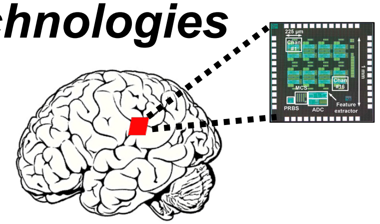
Integrated Neurotechnologies Laboratory



Analog IC design (EE-320), Lecture 4

Prof. Mahsa Shoaran

Institute of Electrical and Micro Engineering, School of Engineering, EPFL

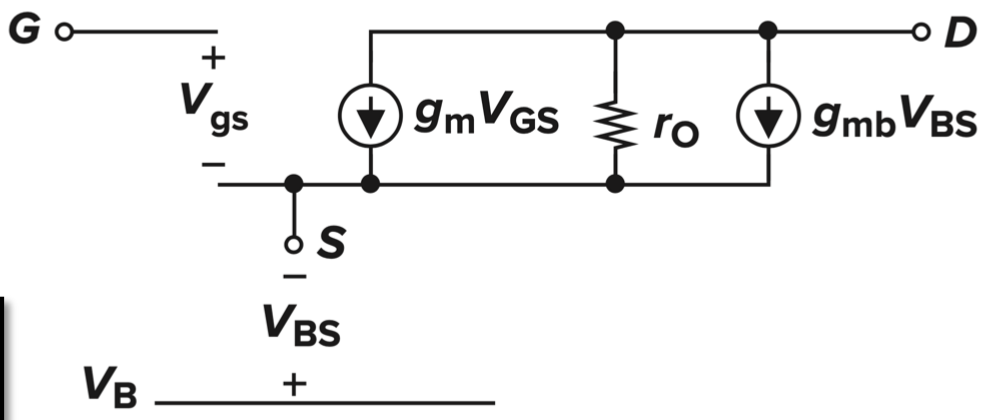
Review: Small-signal model

Saturation

$$I_D \approx \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \frac{2I_D}{V_{GS}-V_{TH}}$$



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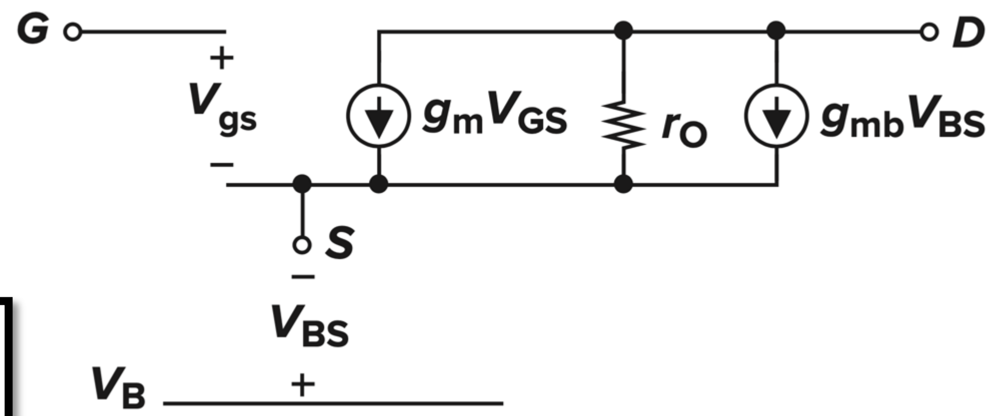
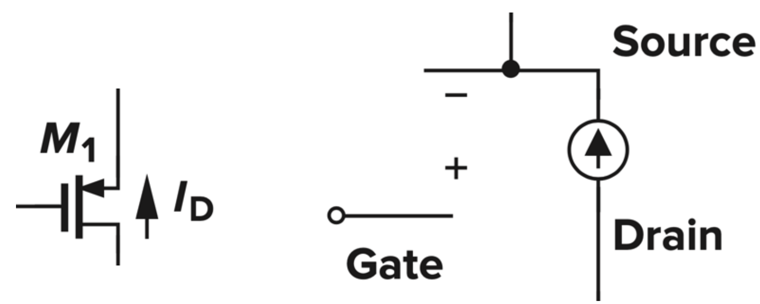


$$\boxed{g_m = \frac{\partial I_D}{\partial V_{GS}}}$$

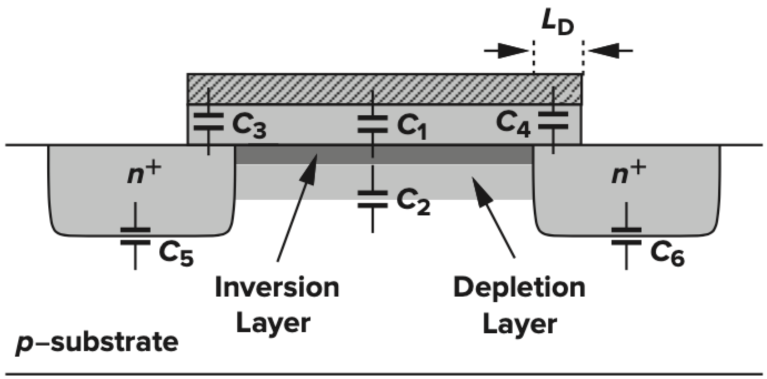
$$\boxed{\begin{aligned} g_m &= \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} \\ &= \frac{2I_D}{V_{GS}-V_{TH}} \end{aligned}}$$

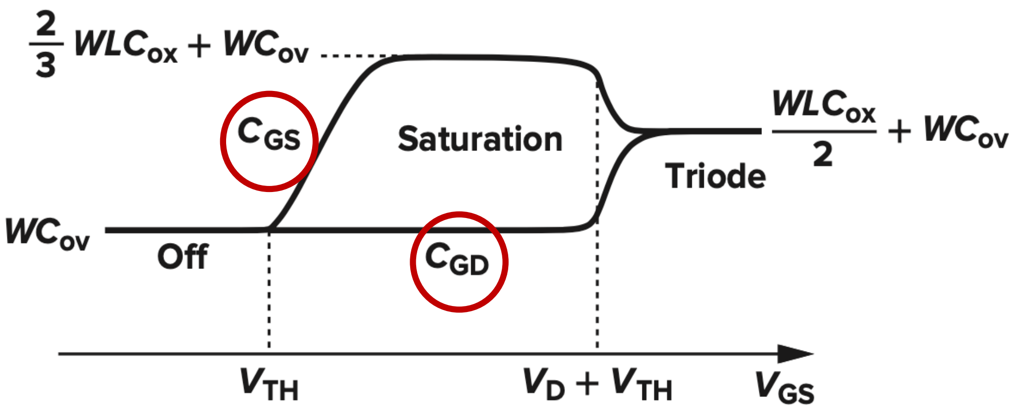
$$\boxed{r_O \approx \frac{1}{\lambda I_D}}$$

