

Analog IC design (EE-320), Lecture 3

Prof. Mahsa Shoaran

Institute of Electrical and Micro Engineering, School of Engineering, EPFL

Review: MOS I/V Characteristics

$$V_{DS} \leq V_{GS} - V_{TH}$$

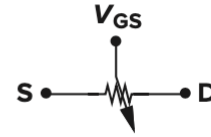
triode

$$\text{or } V_{GD} > V_{TH}$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$V_{DS} \ll 2(V_{GS} - V_{TH}) \longrightarrow R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

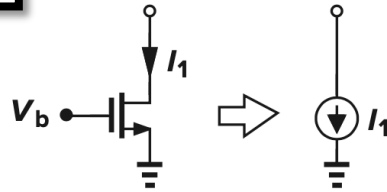
deep triode



$$V_{DS} > V_{GS} - V_{TH}$$

saturation

$$\text{or } V_{GD} < V_{TH}$$



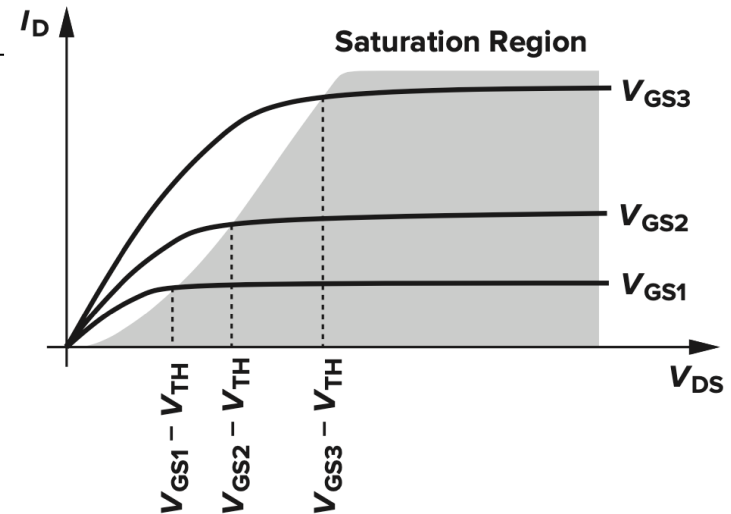
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2$$

PMOS in saturation: $V_{GD} > V_{TH}$

triode: $V_{GD} < V_{TH}$ (V_{TH} is negative)

$$I_D = -\mu_p C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$I_D = -\frac{1}{2} \mu_p C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2$$



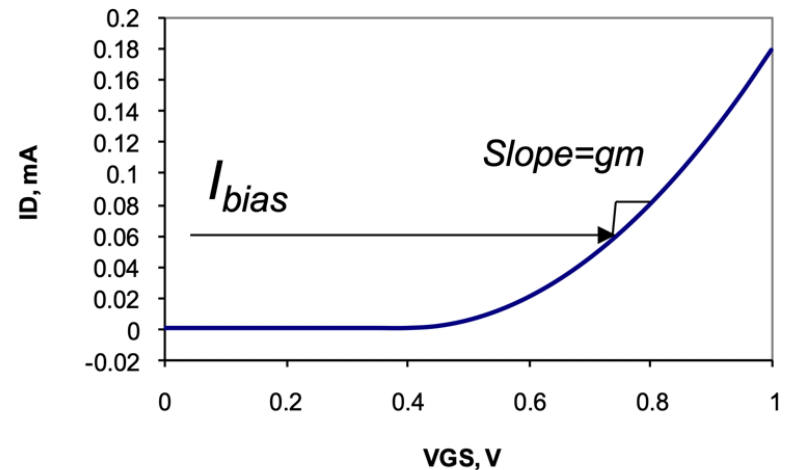
Review: MOS g_m in saturation

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ const.}}$$
$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$
$$= \frac{2I_D}{V_{GS} - V_{TH}}$$



$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$



Review: MOS g_m in saturation, second-order effects

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ const.}}$$

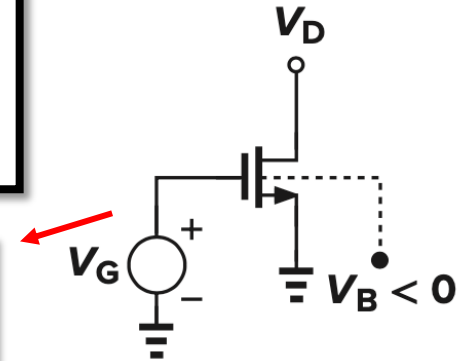
$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

$$= \frac{2I_D}{V_{GS} - V_{TH}}$$

Body
Effect

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|} \right)$$



Review: MOS g_m in saturation, second-order effects

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ const.}}$$

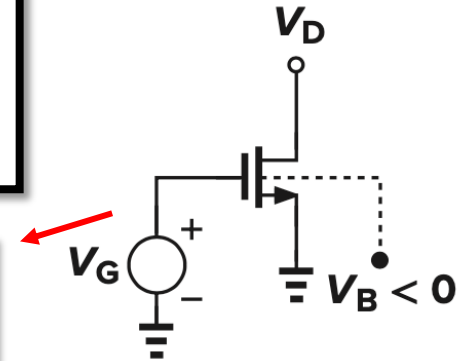
$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

$$= \frac{2I_D}{V_{GS} - V_{TH}}$$

Body
Effect

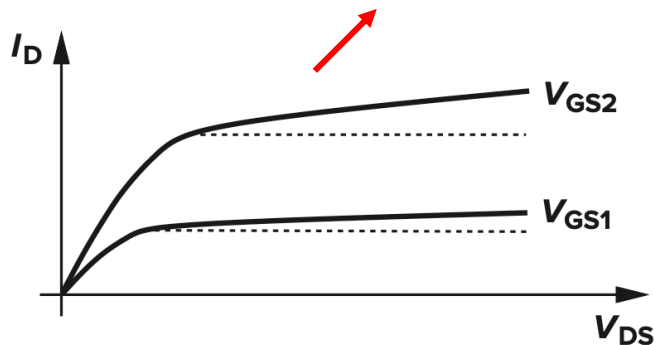
$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|} \right)$$



Channel-Length Modulation

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) (1 + \lambda V_{DS})$$
$$= \sqrt{2\mu_n C_{ox} (W/L) I_D} (1 + \lambda V_{DS})$$



Review: MOS g_m in saturation, second-order effects

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ const.}}$$

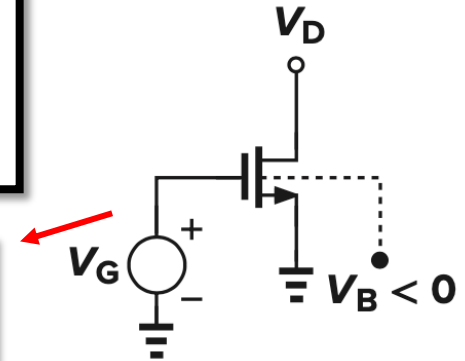
$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

$$= \frac{2I_D}{V_{GS} - V_{TH}}$$

Body
Effect

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|} \right)$$

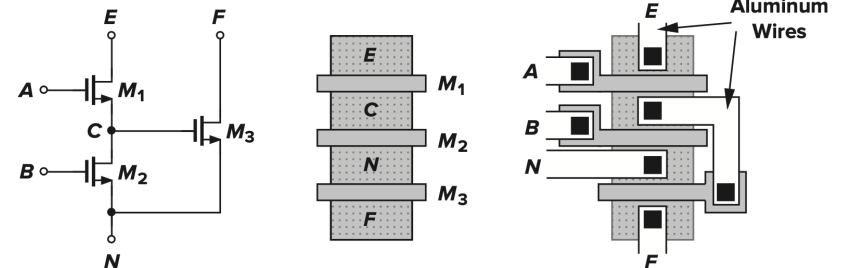
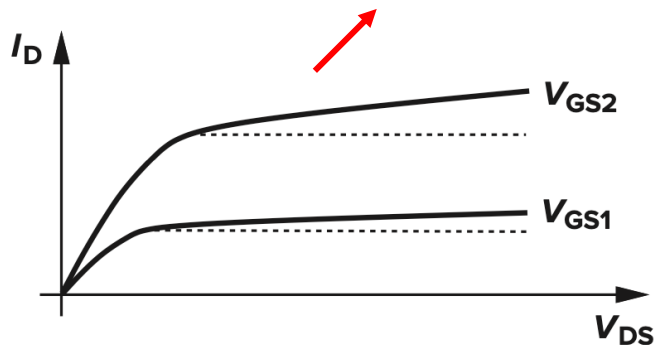


Channel-Length Modulation

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) (1 + \lambda V_{DS})$$

$$= \sqrt{2\mu_n C_{ox} (W/L) I_D} (1 + \lambda V_{DS})$$

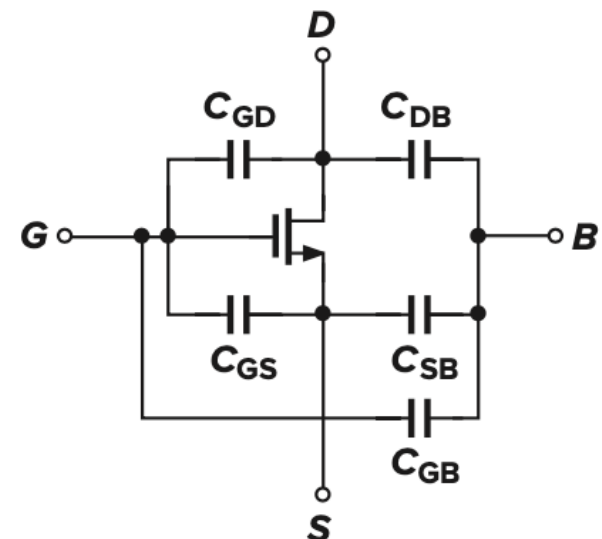


MOS Device Capacitances

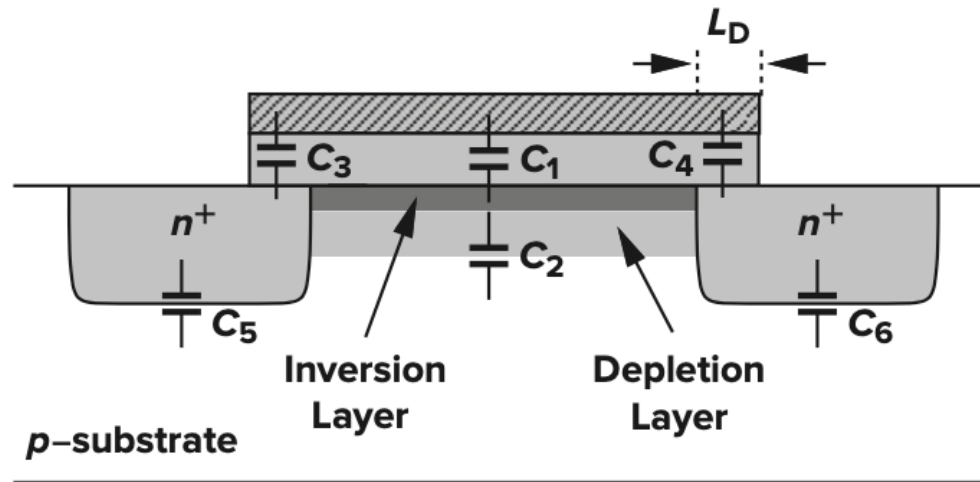
- The basic quadratic I/V relationships along with corrections for body effect and channel-length modulation, provide some understanding of the **low-frequency** behavior of CMOS circuits
- In many analog circuits, however, the **capacitances** associated with the devices must also be taken into account to predict the **high-frequency** behavior as well

