

EE-334

Digital System Design

Discussion for Exercise 5
Mandelbrot Datapath

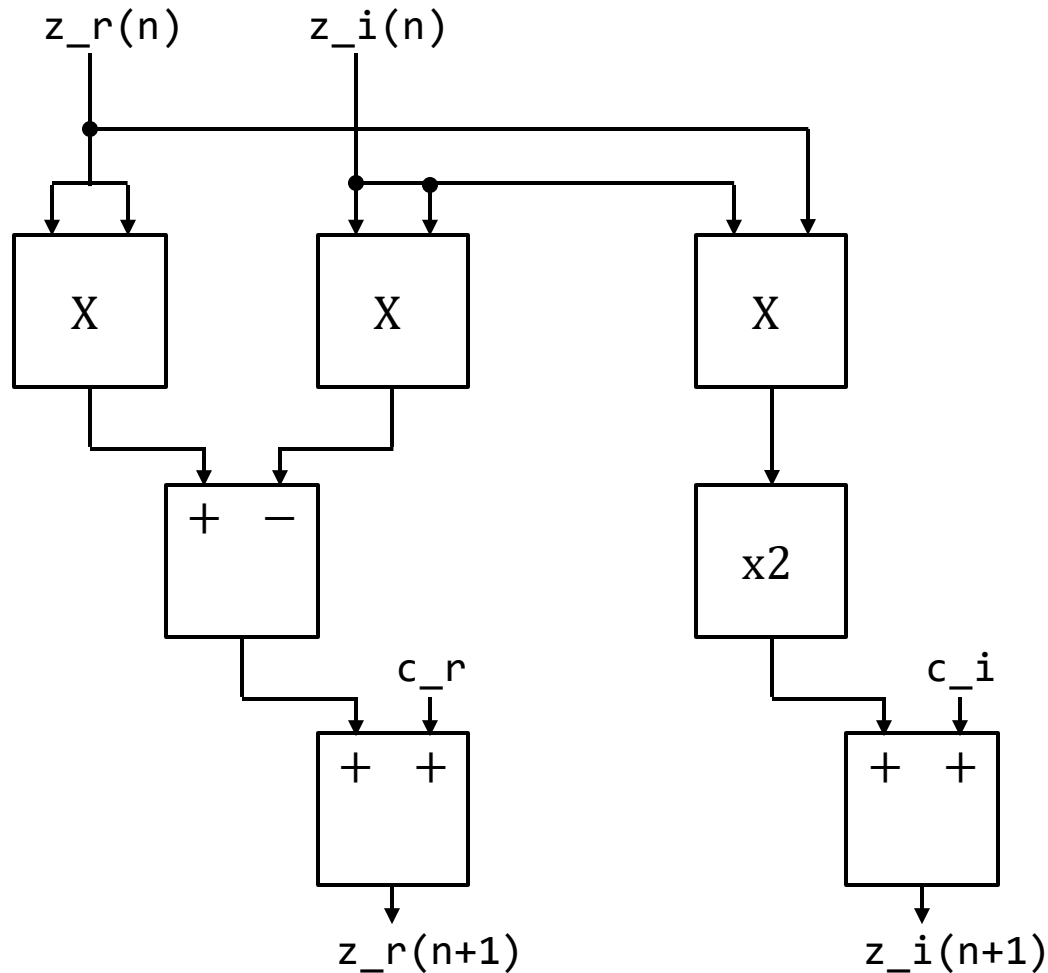
Andreas Burg

Mandelbrot Datapath

- Implement the datapath of a Mandelbrot generator:
 - Algorithm with **variable number of iterations**: this **can not be mapped to an isomorphic architecture**
 - **We assume the maximum number of iterations (MAX_ITER=4)**
- Algorithm specification in pseudo code:

```
GIVEN AT DESIGN TIME : MAX_ITER
INPUTS : c_r , c_i ;
OUTPUT: z_r , z_i ;
z_r=c_r;
z_i=c_i;
for (n=0; n<MAX_ITER; n++) {
    z_r = z_r * z_r - z_i * z_i + c_r;
    z_i = 2*z_r * z_i + c_i;
}
```

Mandelbrot Datapath (Single Iteration)



GIVEN AT DESIGN TIME : MAX ITER

INPUTS : c_r , c_i ;

OUTPUT: z_r , z_i ;

$z_r = c_r$;

$z_i = c_i$;

for ($n=0$; $n < \text{MAX ITER}$; $n++$) {

$z_r = z_r * z_r - z_i * z_i + c_r$;

$z_i = 2 * z_r * z_i + c_i$;

}

Comb. area per iteration:

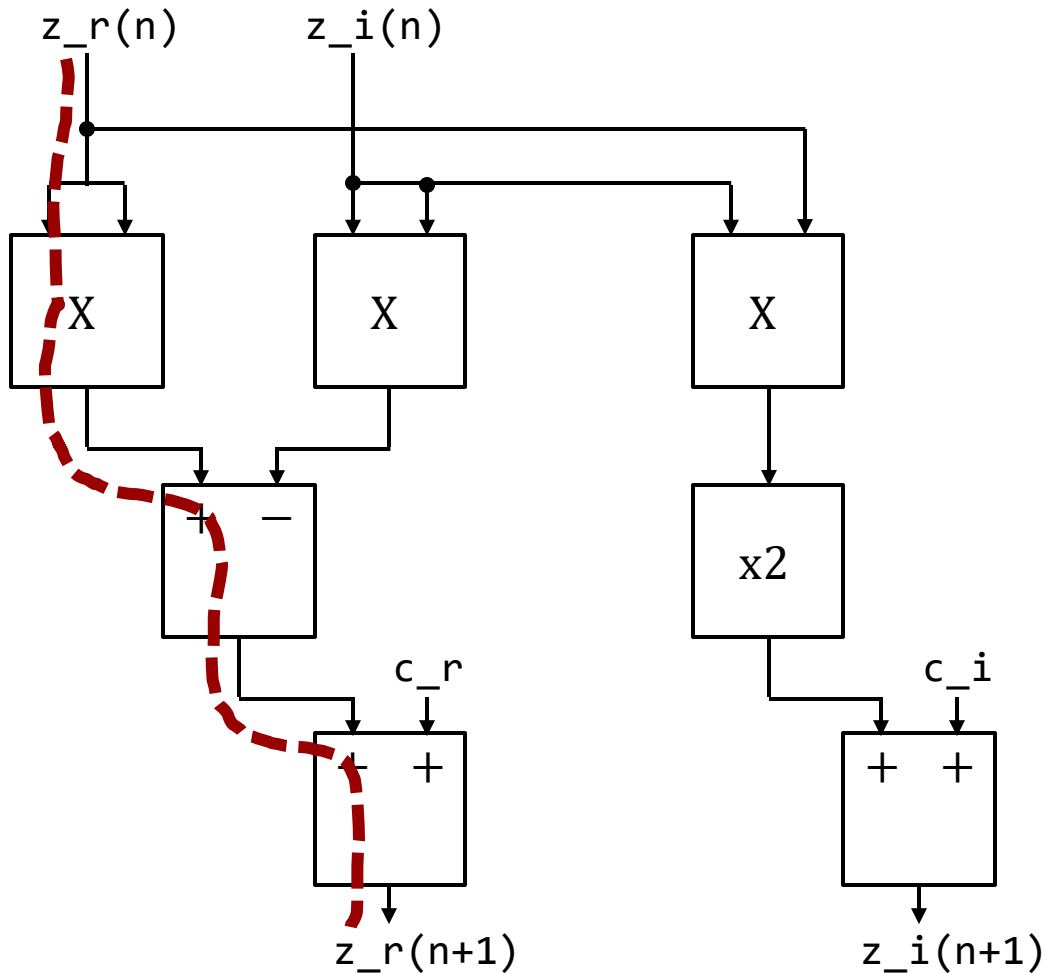
3 Mult (Squarer): 3x3'600

3 Add/Sub : 3x3'600

1 Mult-2 (Shift) : 0

21'600

Mandelbrot Datapath (Single Iteration)



GIVEN AT DESIGN TIME : MAX ITER

INPUTS : c_r , c_i ;

OUTPUT: z_r , z_i ;

$z_r = c_r$;

$z_i = c_i$;

for ($n=0$; $n < \text{MAX ITER}$; $n++$) {

$z_r = z_r * z_r - z_i * z_i + c_r$;

$z_i = 2 * z_r * z_i + c_i$;