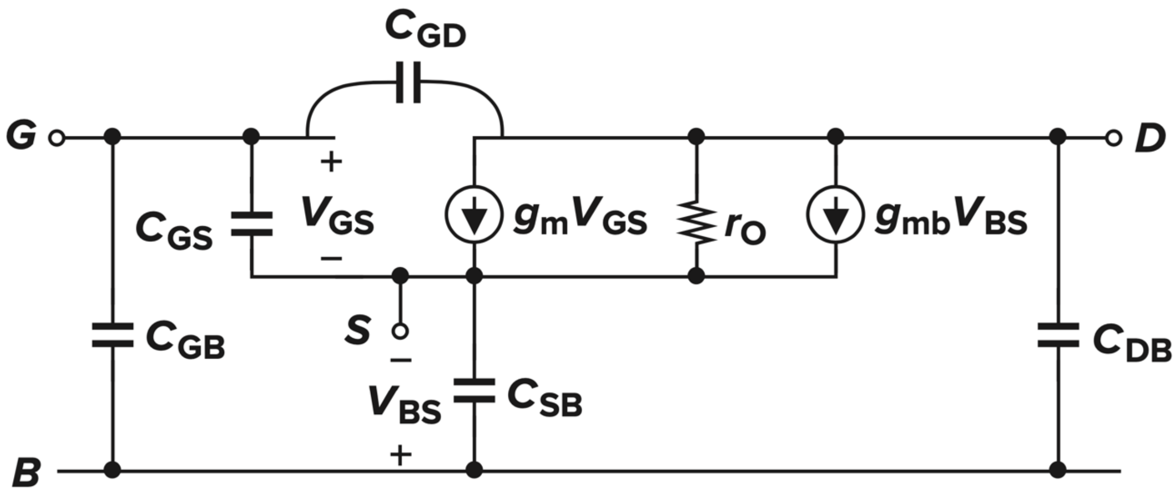
$$\boxed{\begin{aligned} g_{mb} &= \frac{\partial I_D}{\partial V_{BS}} \\ &= g_m \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}} \\ &= \eta g_m \end{aligned}}$$



Review: MOS Device Capacitances, layout

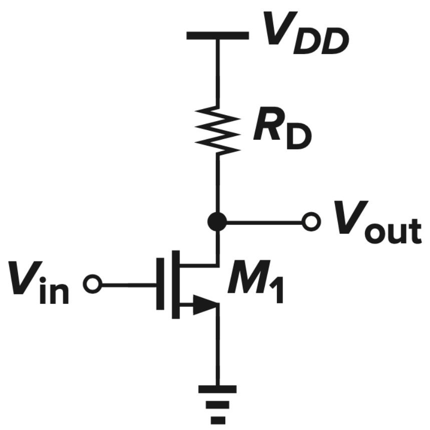






Common-Source Stage - with Resistive Load

- The **common-source** topology: receives the input at the gate and produces the output at the **drain**

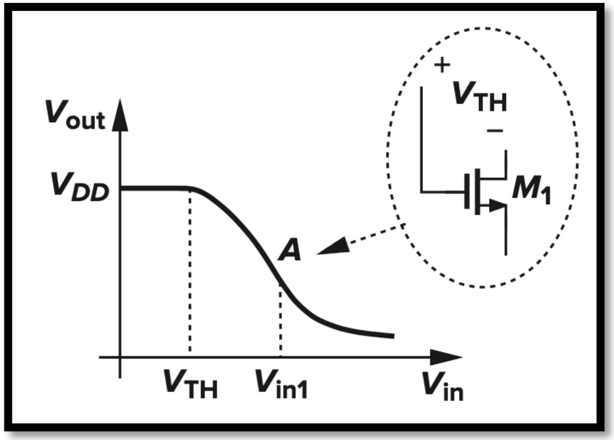


$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 \qquad \textit{(saturation)}$$

$$V_{in1} - V_{TH} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{TH})^2$$

(triode)

$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{in} - V_{TH})V_{out} - V_{out}^2\right]$$



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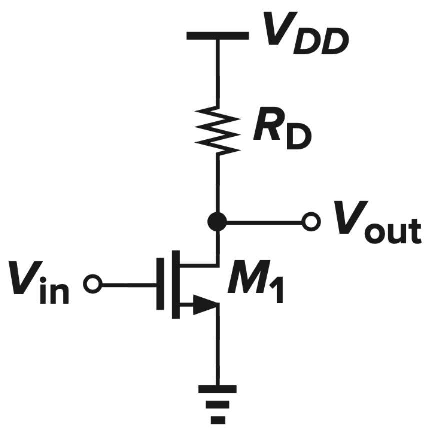

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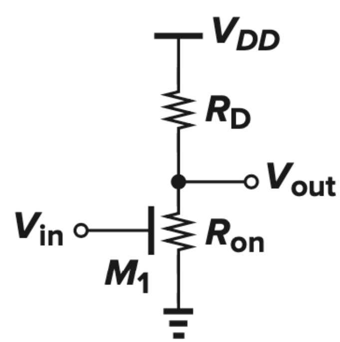
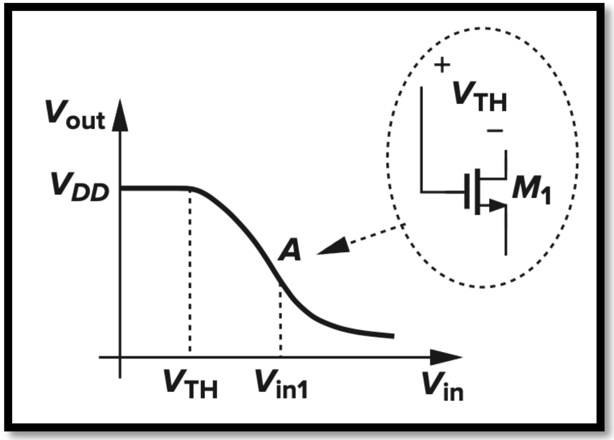
$$V_{out} \ll 2(V_{in} - V_{TH})$$

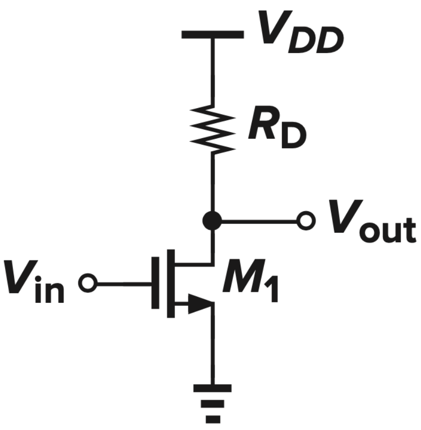
$$V_{out} = V_{DD}\frac{R_{on}}{R_{on} + R_D}$$

$$= \frac{V_{DD}}{1 + \mu_n C_{ox}\frac{W}{L}R_D(V_{in} - V_{TH})}$$





Common-Source Stage - with Resistive Load



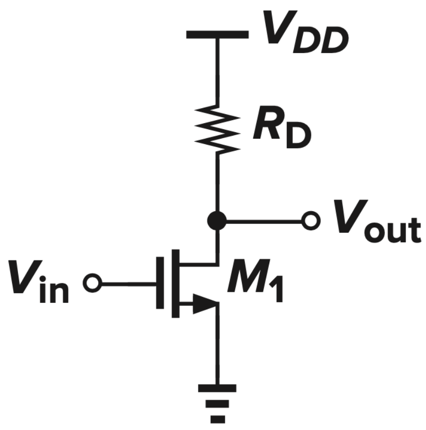
$$\boxed{V_{out} > V_{in} - V_{TH}}$$

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}}$$

$$= -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$$

$$= -g_m R_D$$



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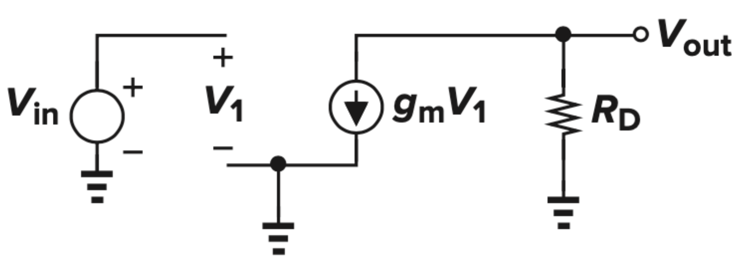
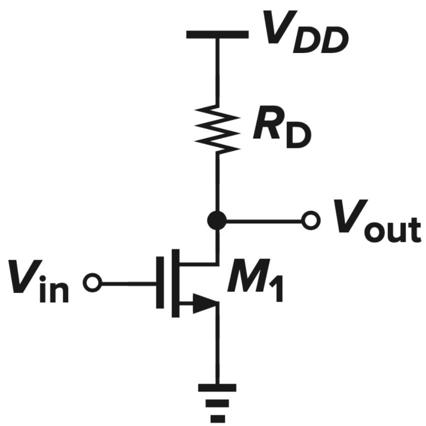
small-signal model:



Common-Source Stage - with Resistive Load



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$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

$$\begin{aligned} A_v &= \frac{\partial V_{out}}{\partial V_{in}} \\ &= -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH}) \\ &= -g_m R_D \end{aligned}$$

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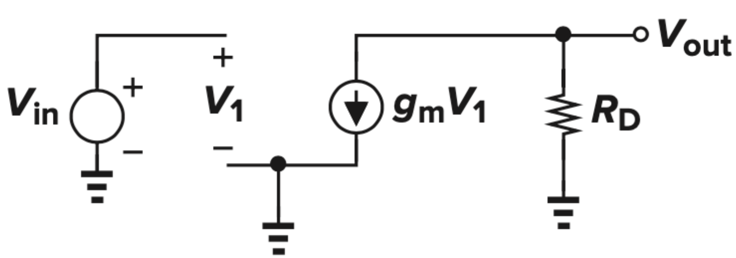
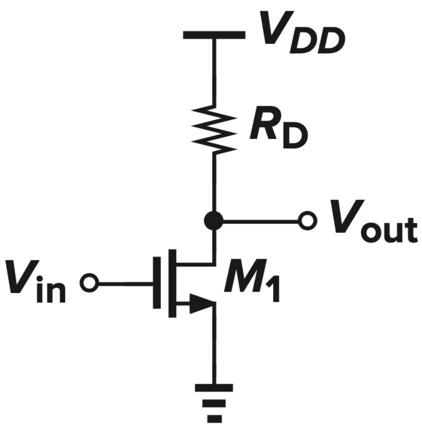


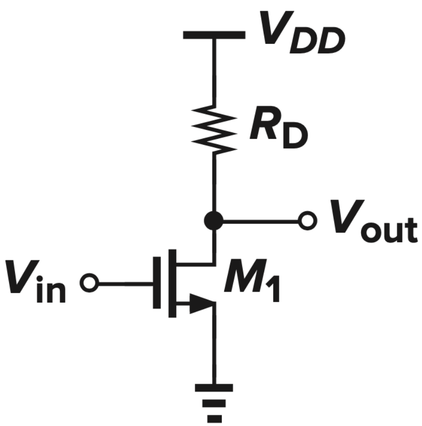
$$A_v = -\sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \frac{V_{RD}}{I_D}$$

$$A_v = -\sqrt{2 \mu_n C_{ox} \frac{W}{L}} \frac{V_{RD}}{\sqrt{I_D}}$$

Example: I_D and g_m as a function of V_{in}?



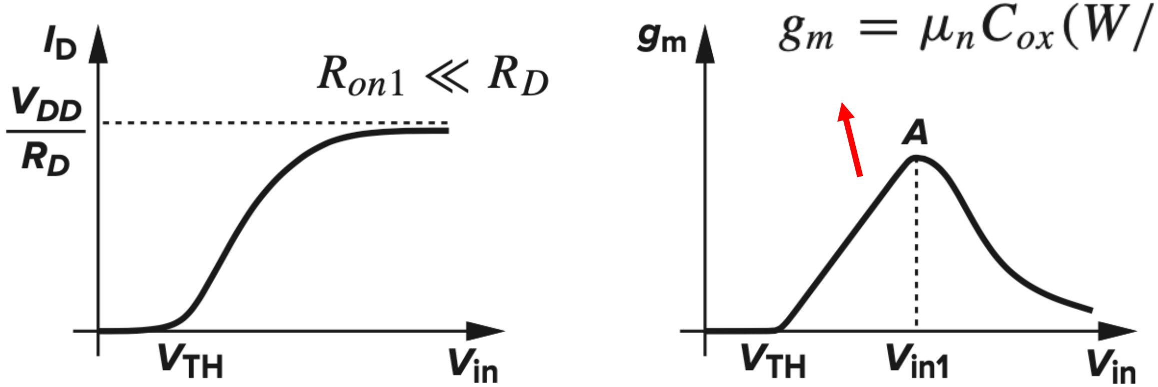
Example: I_D and g_m as a function of V_{in}?



(saturation)

$$g_m = \mu_n C_{ox}(W/L)(V_{GS} - V_{TH})$$

$$R_{on1} \ll R_D$$



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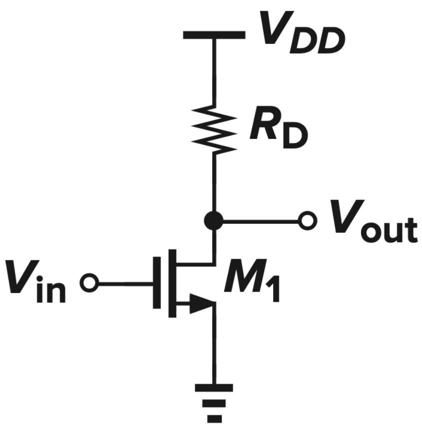


(triode)

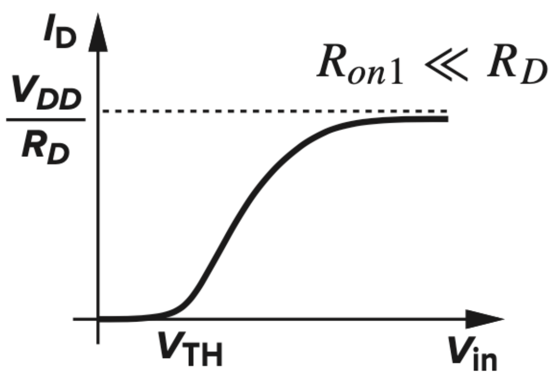
$$g_m = \mu_n C_{ox}(W/L)V_{DS}$$

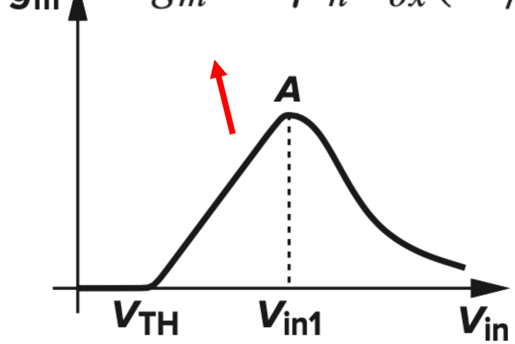
$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{in} - V_{TH})V_{out} - V_{out}^2\right]$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{-\mu_n C_{ox}(W/L)R_D V_{out}}{1 + \mu_n C_{ox}(W/L)R_D \underbrace{(V_{in} - V_{TH} - V_{out})}_{\text{(point } A\text{)}}}$$