MOS Device Capacitances

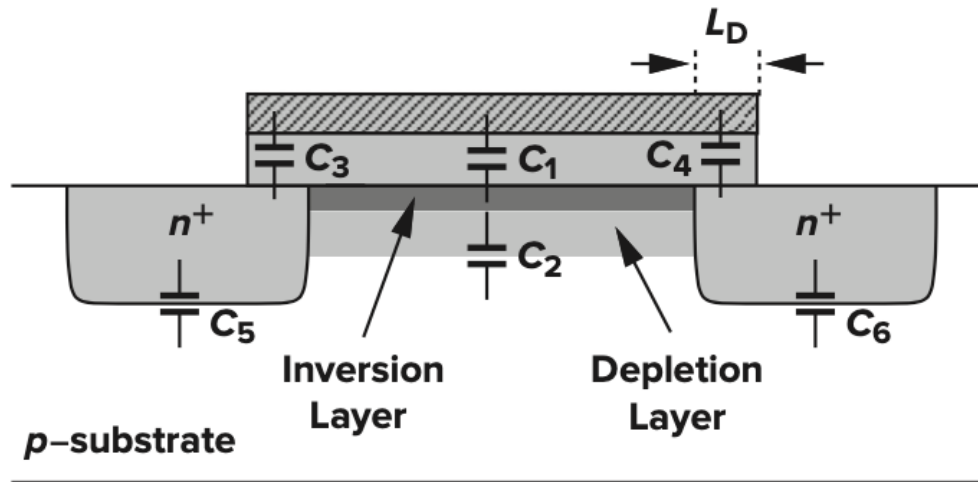
- The basic quadratic I/V relationships along with corrections for body effect and channel-length modulation, provide some understanding of the **low-frequency** behavior of CMOS circuits
- In many analog circuits, however, the **capacitances** associated with the devices must also be taken into account to predict the **high-frequency** behavior as well
- Capacitance between each two of the four terminals of a MOS: the value may depend on the bias conditions of the transistor



MOS Device Capacitances



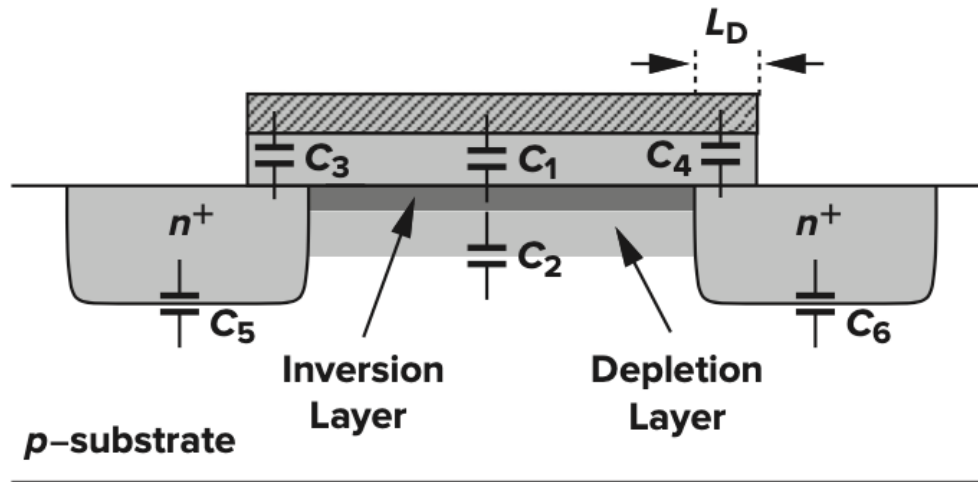
MOS Device Capacitances



(1) the **oxide** capacitance between the **gate** and the **channel**

$$C_1 = WLC_{ox}$$

MOS Device Capacitances



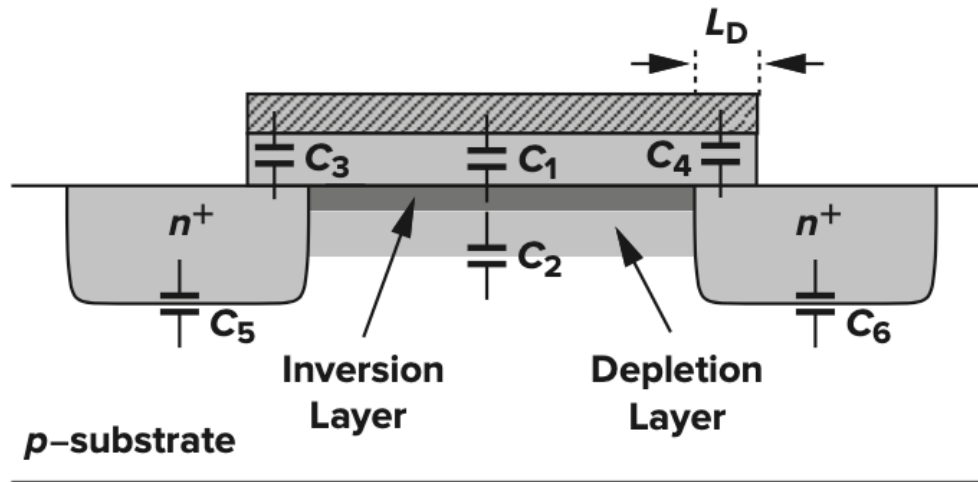
(1) the **oxide** capacitance between the **gate** and the **channel**

$$C_1 = WLC_{ox}$$

(2) the **depletion** capacitance between the channel and the substrate

$$C_2 = WL\sqrt{q\epsilon_{si}N_{sub}/(4\Phi_F)}$$

MOS Device Capacitances



(1) the **oxide** capacitance between the **gate** and the **channel**

$$C_1 = WLC_{ox}$$

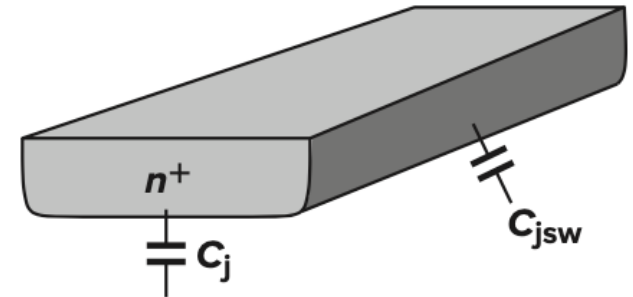
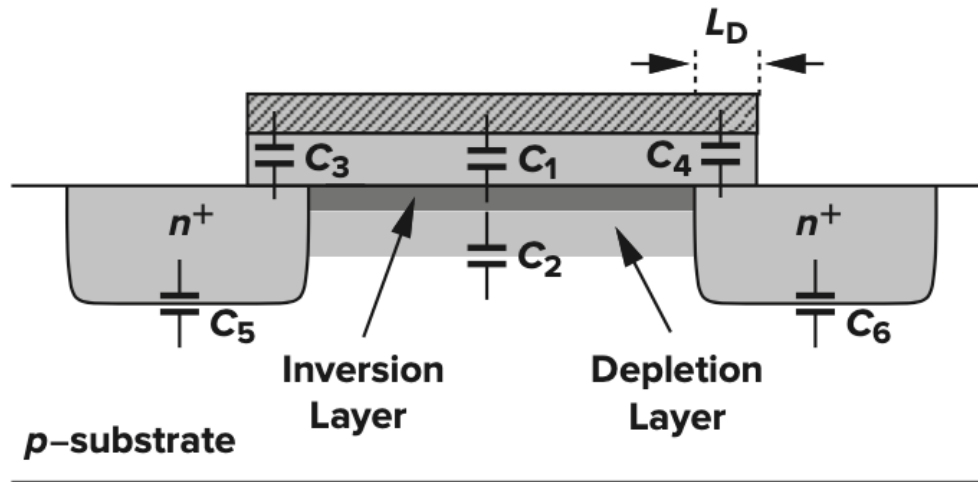
(2) the **depletion** capacitance between the channel and the substrate

$$C_2 = WL\sqrt{q\epsilon_{si}N_{sub}/(4\Phi_F)}$$

(3) the capacitance due to **overlap** of the **gate** poly with **source** and **drain**, C_3 and C_4

The overlap capacitance **per unit width**: C_{ov} in F/m (or fF/ μm). Multiply C_{ov} by W to obtain the **gate-source and gate-drain overlap capacitances**