}

Critical Path

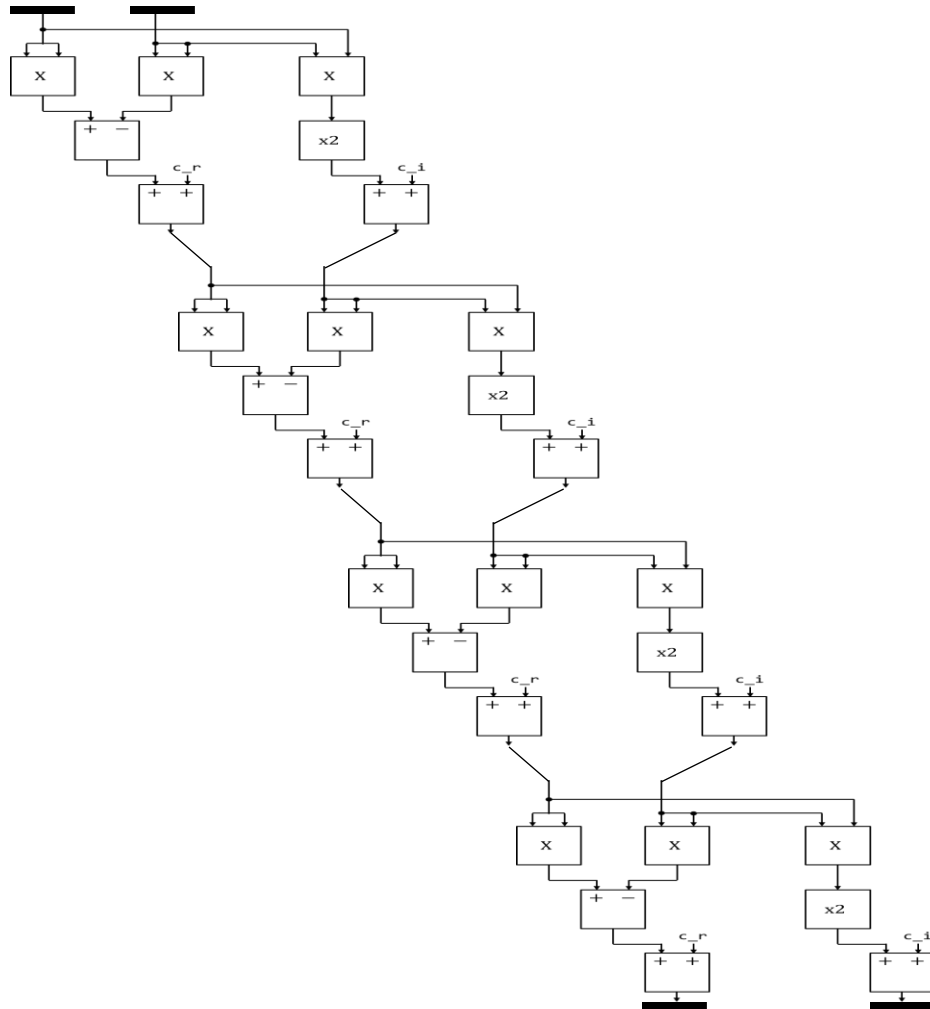
Multiplier 2.3ns

Adder 0.5ns

Adder 0.5ns

3.3ns

Mandelbrot Isomorphic Datapath



```

GIVEN AT DESIGN TIME : MAX ITER
INPUTS : c_r , c_i ;
OUTPUT: z_r , z_i ;
z_r=c_r;
z_i=c_i;
for (n=0; n<MAX ITER; n++) {
    z_r = z_r * z_r - z_i * z_i + c_r;
    z_i = 2*z_r * z_i + c_i;
}
    
```

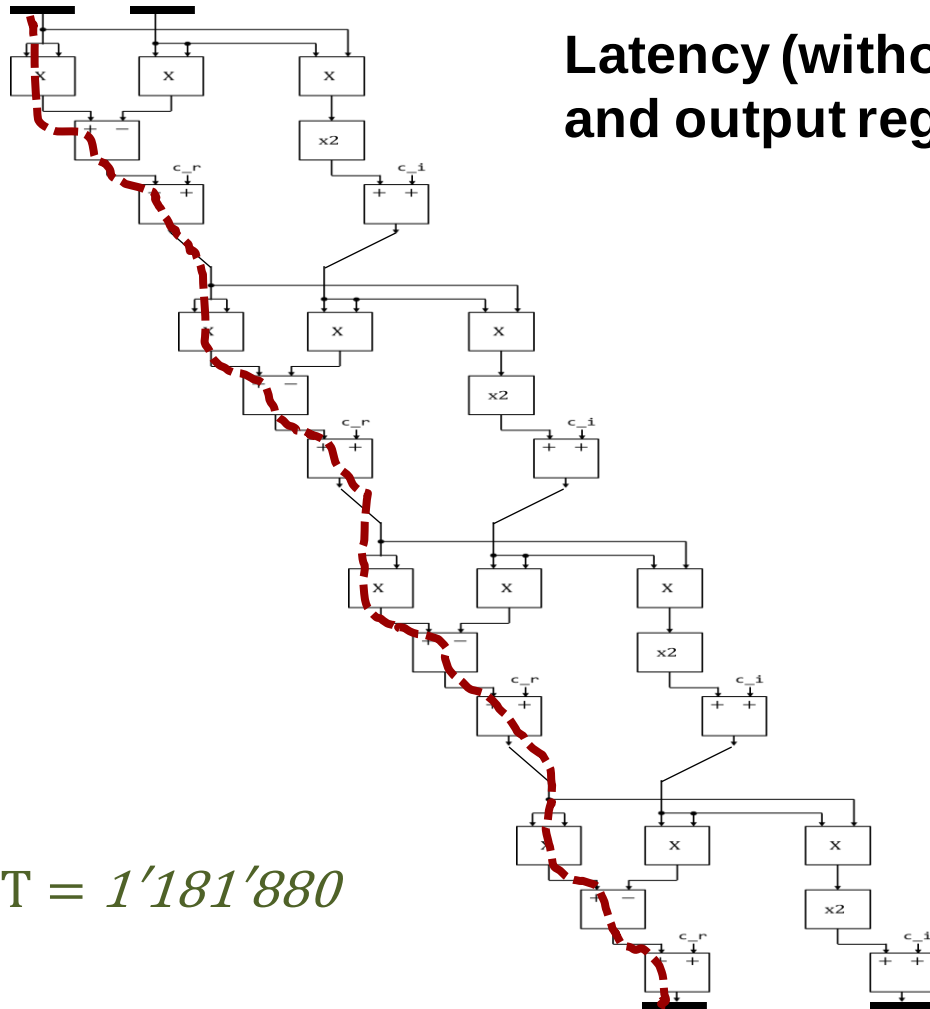
Comb. area per iteration:

Stages 1-4 21'600x4

In/Out Regs 4x450

88'200

Mandelbrot Isomorphic Datapath



Latency (without input
and output registers): 0

$AT = 1'181'880$

GIVEN AT DESIGN TIME : MAX ITER

INPUTS : c_r , c_i ;

OUTPUT: z_r , z_i ;

$z_r = c_r$;

$z_i = c_i$;

for ($n=0$; $n<\text{MAX ITER}$; $n++$) {

$z_r = z_r * z_r - z_i * z_i + c_r$;

$z_i = 2 * z_r * z_i + c_i$;

}

Critical Path

1x Reg CK-Q 0.2 ns

4xMultiplier 4x2.3 ns

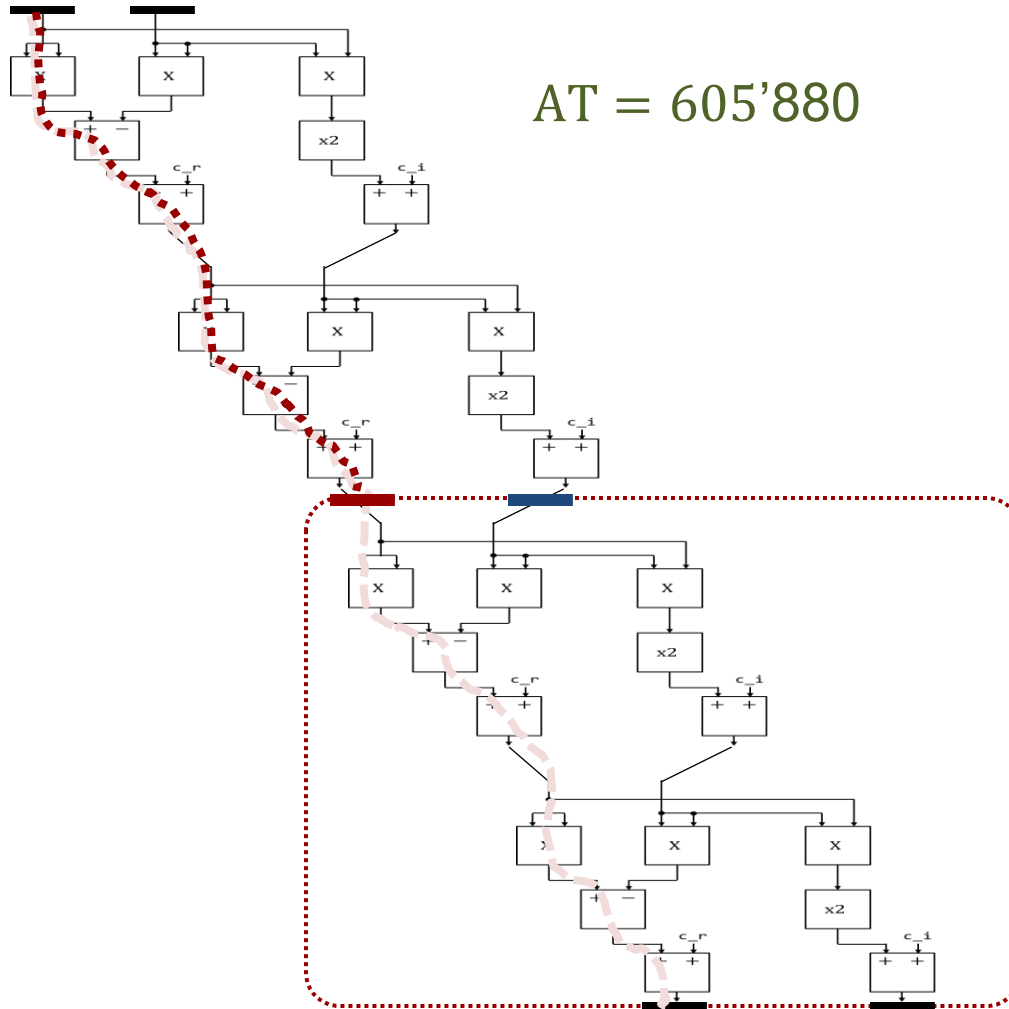
8xAdder 8x0.5 ns

13.4ns

$T_{CLK} > T_{crit} + T_{setup}$

$T_{CLK} > 13.4 + 0.0$

Mandelbrot Pipelined Datapath



Critical Path

1x Reg CK-Q	0.2 ns
2xMultiplier	2x2.3 ns
4xAdder	4x0.5 ns

$$T_{CLK} > T_{crit} + T_{setup}$$

$$T_{CLK} > 6.8 + 0.0$$

6.8ns

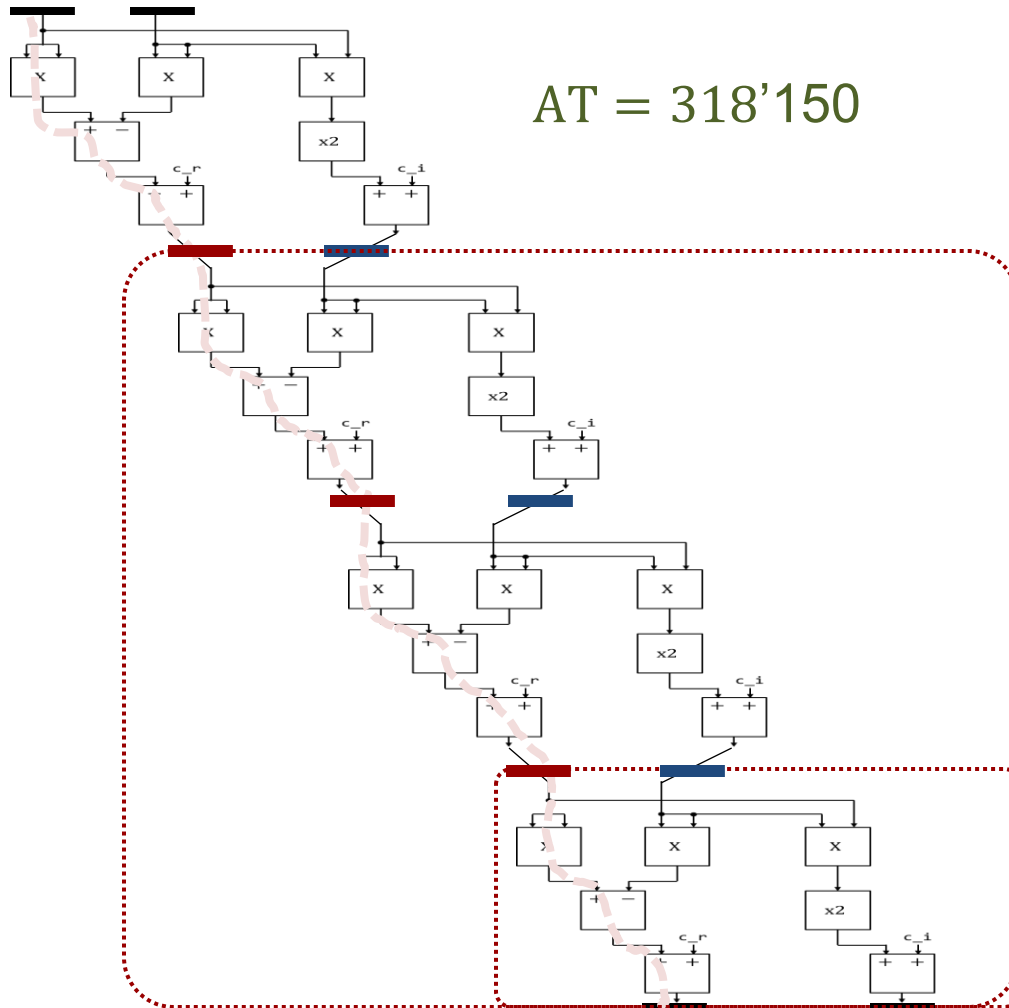
Comb. area per iteration:

Stages 1-4	21'600x4
Pipeline Regs	2x450
In/Out Regs	4x450

89'100

Clock frequency improvement
(compared to no pipelining): 1.97x

Mandelbrot Pipelined Datapath



Critical Path

1x Reg CK-Q	0.2 ns
1xMultiplier	1x2.3 ns
2xAdder	2x0.5 ns

$$T_{CLK} > T_{crit} + T_{setup}$$

$$T_{CLK} > 3.5 + 0.0$$

3.5ns

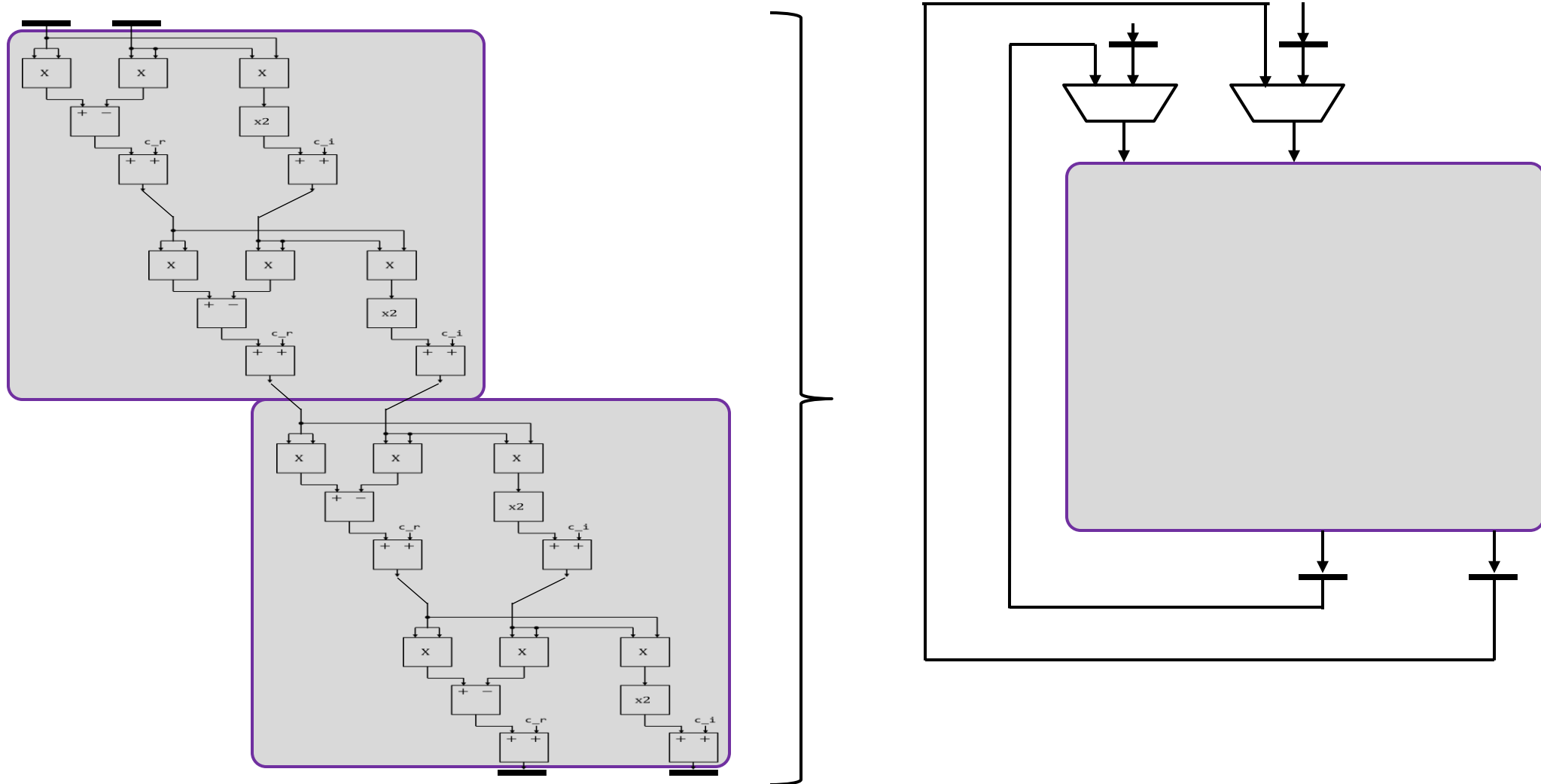
Comb. area per iteration:

Stages 1-4	21'600x4
Pipeline Regs	6x450
In/Out Regs	4x450

90'900

Clock frequency improvement
(compared to 1x pipelining): 1.94x

Mandelbrot Iterative Decomposition (2x)



Mandelbrot Iterative Decomposition (2x)

Critical Path

1x Reg CK-Q	0.2 ns
1x MUX	0.1 ns
2xMultiplier	2x2.3 ns
4xAdder	4x0.5 ns

6.9ns

Comb. area:

Pair of Stages	21'600x2
MUX	2x450
In/Out Regs	4x450

45'900

$$\Gamma = 2$$

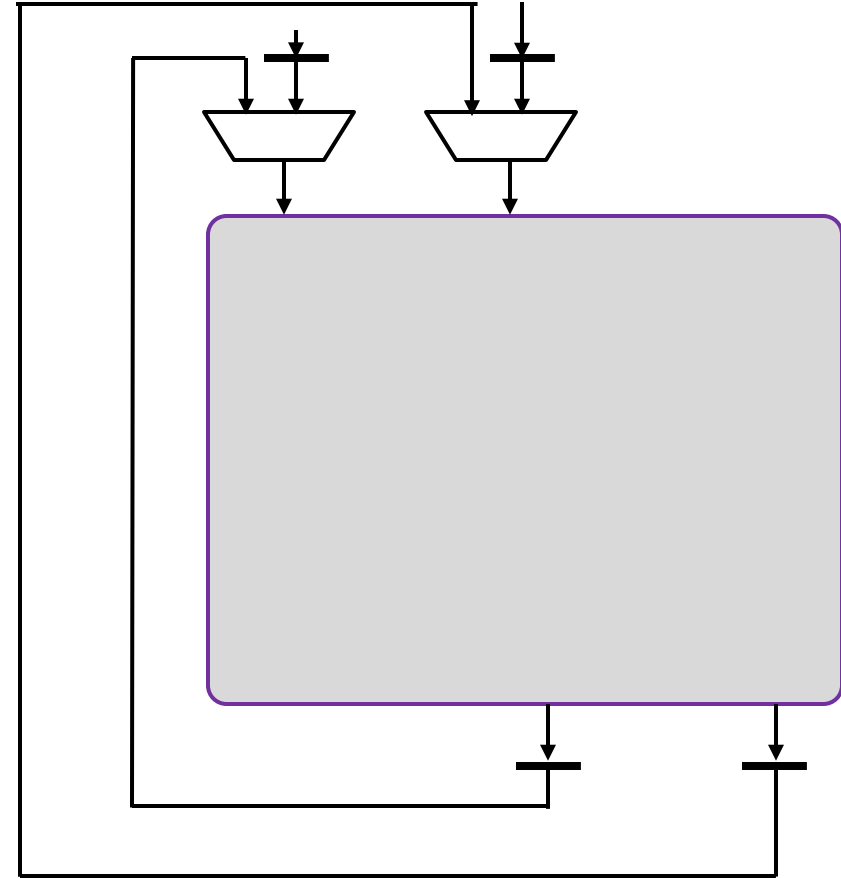
$$T_{CLK} > T_{crit} + T_{setup}$$

$$T_{CLK} > 6.9 + 0.0$$

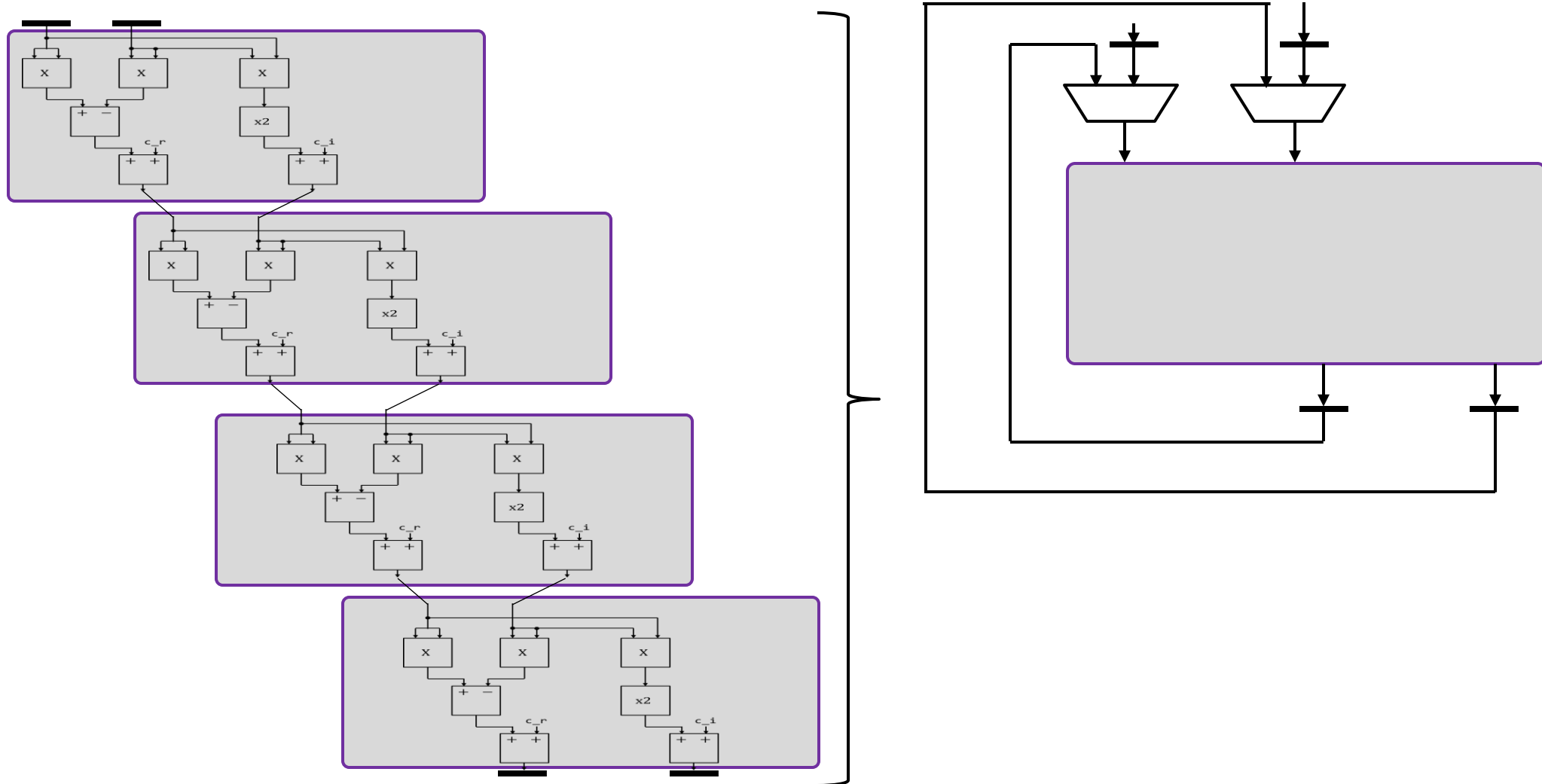
$$T_{pd} = \Gamma * T_{clk}$$

$$T_{pd} = 2 * 6.9 = 13.8ns$$

$$AT = 633'420$$



Mandelbrot Iterative Decomposition (4x)



Mandelbrot Iterative Decomposition (4x)

Critical Path

1x Reg CK-Q	0.2 ns
1x MUX	0.1 ns
1xMultiplier	1x2.3 ns
2xAdder	2x0.5 ns

3.6ns

Comb. area:

One Stages	21'600
MUX	2x450