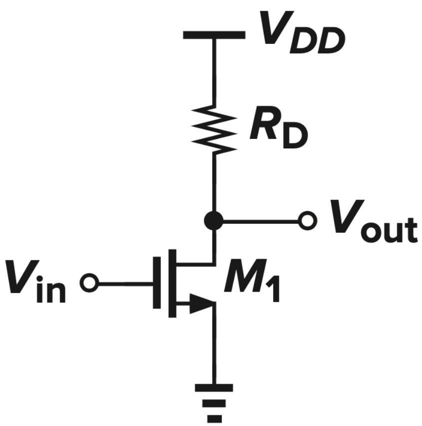
(saturation)

$$g_m = \mu_n C_{ox}(W/L)(V_{GS} - V_{TH})$$





CS stage including channel-length modulation



$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2(1 + \lambda V_{out})$$

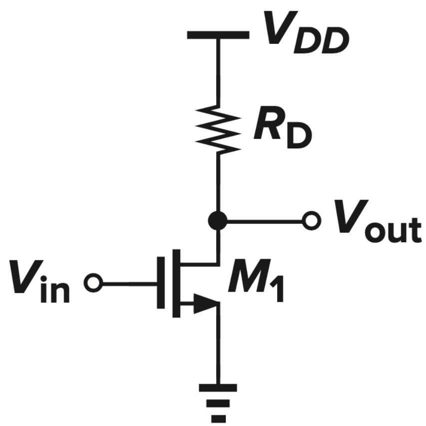
$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})(1 + \lambda V_{out})$$

$$-R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}}$$

$$A_v = -R_D g_m - \frac{R_D}{r_O} A_v$$

$$= -g_m \frac{r_O R_D}{r_O + R_D}$$

CS stage including channel-length modulation



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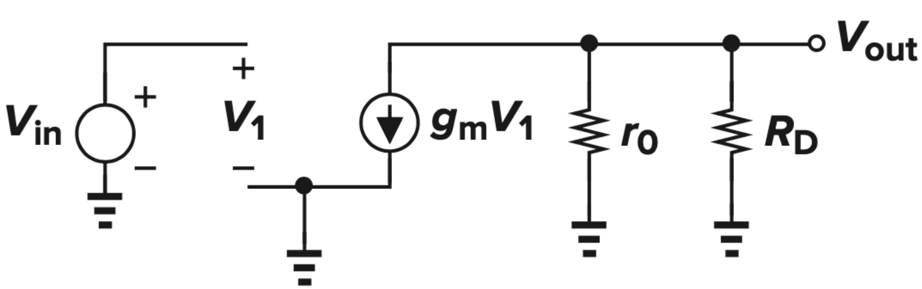
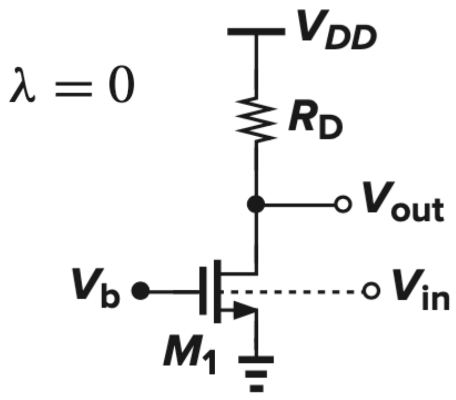
$$= -g_m\frac{r_O R_D}{r_O + R_D}$$



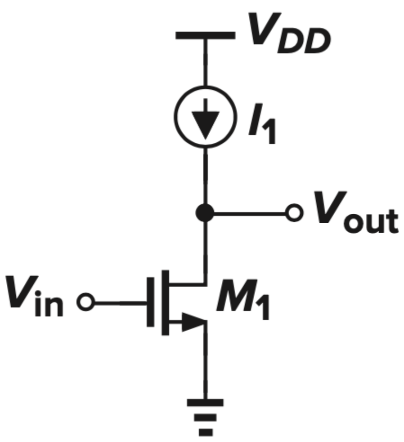
$$g_m V_1(r_O \| R_D) = -V_{out}$$

$$V_{out}/V_{in} = -g_m(r_O \| R_D)$$

Example: Voltage gain?



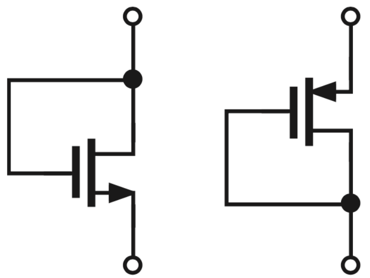
$$A_v = -g_{mb} R_D$$

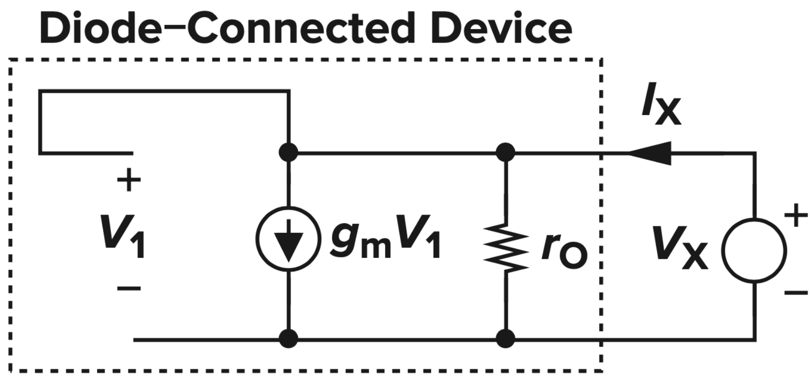


$$A_v = -g_m r_O$$

CS Stage - with Diode-Connected Load

- It is difficult to fabricate resistors with controlled values or a reasonable physical size
  - use transistors with **gain** and **drain** shorted





$$V_X/I_X = (1/g_m)\|r_O \approx 1/g_m$$

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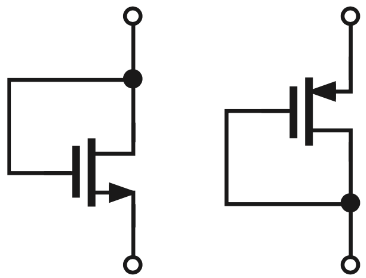
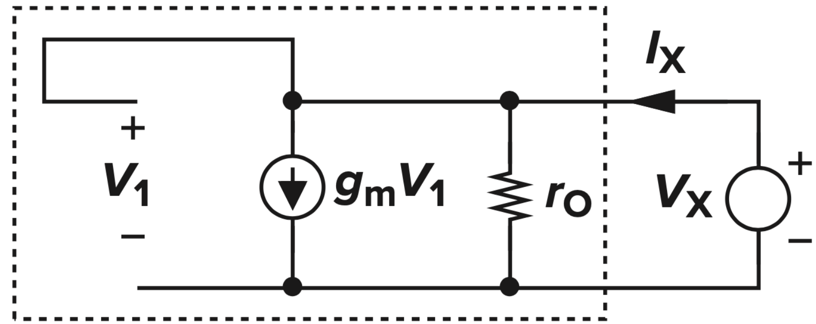
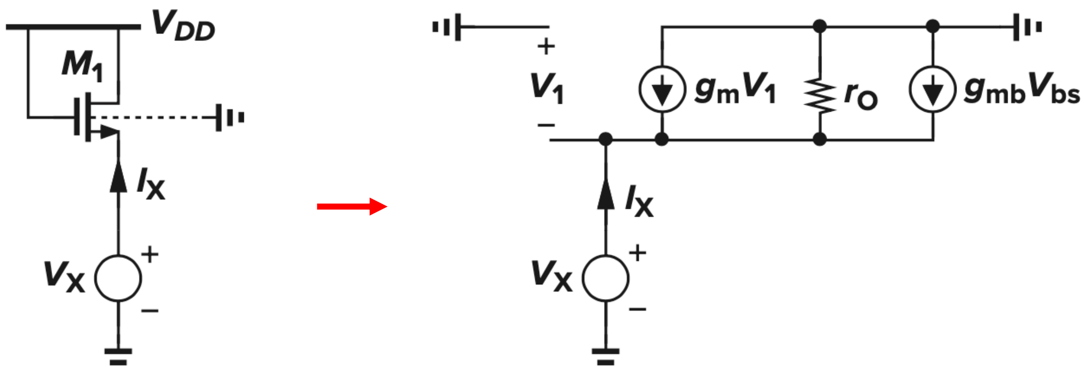