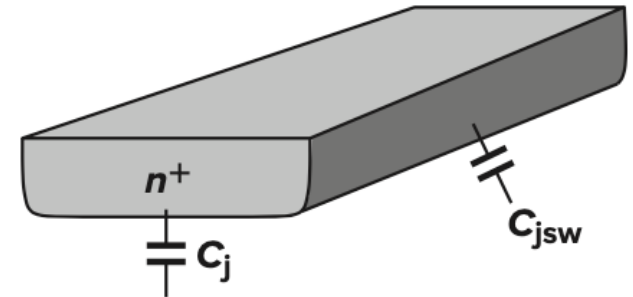
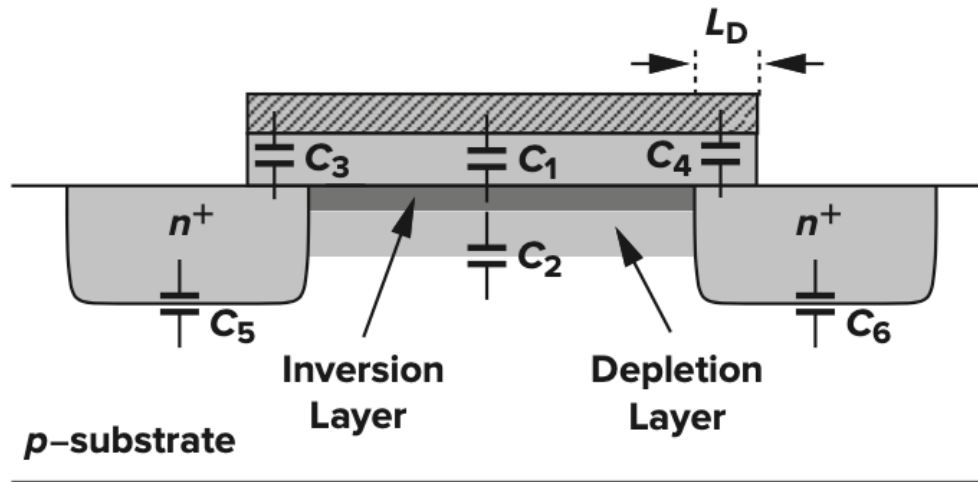
MOS Device Capacitances



(4) The **junction capacitance** between the **S/D** areas and the **substrate**, it has two components: the **bottom-plate capacitance** at the bottom of the junction C_j , and the **sidewall capacitance** due to the perimeter of the junction, C_{jsw}

- C_j and C_{jsw} as capacitance per unit *area* (in F/m²) and unit *length* (in F/m)
- C_j is multiplied by the S/D **area**, and C_{jsw} by the S/D **perimeter**

MOS Device Capacitances

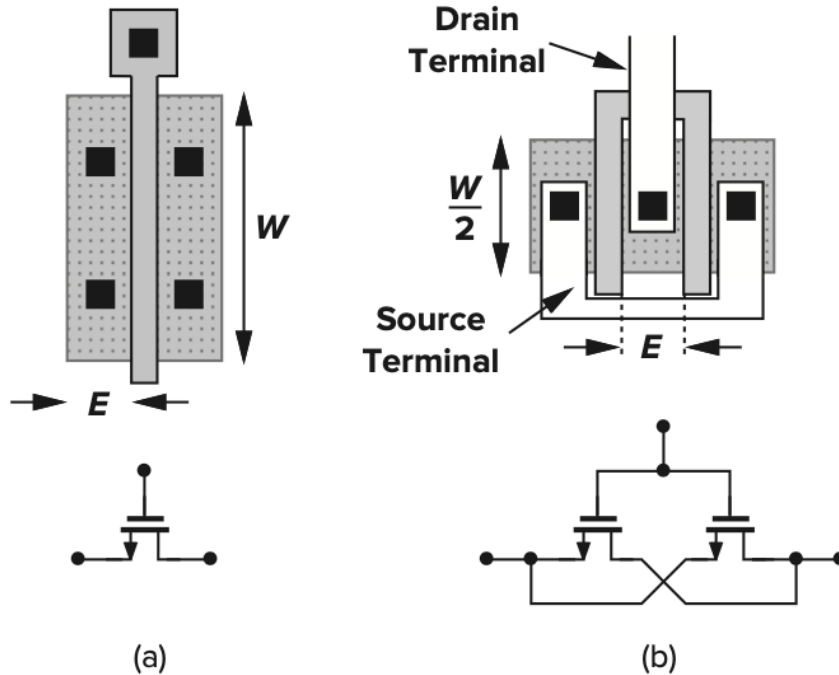


(4) The **junction capacitance** between the **S/D** areas and the **substrate**, it has two components: the **bottom-plate capacitance** at the bottom of the junction C_j , and the **sidewall capacitance** due to the perimeter of the junction, C_{jsw}

- C_j and C_{jsw} as capacitance per unit *area* (in F/m²) and unit *length* (in F/m)
- C_j is multiplied by the S/D **area**, and C_{jsw} by the S/D **perimeter**
- Each **junction capacitance** can be expressed as $C_j = C_{j0}/[1 + V_R/(\Phi_B)]^m$ where V_R is the reverse voltage across the junction, Φ_B is the junction built-in potential, and m is a power typically in the range of 0.3 and 0.4

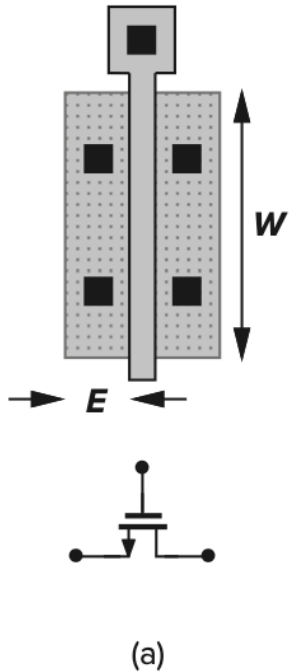
Example: MOS Device Capacitances

- ❖ Calculate the source and drain junction capacitances of the structures below:



Example: MOS Device Capacitances

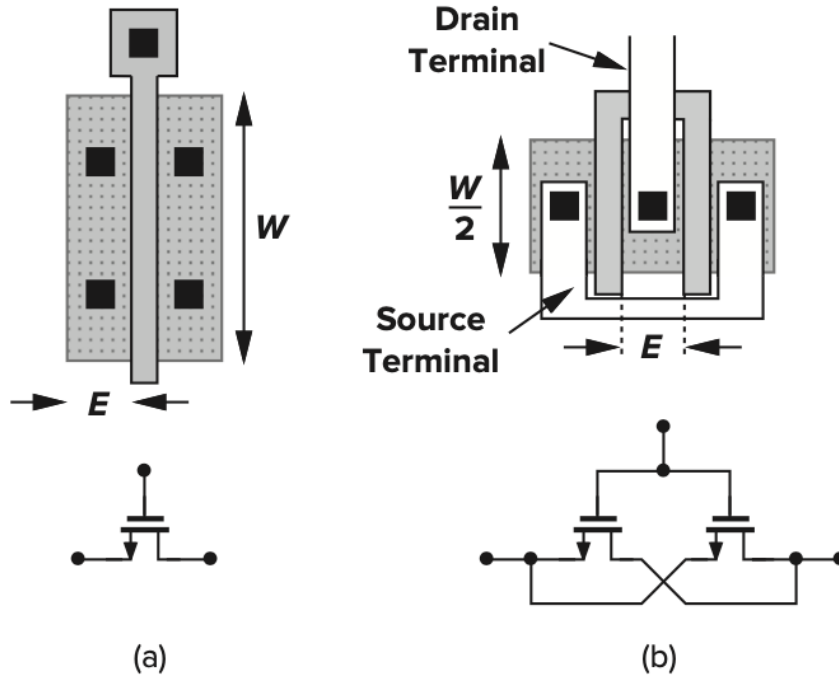
- ❖ Calculate the source and drain junction capacitances of the structures below:



$$C_{DB} = C_{SB} = WEC_j + 2(W + E)C_{jsw}$$

Example: MOS Device Capacitances

- ❖ Calculate the source and drain junction capacitances of the structures below:



$$C_{DB} = \frac{W}{2} E C_j + 2 \left(\frac{W}{2} + E \right) C_{jsw}$$

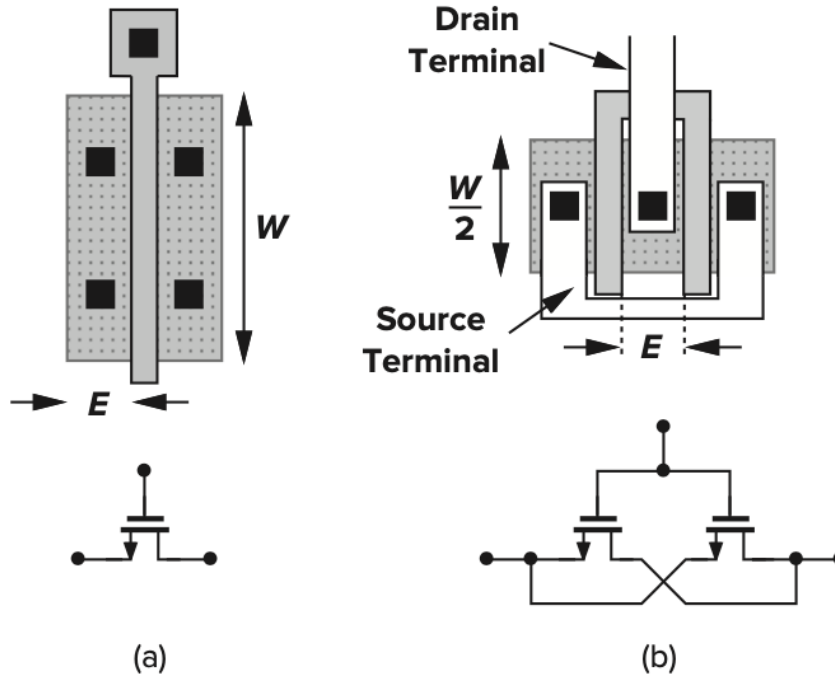
$$C_{SB} = 2 \left[\frac{W}{2} E C_j + 2 \left(\frac{W}{2} + E \right) C_{jsw} \right]$$

$$= W E C_j + 2(W + 2E) C_{jsw}$$

$$C_{DB} = C_{SB} = W E C_j + 2(W + E) C_{jsw}$$

Example: MOS Device Capacitances

- ❖ Calculate the source and drain junction capacitances of the structures below:



$$C_{DB} = \frac{W}{2} E C_j + 2 \left(\frac{W}{2} + E \right) C_{jsw}$$

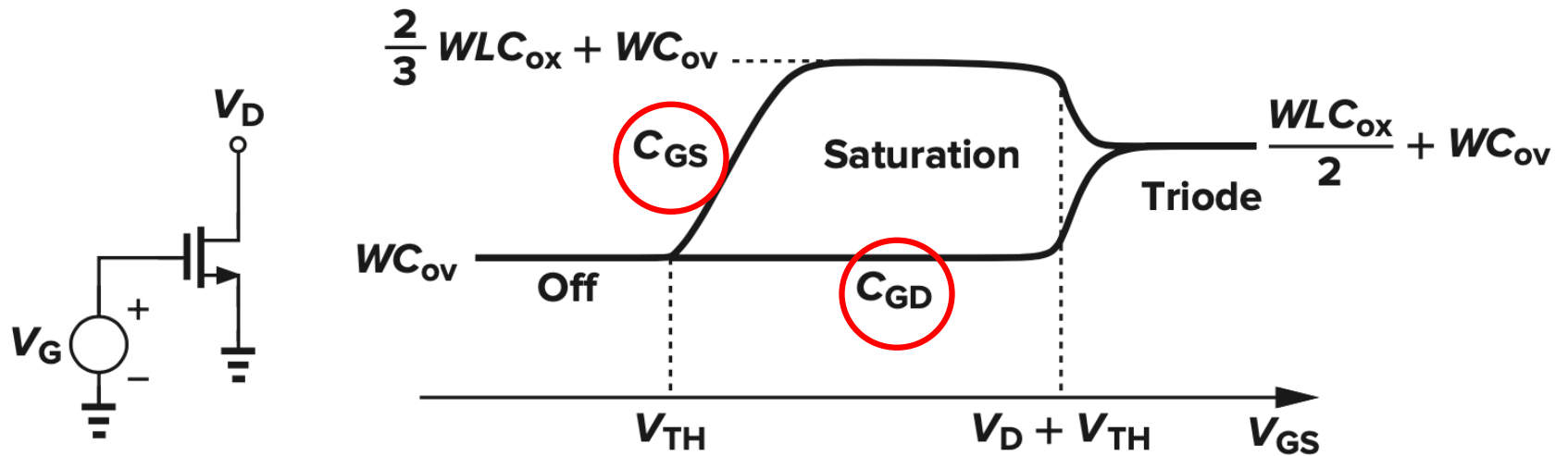
$$C_{SB} = 2 \left[\frac{W}{2} E C_j + 2 \left(\frac{W}{2} + E \right) C_{jsw} \right]$$

$$= W E C_j + 2(W + 2E) C_{jsw}$$

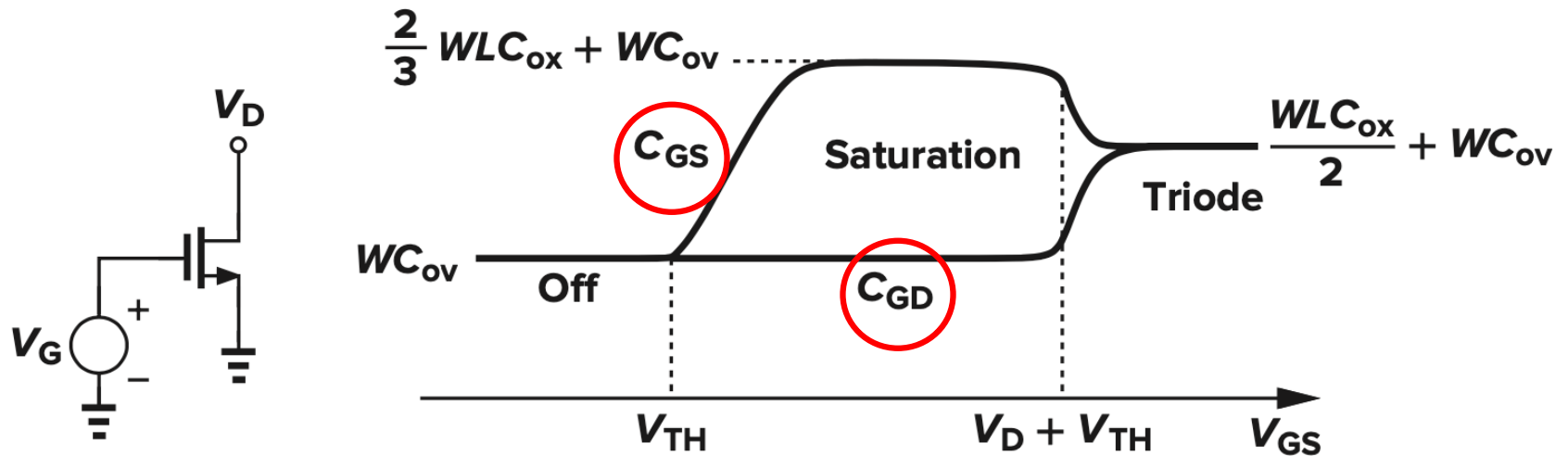
$$C_{DB} = C_{SB} = W E C_j + 2(W + E) C_{jsw}$$

a "folded" structure (b) has substantially **less drain junction capacitance** than (a) while providing the same W/L

MOS Capacitances in different regions



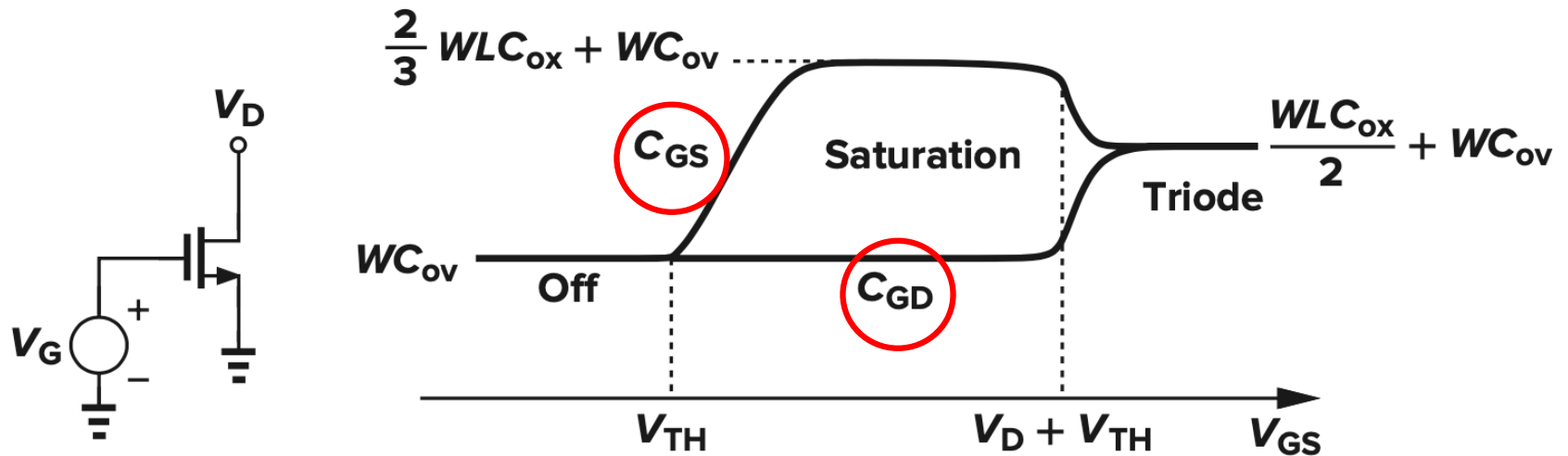
MOS Capacitances in different regions



- If the device is **off**, $C_{GD} = C_{GS} = C_{ov}W$, and the **gate-bulk** capacitance consists of the series combination of the gate-oxide capacitance and the depletion-region capacitance, i.e., $C_{GB} = (W L C_{ox})C_d / (W L C_{ox} + C_d)$, where L is the effective length

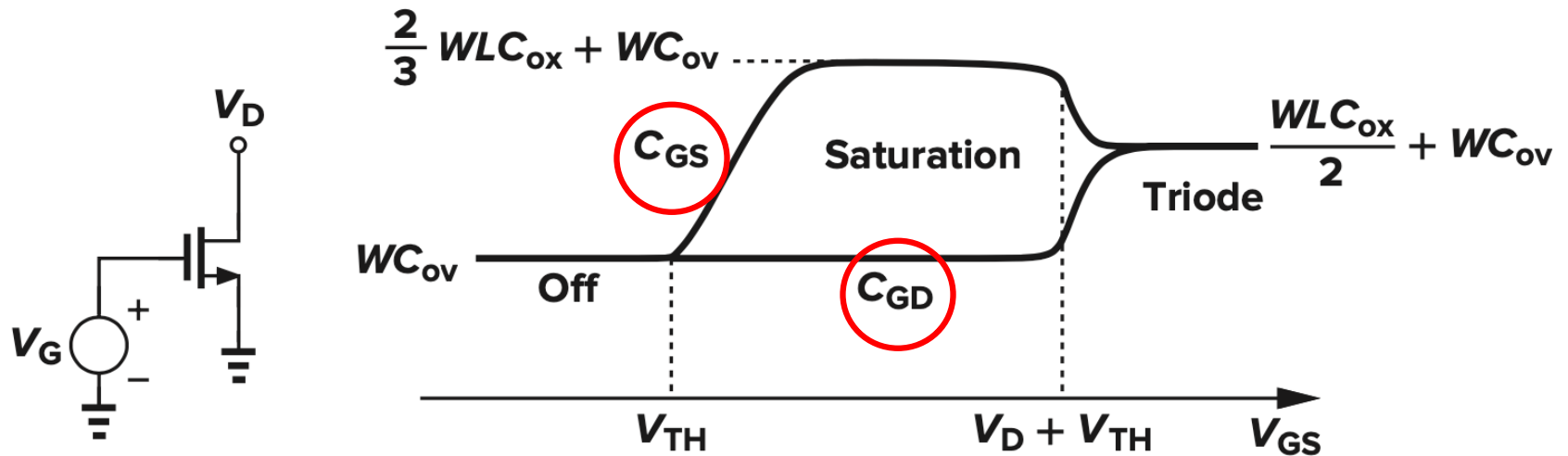
$$C_d = W L \sqrt{q \epsilon_{si} N_{sub} / (4 \Phi_F)}$$

MOS Capacitances in different regions



- If the device is **off**, $C_{GD} = C_{GS} = C_{ov}W$, and the **gate-bulk** capacitance consists of the series combination of the gate-oxide capacitance and the depletion-region capacitance, i.e., $C_{GB} = (WLC_{ox})C_d / (WLC_{ox} + C_d)$, where L is the effective length
- The value of C_{SB} and C_{DB} is a function of the S and D voltages wrt the substrate

