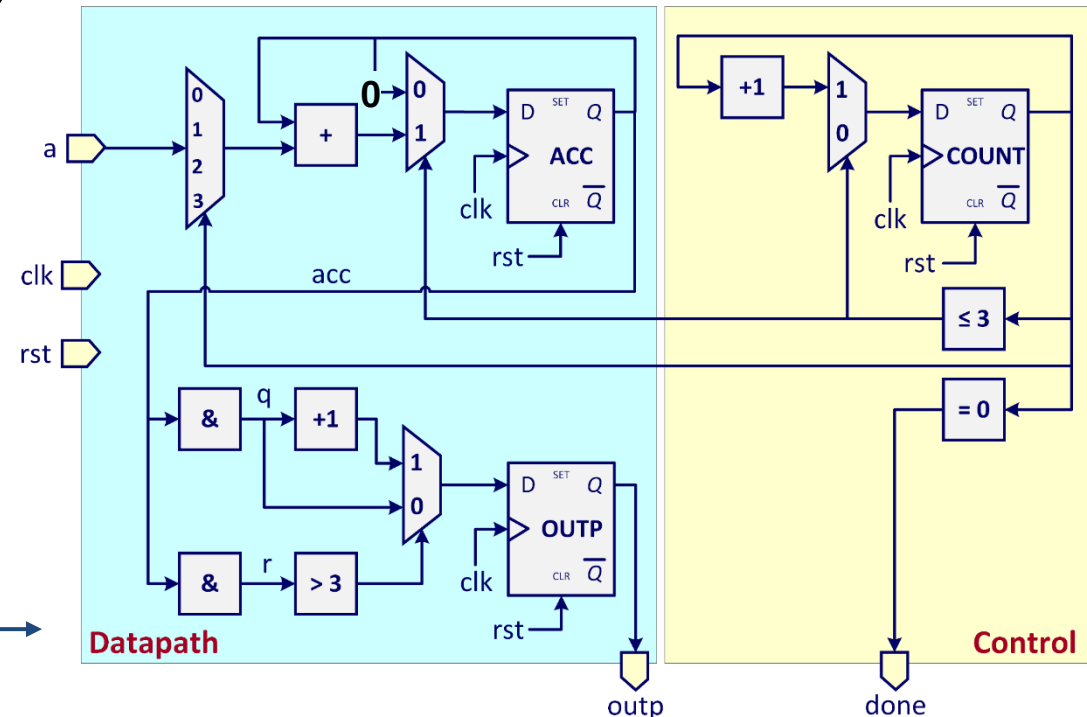
```
int sigproc(int a[]) {  
    int q, r;  
    int sum = a[0];  
    for (int i=1; i<=3; i++) {  
        sum += a[i];  
    }  
    q = sum/8;  
    r = sum % 8;  
    if (r > 3) q += 1;  
    return q;  
}
```

#	Operations	REG
0	sum_nxt=0+a[0]	sum
1	sum_nxt=sum+a[1]	sum
2	sum_nxt=sum+a[2]	sum
3	sum_nxt=sum+a[3]	sum
4	q=sum/8; q'=q+1; r=sum%8; cond=r>3 out_nxt=sel(q,q';cond)	out

Datapath Design Procedure Example

- **Step-1:** identify **functional units**
 - Adder, Div-by-8 (shift-right-3), Modulo-8 (extract 3 LSBs)
Compare to 3, Add-1
- **Step-2:** define **hardware resources**
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- **Step-3:** schedule operations across cycles and bind to hardware resources
 - For-loop in cycles 0-3, update of q in cycle 4
 - Binding with 1 operation
- **Step-4:** determine storage elements
 - Registers (2) for intermediate sum and result q
- **Step-5:** define interconnect (MUXes) →

```
int sigproc(int a[]) {  
    int q, r;  
    int sum = a[0];  
    for (int i=1; i<=3;i++) {  
        sum += a[i];  
    }  
    q = sum/8;  
    r = sum % 8;  
    if (r > 3) q += 1;  
    return q;  
}
```



```
entity sigproc is
  port (
    signal clk   : in  std_logic;
    signal rst   : in  std_logic;
    signal a     : in  std_logic_vector(31 downto 0);
    signal outp  : out std_logic_vector(7 downto 0)
    signal done  : out std_logic);
end entity sigproc;
```

```
architecture rtl of sigproc is
```

-- for structuring the input a

type au_vector is array (0 to 3) of unsigned(7 downto 0);

signal acc_next, acc_reg : unsigned(7 downto 0);
signal outp_next, outp_reg : unsigned(7 downto 0);
signal count_next, count_reg : unsigned(2 downto 0);
signal q, r : unsigned(7 downto 0);
signal au : au_vector;

...continued next slide...