Diode−Connected Device



$$V_X/I_X = (1/g_m)\|r_O \approx 1/g_m$$



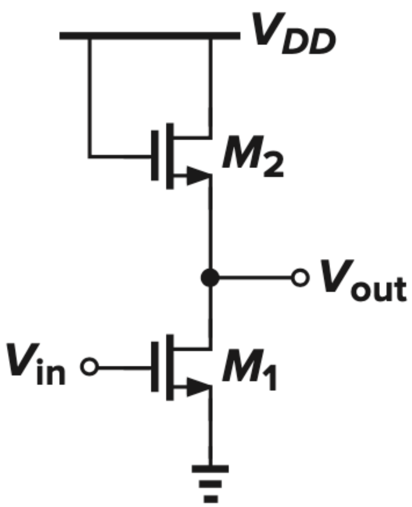
$$(g_m + g_{mb})V_X + \frac{V_X}{r_O} = I_X$$

$$\frac{V_X}{I_X} = \frac{1}{g_m + g_{mb} + r_O^{-1}}$$

$$= \frac{1}{g_m + g_{mb}} \| r_O$$

$$\approx \frac{1}{g_m + g_{mb}}$$

CS Stage - with Diode-Connected Load



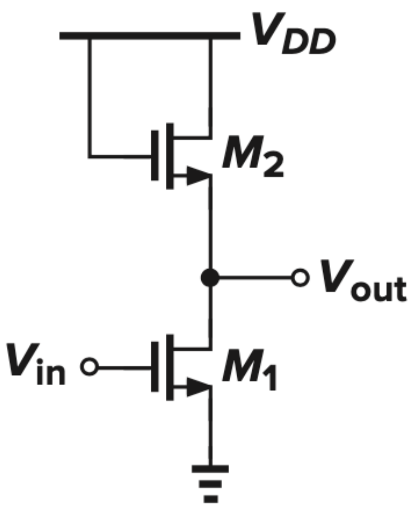
$$A_v = -g_{m1}\frac{1}{g_{m2}+g_{mb2}}$$

$$= -\frac{g_{m1}}{g_{m2}}\frac{1}{1+\eta}$$

$$A_v = -\frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}(W/L)_2 I_{D2}}}\frac{1}{1+\eta}$$

$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta}$$

CS Stage - with Diode-Connected Load



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$$= -\frac{g_{m1}}{g_{m2}}\frac{1}{1+\eta}$$

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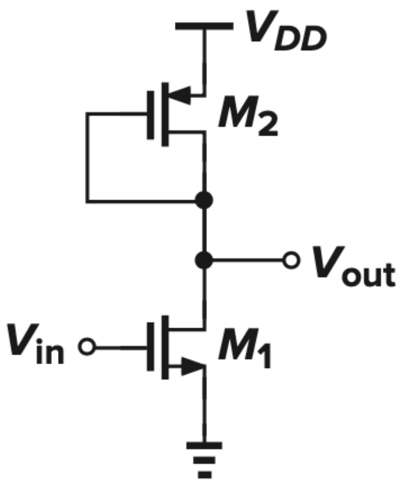
$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta}$$

✓ The input-output characteristic is relatively linear

$$\boxed{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{out}-V_{TH2})^2}$$

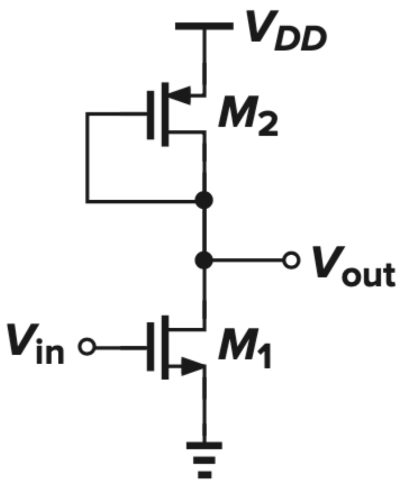
CS Stage - with PMOS Diode-Connected Load



(assuming no r_o)

$$A_v = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$$

CS Stage - with PMOS Diode-Connected Load



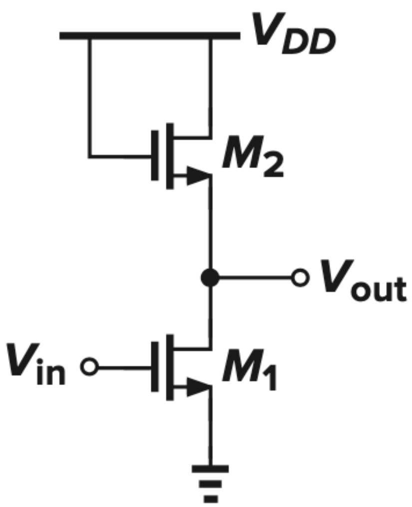
(assuming no r_o)

$$A_v = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$$

- A **high gain** requires a "**strong**" input device and a "**weak**" load device
  - disproportionately wide or long transistors >> large input or load cap
  - another limitation: reduction in allowable voltage swings

$$\mu_n \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 = \mu_p \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})^2 \rightarrow \boxed{\frac{|V_{GS2} - V_{TH2}|}{V_{GS1} - V_{TH1}} = A_v}$$

CS Stage - with Diode-Connected Load



- Including channel-length modulation:

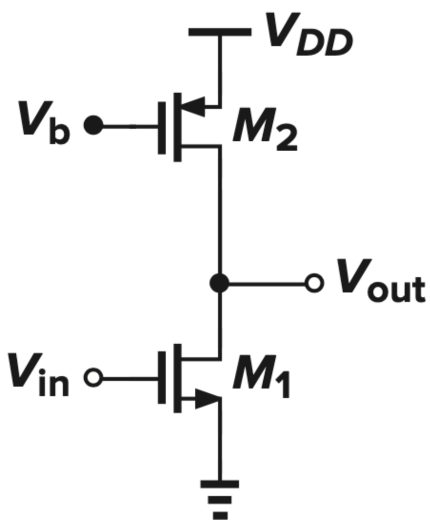
$$A_v = -g_{m1}\left(\frac{1}{g_{m2}}\|r_{O1}\|r_{O2}\right)$$

CS Stage with Current-Source Load

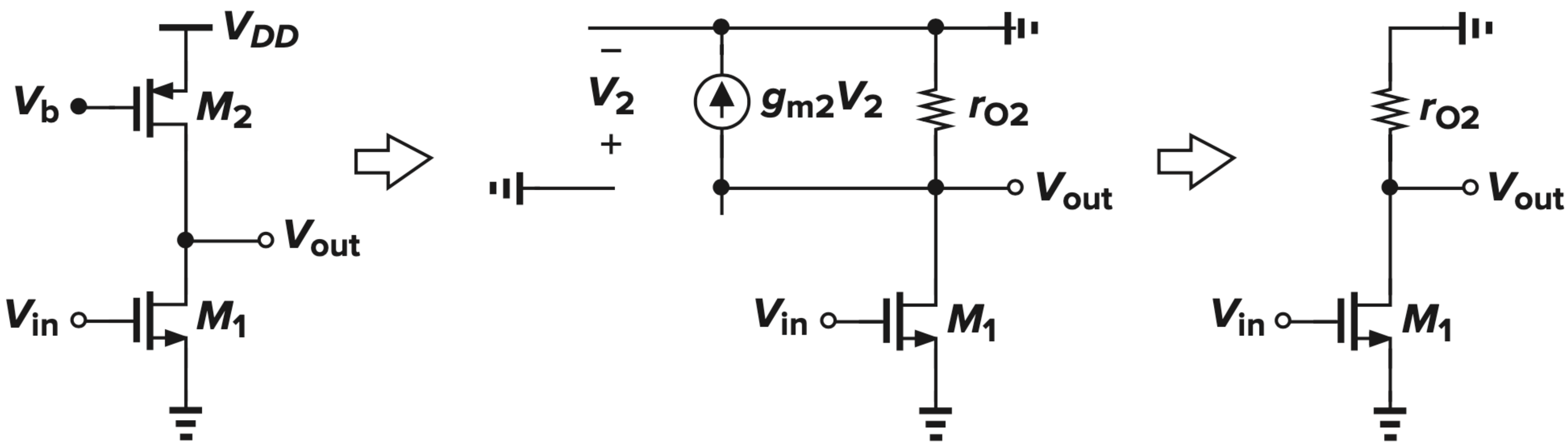
- To achieve a large voltage gain in a single stage, $A_v = -g_m R_D$ suggests that we should increase the **load impedance** of the CS stage