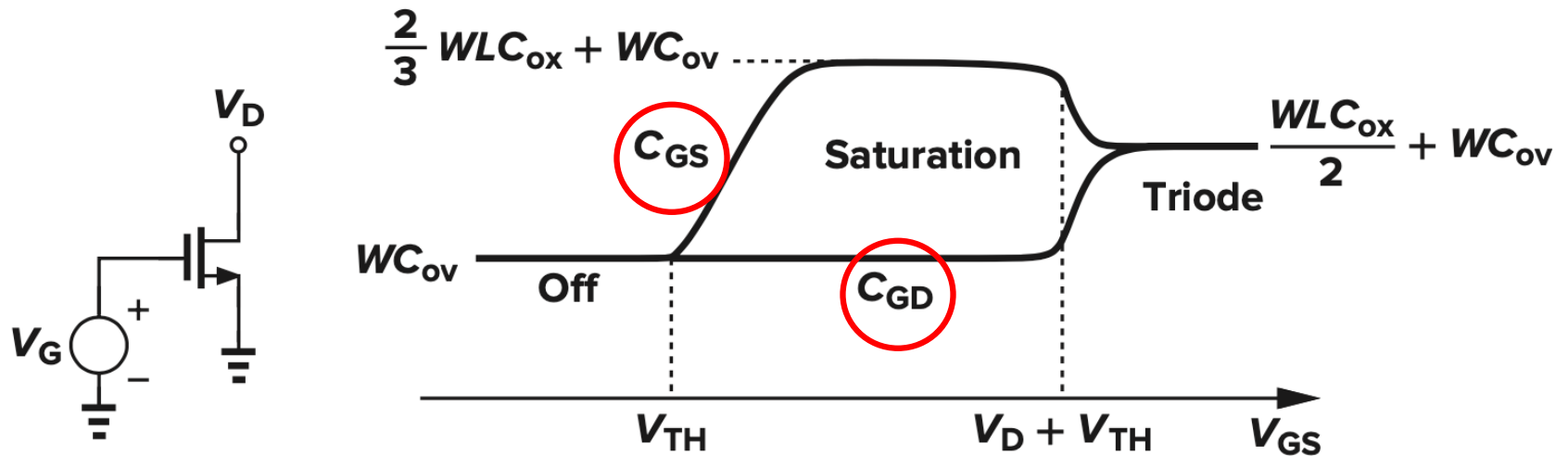
MOS Capacitances in different regions



- If the device is **off**, $C_{GD} = C_{GS} = C_{ov}W$, and the **gate-bulk** capacitance consists of the series combination of the gate-oxide capacitance and the depletion-region capacitance, i.e., $C_{GB} = (WLC_{ox})C_d / (WLC_{ox} + C_d)$, where L is the effective length
- The value of **C_{SB} and C_{DB}** is a function of the S and D voltages wrt the substrate
- In **deep triode**, (S and D at approximately equal voltages), the **gate-channel** capacitance, WLC_{ox} , is divided equally between the G and S and the G and D, since a change of G voltage draws equal charge from S and D:

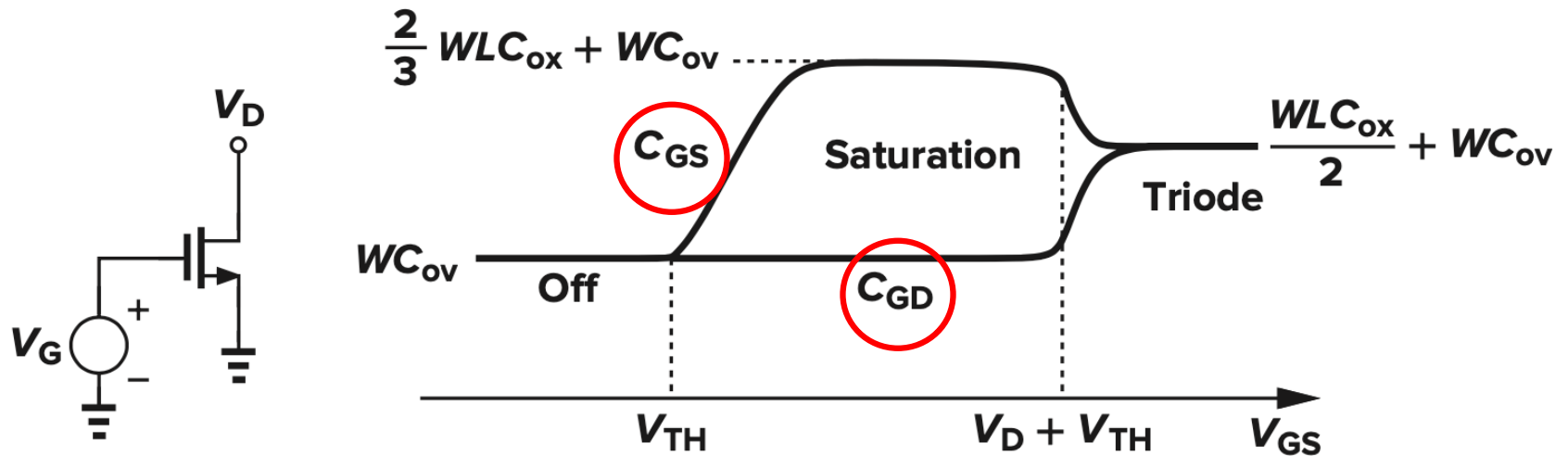
$$C_{GD} = C_{GS} = WLC_{ox}/2 + WC_{ov}$$

MOS Capacitances in different regions



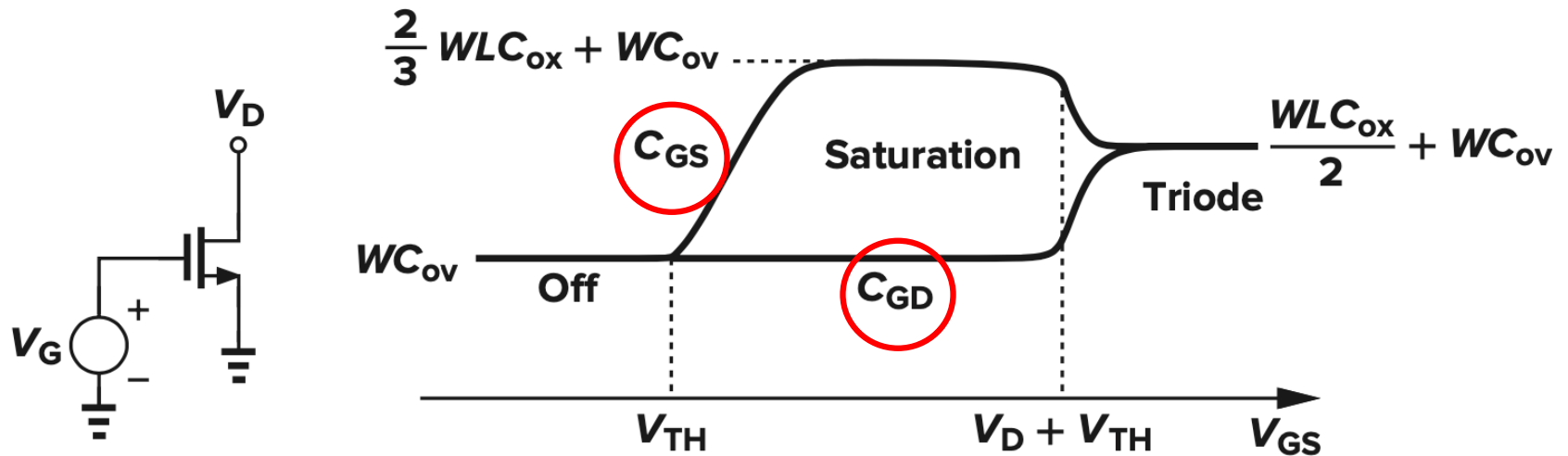
- If in **saturation**, the MOS exhibits a **gate-drain** capacitance roughly equal to WC_{ov}

MOS Capacitances in different regions



- If in **saturation**, the MOS exhibits a **gate-drain** capacitance roughly equal to WC_{ov}
- For C_{GS} , the potential difference between gate and channel varies from V_{GS} at the source to V_{TH} at the pinch-off point, resulting in a nonuniform vertical electric field in the gate oxide from S to D
- It can be proved that the equivalent capacitance, excluding the gate-source overlap capacitance, equals $(2/3)WLC_{ox} \rightarrow \boxed{C_{GS} = 2WL_{eff}C_{ox}/3 + WC_{ov}}$

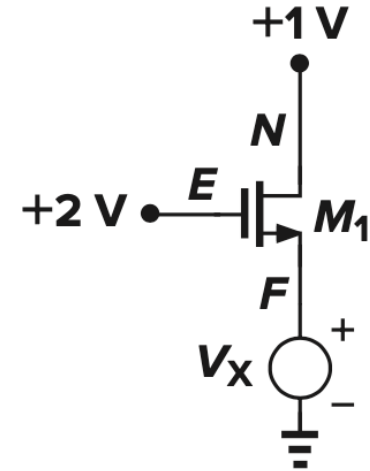
MOS Capacitances in different regions



- If in **saturation**, the MOS exhibits a **gate-drain** capacitance roughly equal to WC_{ov}
- For C_{GS} , the potential difference between gate and channel varies from V_{GS} at the source to V_{TH} at the pinch-off point, resulting in a nonuniform vertical electric field in the gate oxide from S to D
- It can be proved that the equivalent capacitance, excluding the gate-source overlap capacitance, equals $(2/3)WLC_{ox} \rightarrow \boxed{C_{GS} = 2WL_{eff}C_{ox}/3 + WC_{ov}}$
- The **gate-bulk** capacitance is usually neglected in **triode** and **saturation** because the inversion layer acts as a “shield” between G and B

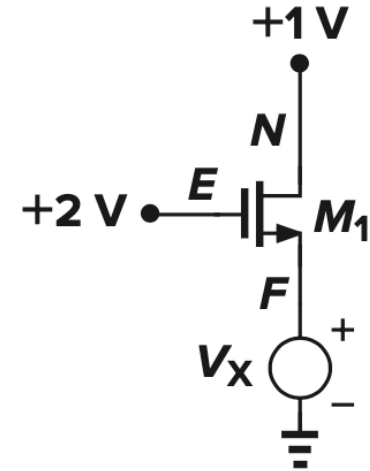
Example: MOS Capacitances in different regions

- ❖ Sketch the capacitances of M_1 as V_X varies from zero to 3 V. Assume that $V_{TH} = 0.3$ V and $\lambda = \gamma = 0$.