Example Implementation in VHDL

...continued from previous slide...

```
begin -- architecture rtl
```

```
  REG : process (clk, rst) is
  begin
```

```
    if rst = '1' then
```

```
      acc_reg  <= (others => '0');
```

```
      outp_reg <= (others => '0');
```

```
      count_reg <= (others => '0');
```

```
    elsif rising_edge(clk) then
```

```
      acc_reg  <= acc_next;
```

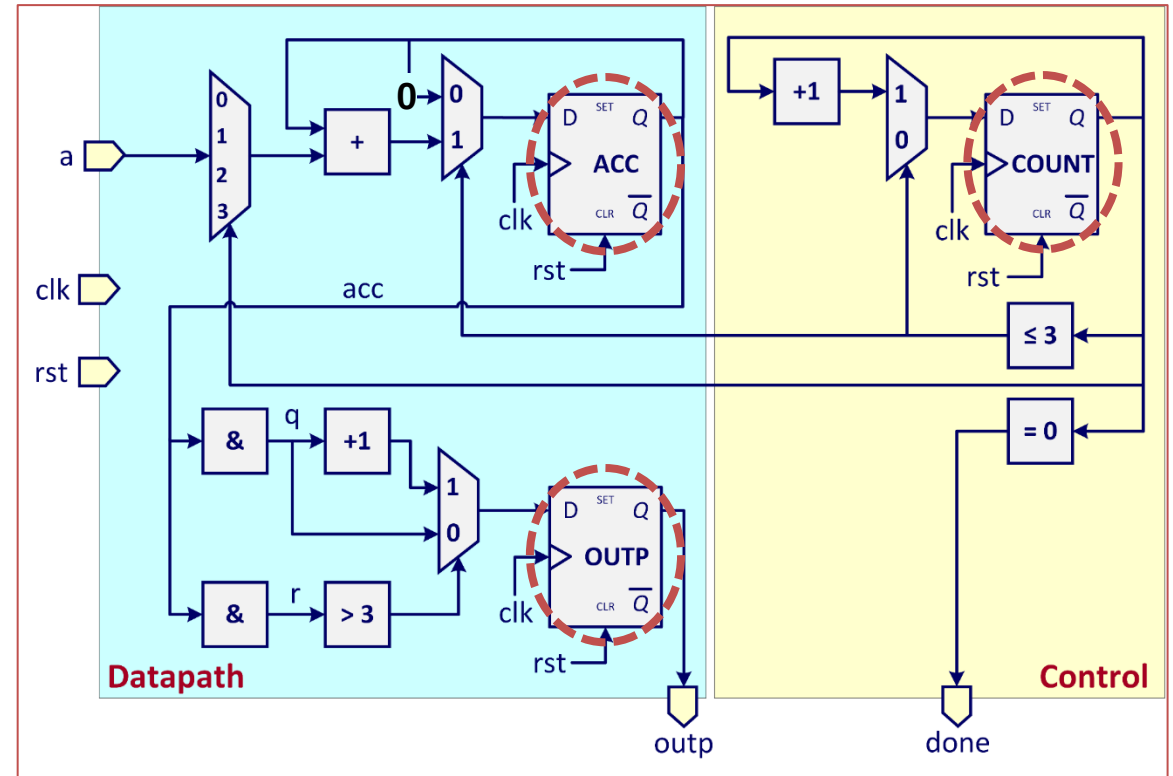
```
      outp_reg <= outp_next;
```

```
      count_reg <= count_next;
```

```
    end if;
```

```
  end process REG;
```

...continued next slide ...



Example Implementation in VHDL

...continued from previous slide...

```
count_next <= count_reg + 1 when count_reg <= 3 else  
    (others => '0');  
done <= '1' when count_reg = 0 else  
    '0';
```

```
A_SPLIT : for i in 0 to 3 generate  
    au(i) <= unsigned(a((i+1)*8-1 downto i*8));  
end generate A_SPLIT;
```

```
acc_next <= acc_reg + au(to_integer(count_reg))  
    when count_reg <= 3 else  
    (others => '0');
```

```
q <= "000" & acc_reg(7 downto 3); -- /8  
r <= "00000" & acc_reg(2 downto 0); -- % 8  
outp_next <= q + 1 when r > 3 else  
    q;
```

```
outp <= std_logic_vector(outp_reg);  
end architecture rtl;
```