- With a resistor or diode-connected load, increasing the load resistance translates to a large dc drop across the load, limiting the output voltage **swing**

- A more practical approach is to replace the load with a device that does not obey Ohm's law, such as a **current source**



CS Stage with Current-Source Load



$$A_v = -g_{m1}(r_{O1} \| r_{O2})$$

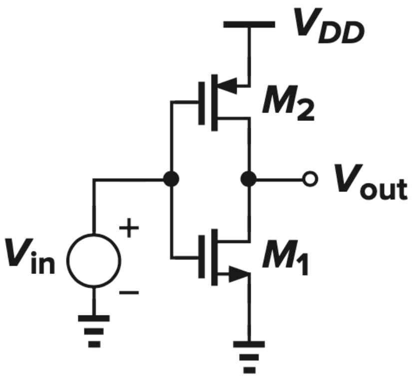
- The output impedance and minimum required $|V_{DS}|$ of M_2 less strongly coupled than the value and voltage drop of a resistor

$$|V_{DS2,min}| = |V_{GS2} - V_{TH2}|$$

- If ro_2 is not sufficiently high, the length and width of M_2 can be increased while maintaining the same overdrive voltage
  - penalty: the larger capacitance introduced by M_2 at the output node

CS Stage with Active Load

- Replace the constant current source with an amplifying device



CS Stage with Active Load

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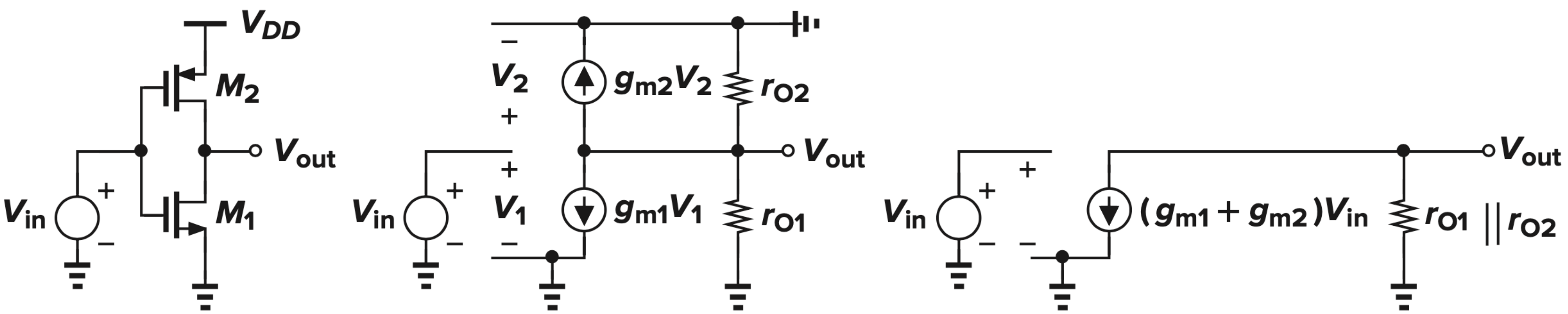

$$\boxed{A_v = -(g_{m1} + g_{m2})(r_{O1}||r_{O2})}$$

CS Stage with Active Load

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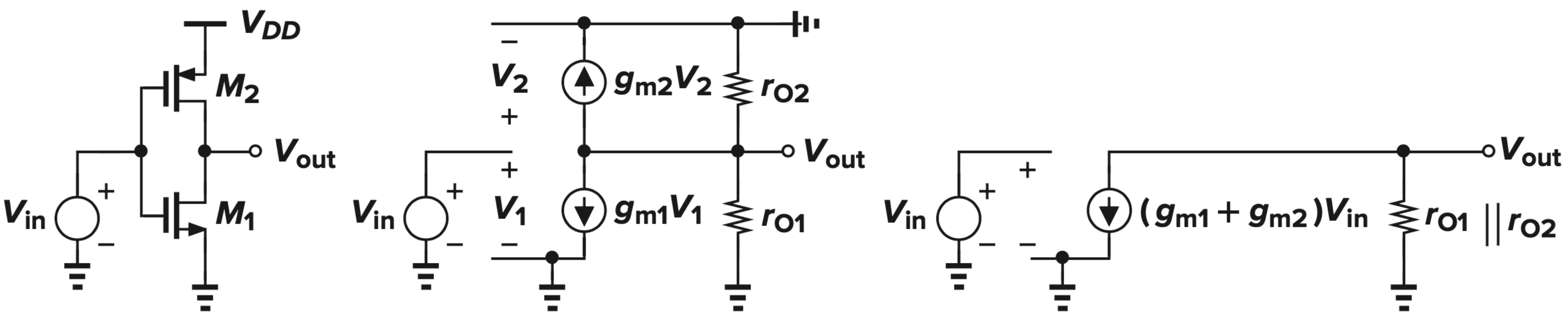


$$\boxed{A_v = -(g_{m1} + g_{m2})(r_{O1}||r_{O2})}$$

- *a "**complementary CS stage**"*

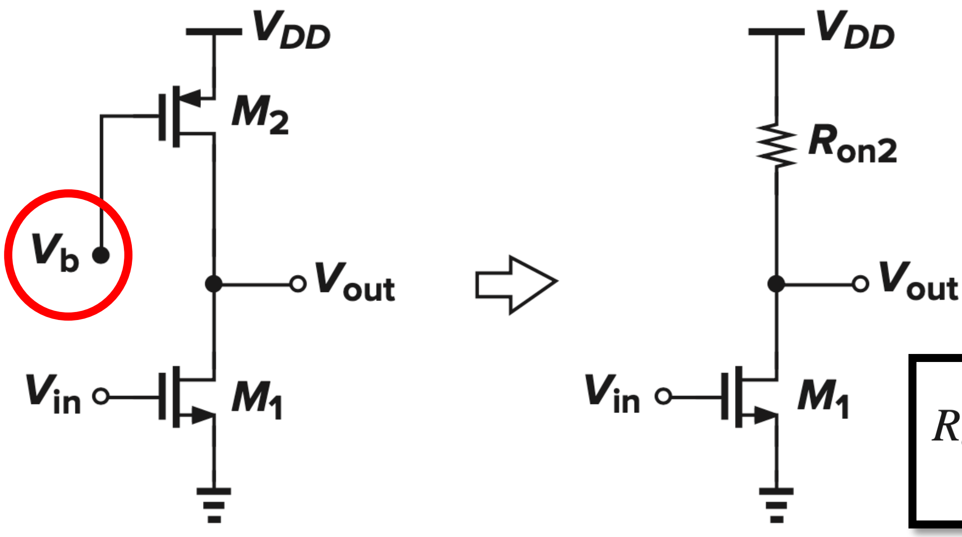
× the bias current of the two transistors is a strong function of PVT

$$V_{GS1} + |V_{GS2}| = V_{DD}$$

× it amplifies supply voltage variations ("supply noise")