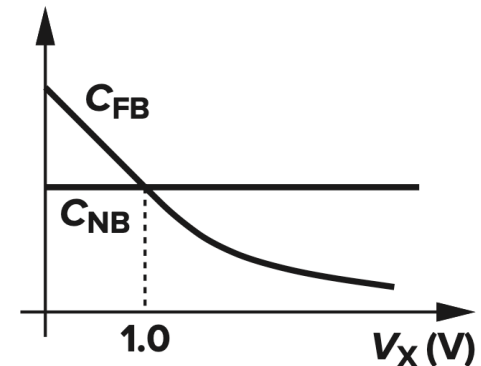


Example: MOS Capacitances in different regions

- ❖ Sketch the capacitances of M_1 as V_X varies from zero to 3 V. Assume that $V_{TH} = 0.3$ V and $\lambda = \gamma = 0$.

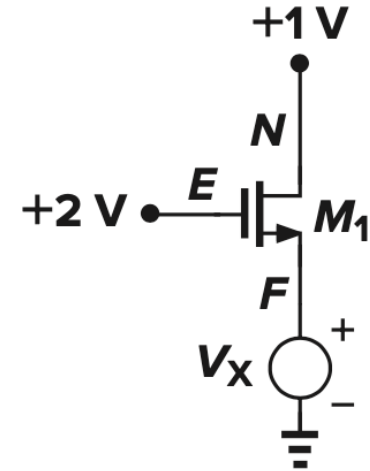


$$C_j = C_{j0}/[1 + V_R/(\Phi_B)]^m$$

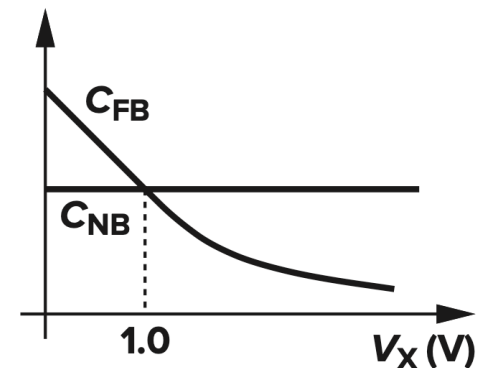
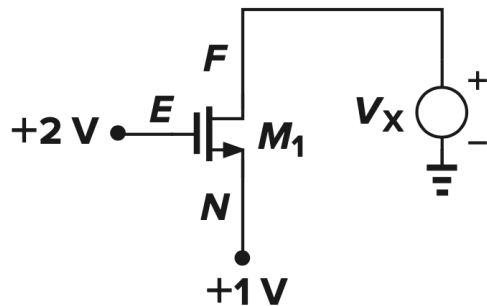


Example: MOS Capacitances in different regions

- ❖ Sketch the capacitances of M_1 as V_X varies from zero to 3 V. Assume that $V_{TH} = 0.3$ V and $\lambda = \gamma = 0$.

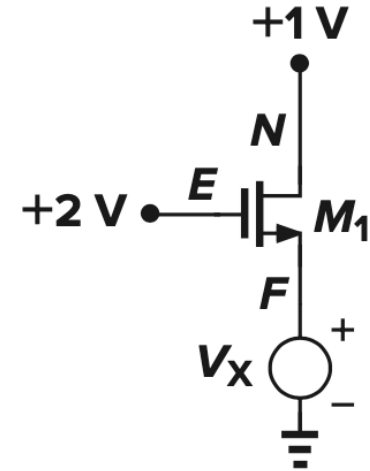


As V_X exceeds 1 V, the role of the S and D is exchanged:

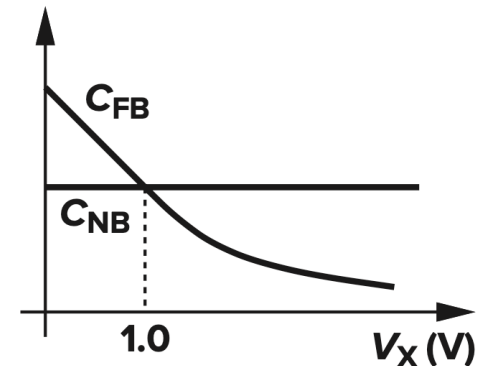
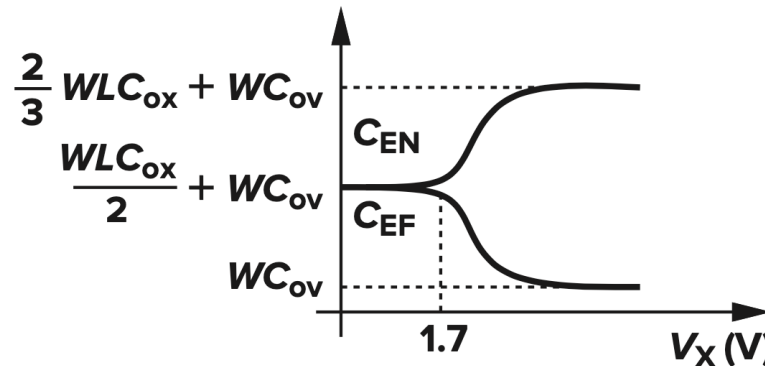
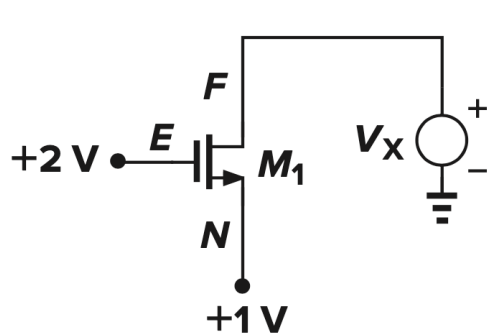


Example: MOS Capacitances in different regions

- ❖ Sketch the capacitances of M_1 as V_X varies from zero to 3 V. Assume that $V_{TH} = 0.3$ V and $\lambda = \gamma = 0$.



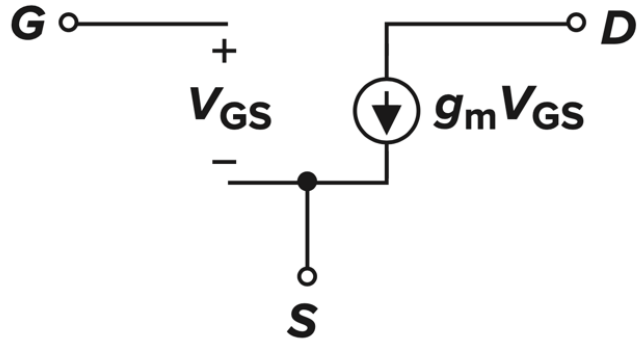
As V_X exceeds 1 V, the role of the S and D is exchanged:



Small-Signal model of MOS

- The “**large-signal**” behavior of NMOS devices, to predict the drain current for arbitrary voltages applied to the G, S, and D, but
 - nonlinear nature of large-signal models: difficult to analyze
 - if the perturbation in bias conditions is **small**, a “**small-signal**” model (an approximation of the large-signal model around the operating point), can simplify the calculations
- Since in many analog circuits, MOSFETs are biased in the saturation region, we derive the **small-signal model** for **saturation**
- For MOS operating as a switch: a **linear resistor** together with capacitances

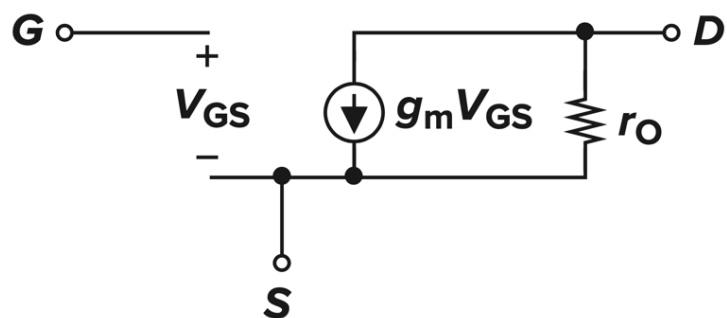
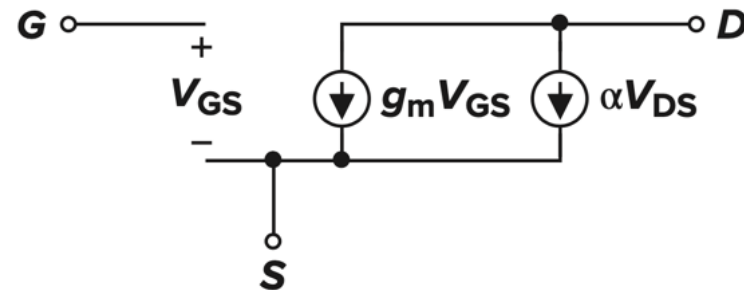
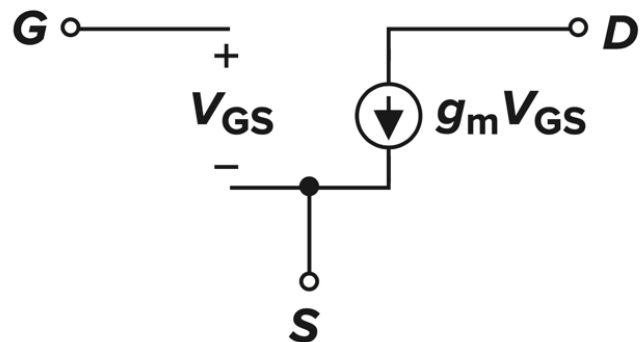
Small-Signal model of MOS