In/Out Regs	4x450

24'300

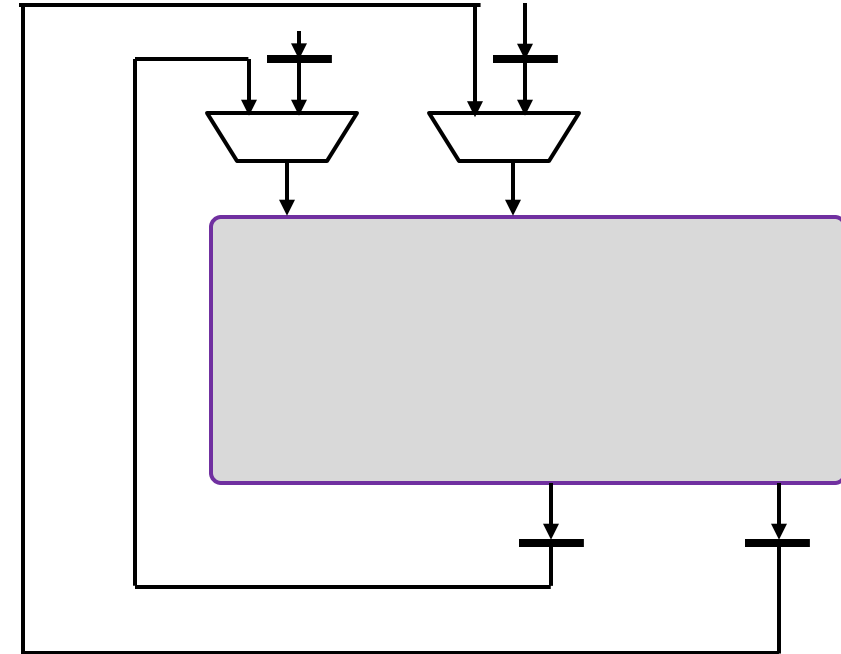
$$\Gamma = 4$$

$$T_{CLK} > T_{crit} + T_{setup}$$

$$T_{CLK} > 3.6 + 0.0$$

$$T_{pd} = \Gamma * T_{clk}$$

$$T_{pd} = 4 * 3.6 = 14.4ns$$



$$AT = 349'920$$

Architecture Comparison

Architecture	Area	Time per data item	AT-product
Isomorphic	88'200	13.4 ns	1'181'880
Pipelined (2 stages)	89'100	6.8 ns	605'880
Pipelined (4 stages)	90'900	3.5 ns	318'150
Iterative decomposed (2 cycles)	45'900	13.8 ns	633'420
Iterative decomposed (4 cycles)	24'300	14.4 ns	349'920