CS Stage with Triode Load

- A MOS device operating in the deep triode region behaves as a resistor



$$\boxed{R_{on2} = \frac{1}{\mu_p C_{ox}(W/L)_2(V_{DD} - V_b - |V_{THP}|)}}$$

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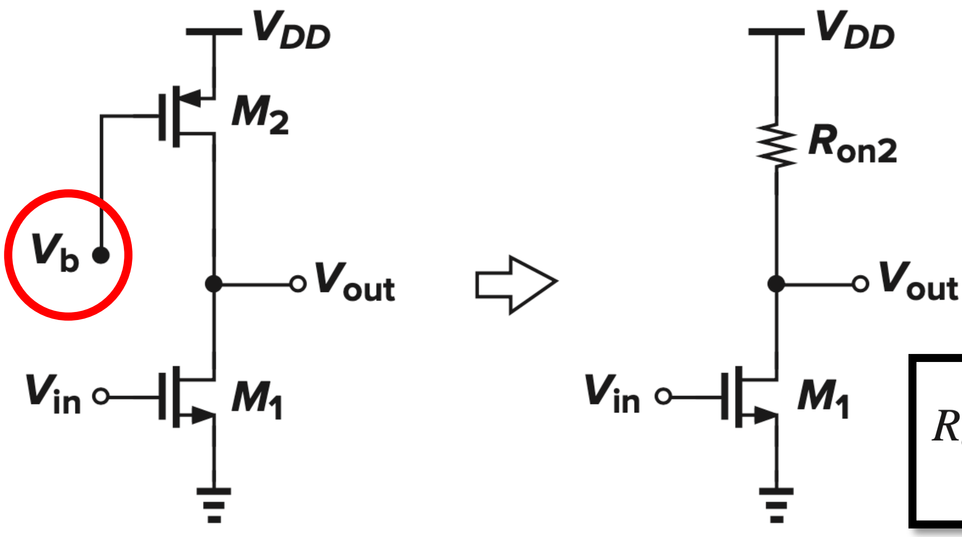


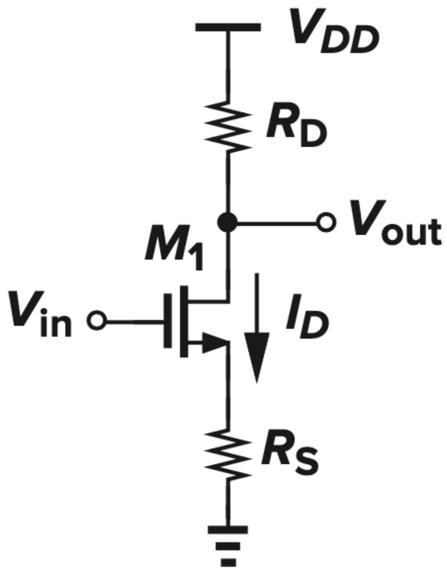
$$R_{on2} = \frac{1}{\mu_p C_{ox}(W/L)_2(V_{DD} - V_b - |V_{THP}|)}$$

× Dependence of R_{on2} on $\mu_p C_{ox}$, V_b, and V_{THP}
  - × $\mu_p C_{ox}$ and V_{THP} vary with process and temperature
  - × generating a precise V_b requires additional complexity

✓ Triode loads consume less voltage headroom than diode-connected

CS Stage with Source Degeneration

- The nonlinear dependence of the drain current upon overdrive: nonlinearity
- Placing a "**degeneration**" **resistor** in series with the **source** terminal makes the input device more **linear** (i.e., make the gain a weaker function of g_m)



$$G_m = \partial I_D / \partial V_{in}$$

$$V_{GS} = V_{in} - I_D R_S$$

$$\rightarrow \quad G_m = \frac{g_m}{1 + g_m R_S}$$

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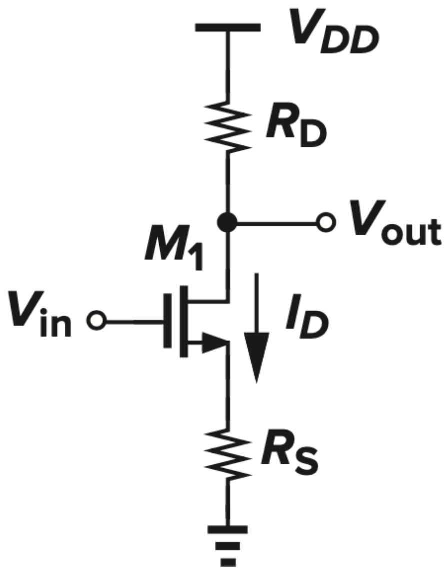

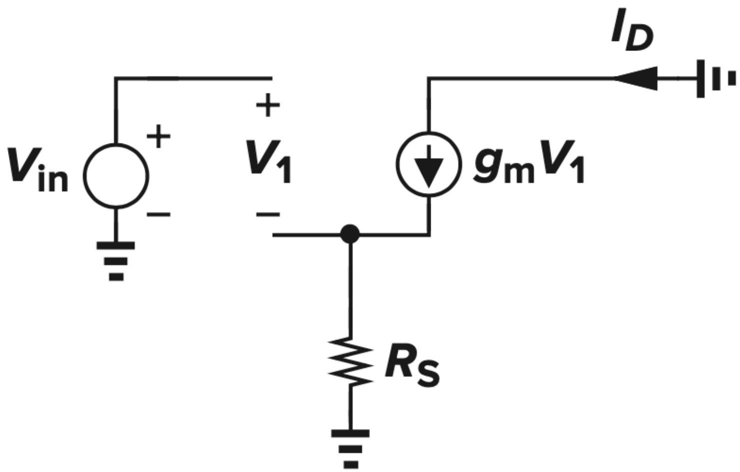

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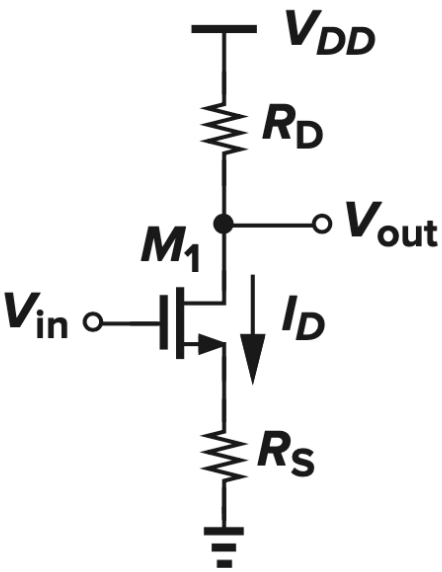
CS Stage with Source Degeneration