Small-Signal model of MOS

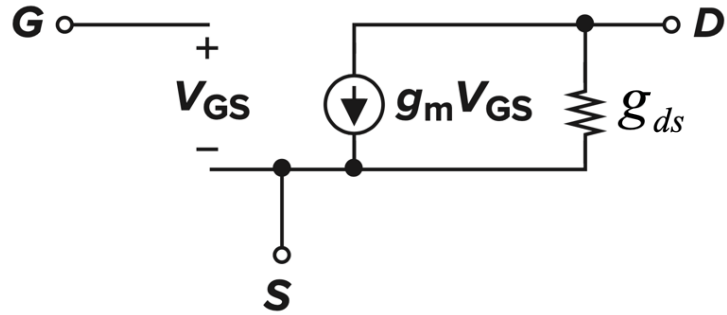


Small-Signal model of MOS

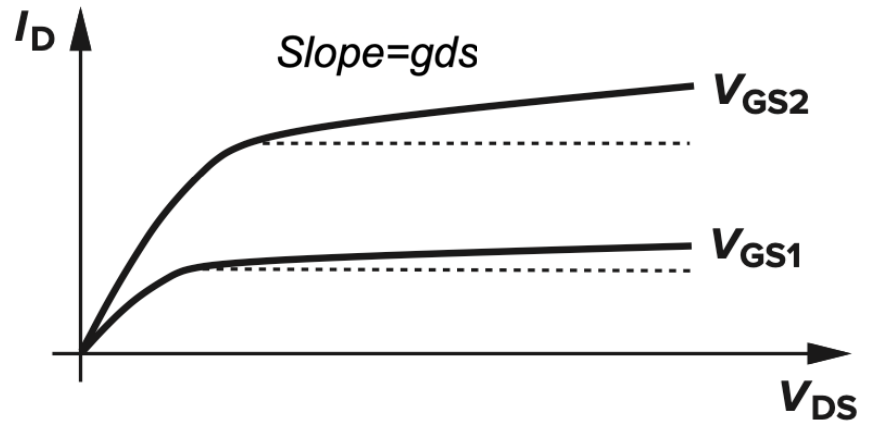


$$\begin{aligned}
 r_O &= \frac{\partial V_{DS}}{\partial I_D} \\
 &= \frac{1}{\partial I_D / \partial V_{DS}} \\
 &= \frac{1}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot \lambda} \\
 &\approx \frac{1 + \lambda V_{DS}}{\lambda I_D} \\
 &\approx \frac{1}{\lambda I_D}
 \end{aligned}$$

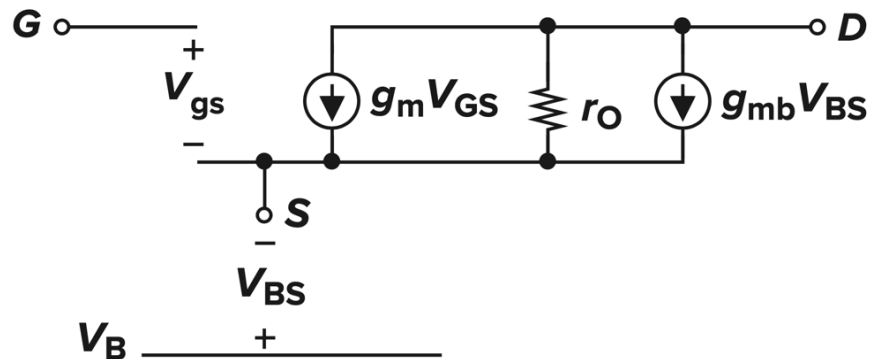
Small-Signal model of MOS (less common)



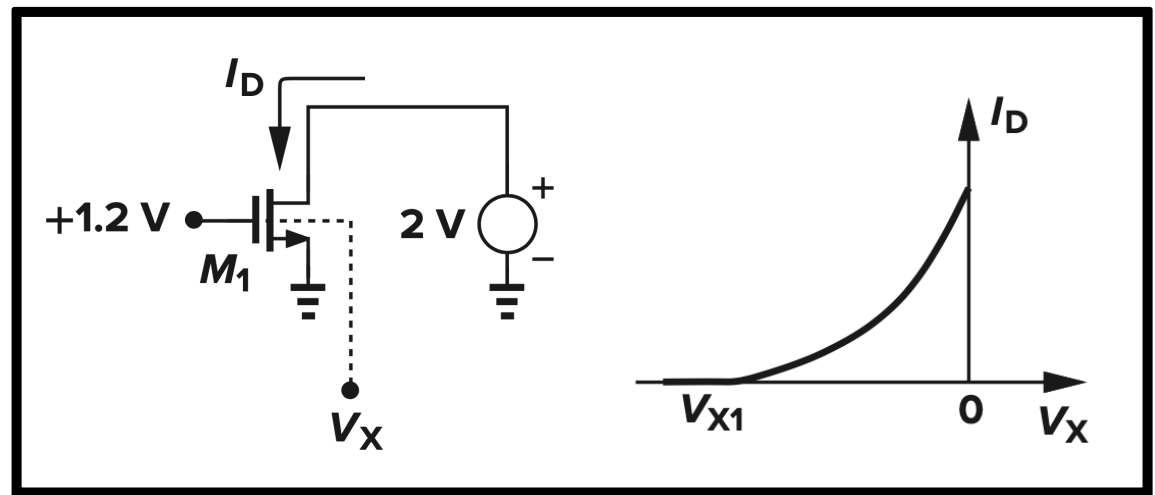
$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}}$$



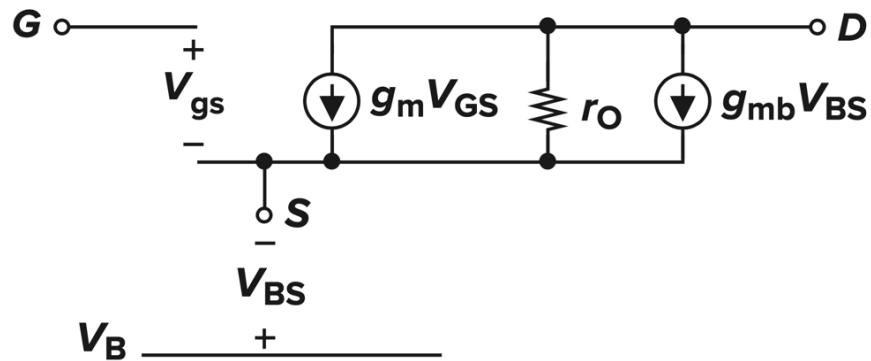
Small-Signal model of MOS with body effect



$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|} \right)$$

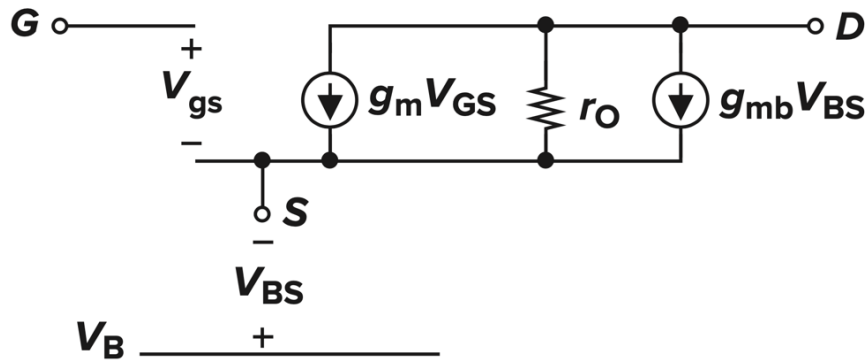


Small-Signal model of MOS with body effect



$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}$$
$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \left(-\frac{\partial V_{TH}}{\partial V_{BS}} \right)$$

Small-Signal model of MOS with body effect



$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}$$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \left(-\frac{\partial V_{TH}}{\partial V_{BS}} \right)$$

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|} \right)$$

$$\frac{\partial V_{TH}}{\partial V_{BS}} = -\frac{\partial V_{TH}}{\partial V_{SB}}$$

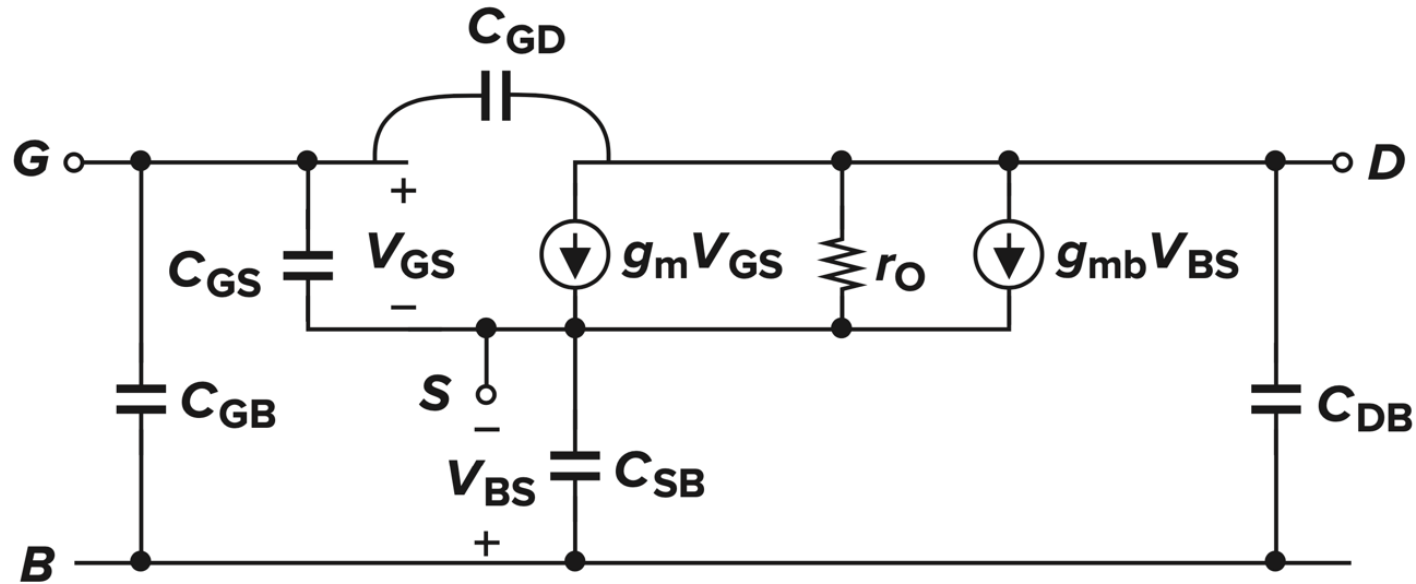
$$= -\frac{\gamma}{2} (2\Phi_F + V_{SB})^{-1/2}$$