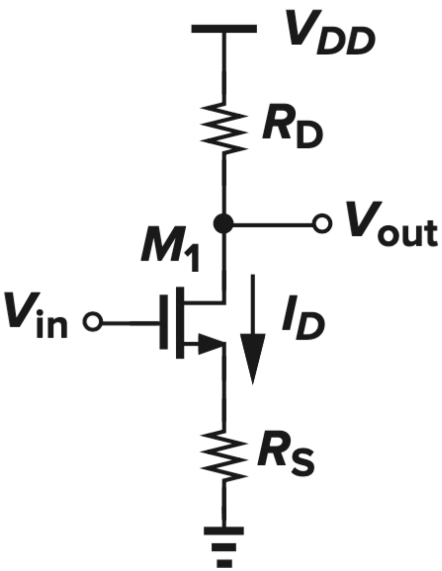


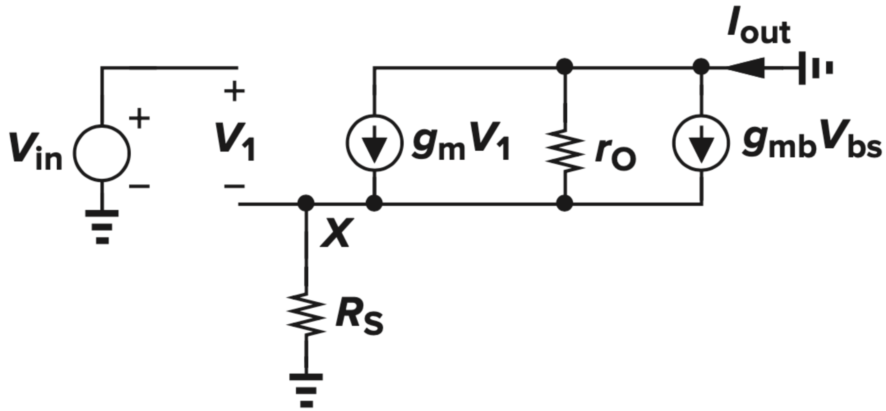
$$G_m = \frac{g_m}{1 + g_m R_S}$$

$$A_v = -G_m R_D = \frac{-g_m R_D}{1 + g_m R_S}$$

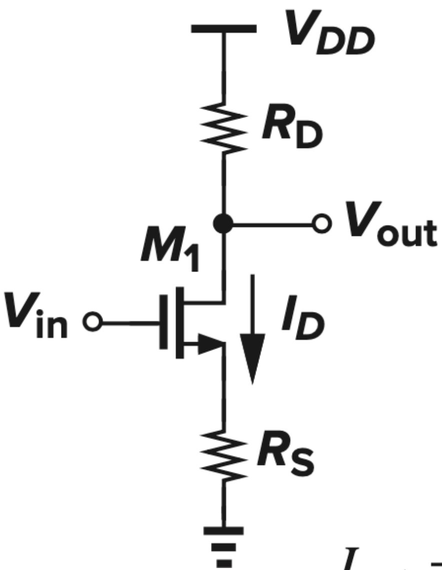
- As R_S increases, G_m becomes a weaker function of g_m and hence the drain current
- For $R_S \gg 1/g_m$, we have $G_m \approx 1/R_S$, i.e., $I_D \approx V_{in}/R_S$: most of the change in V_{in} appears across R_S and the drain current is a "**linearized**" function of the input voltage
- The linearization is obtained at the cost of lower gain

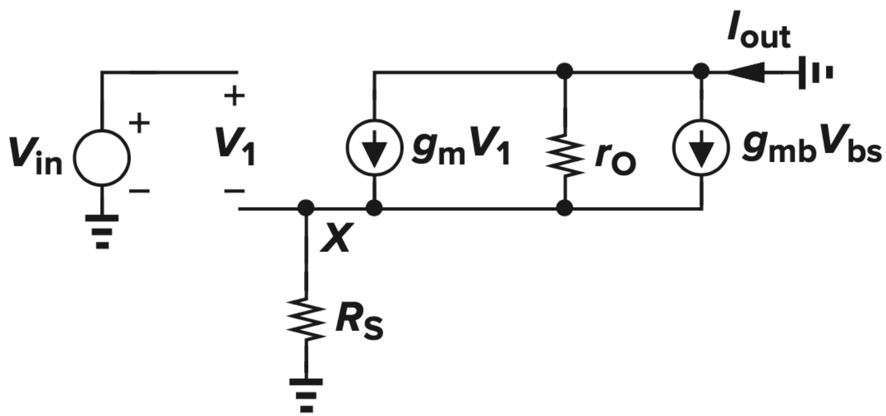
G_m in the presence of g_{mb} and r_o





G_m in the presence of g_{mb} and r_o





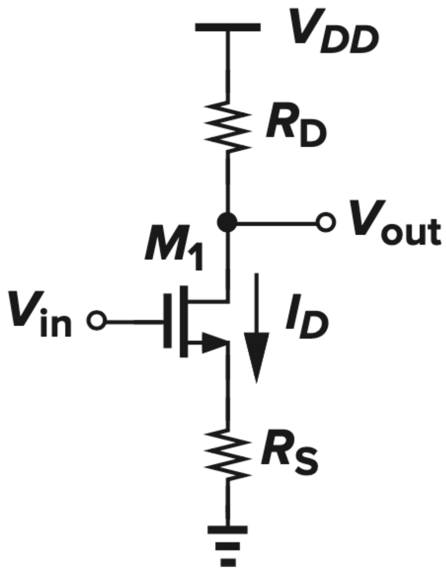
$$I_{out} = g_m V_1 - g_{mb} V_X - \frac{I_{out} R_S}{r_O}$$

$$= g_m (V_{in} - I_{out} R_S) + g_{mb}(-I_{out} R_S) - \frac{I_{out} R_S}{r_O}$$

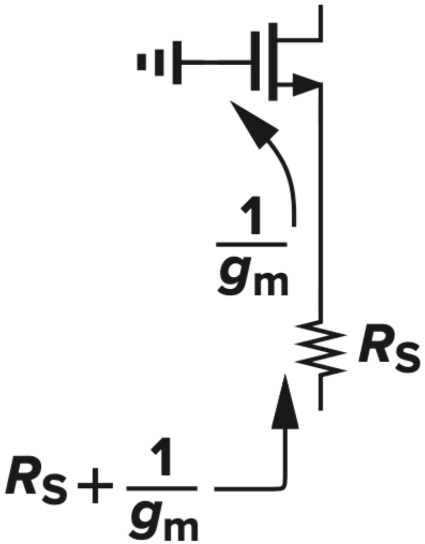
$$G_m = \frac{I_{out}}{V_{in}}$$

$$= \frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O}$$

Gain by Inspection

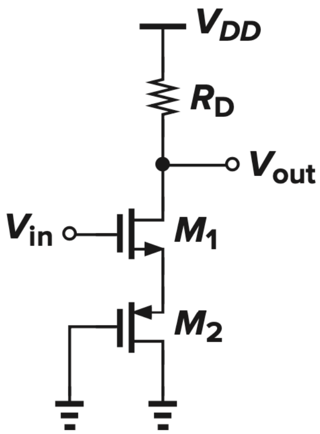


$$A_v = \frac{-g_m R_D}{1 + g_m R_S} \rightarrow \boxed{A_v = -\frac{R_D}{\frac{1}{g_m} + R_S}}$$



Resistance seen in the source path

Example: Calculate the small-signal gain



$$\lambda = \gamma = 0$$

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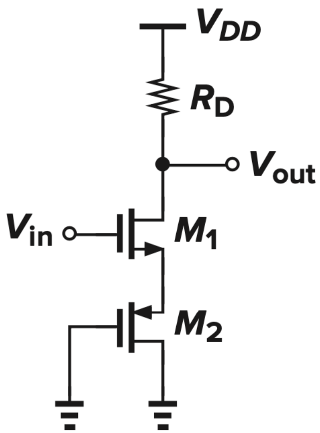

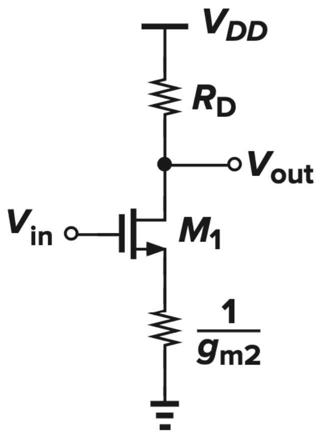

$$A_v = -\frac{R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}$$

$$\lambda = \gamma = 0$$