- $g_m V_{GS}$ and $g_{mb} V_{BS}$ have the same **polarity**

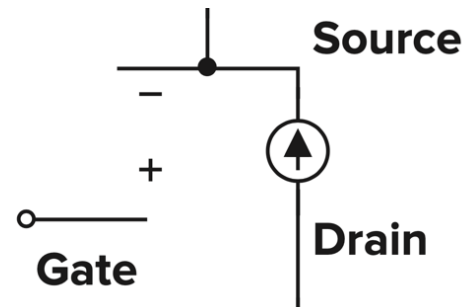
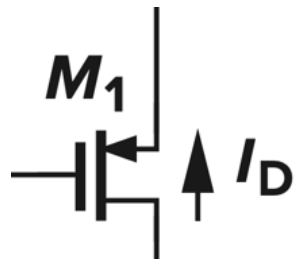
$$g_{mb} = g_m \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

$$= \eta g_m$$

Complete MOS small-signal model



PMOS small-signal model



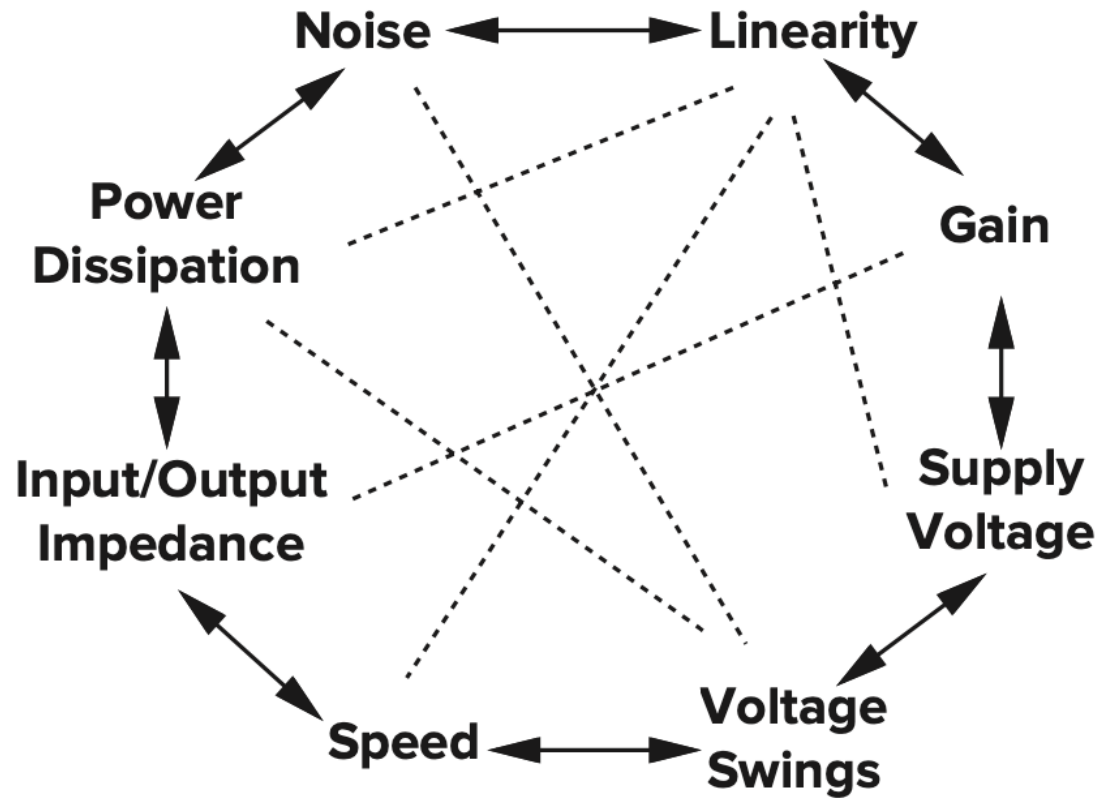
An essential function in most analog circuits

- We amplify an analog or digital signal because
 - it may be too small to drive a load
 - overcome the noise of a subsequent stage
 - provide logical levels to a digital circuit
 - ...

An essential function in most analog circuits

- We amplify an analog or digital signal because
 - it may be too small to drive a load
 - overcome the noise of a subsequent stage
 - provide logical levels to a digital circuit
 - ...
- Four types of amplifiers:
 - **common-source**
 - **common-gate**
 - **source follower**
 - **cascodes**

Analog design tradeoffs



Amplifier topologies

Common-Source Stage

Source Follower

Common-Gate Stage

Cascode

With Resistive Load

With Diode-Connected Load

With Current-Source Load

With Active Load

With Source Degeneration

With Resistive Bias

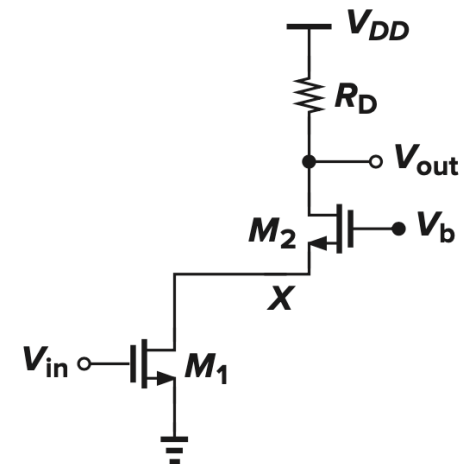
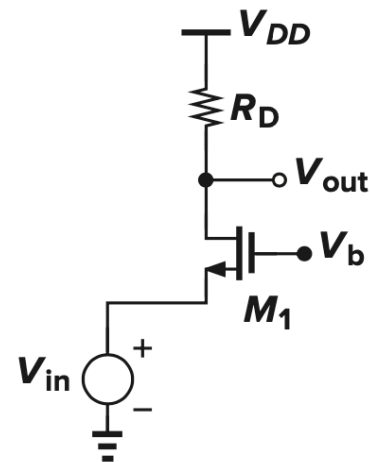
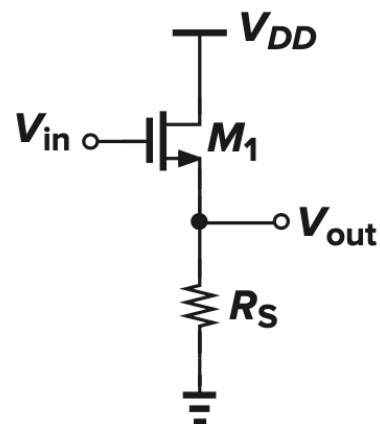
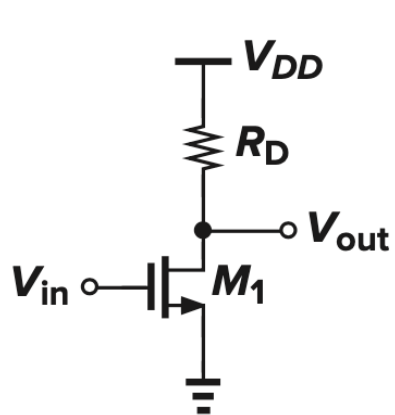
With Current-Source Bias

With Resistive Load

With Current-Source Load

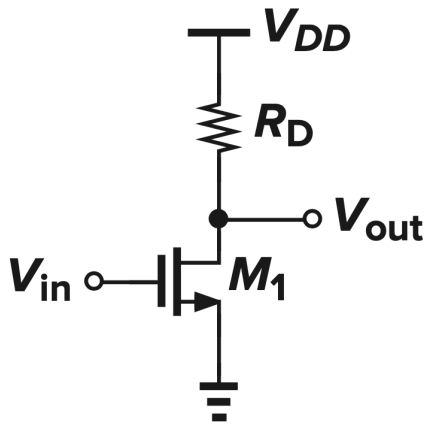
Telescopic

Folded



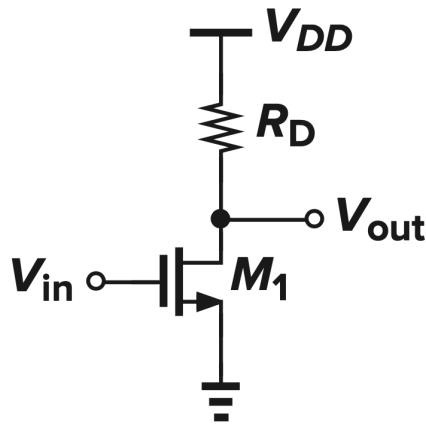
Common-Source Stage - with Resistive Load

- The **common-source** topology: receives the input at the **gate** and produces the output at the **drain**



Common-Source Stage - with **Resistive** Load

- The **common-source** topology: receives the input at the **gate** and produces the output at the **drain**

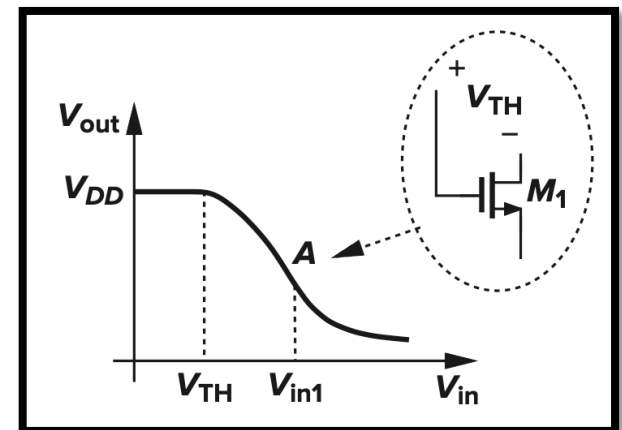


$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \quad (\text{saturation})$$

$$V_{in1} - V_{TH} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2$$

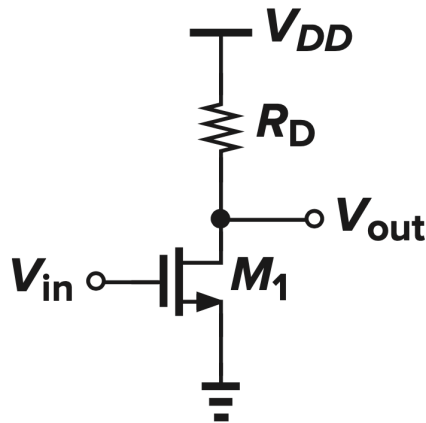
↓ (triode)

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{in} - V_{TH})V_{out} - V_{out}^2]$$



Common-Source Stage - with Resistive Load

- The **common-source** topology: receives the input at the **gate** and produces the output at the **drain**



$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \quad (\text{saturation})$$

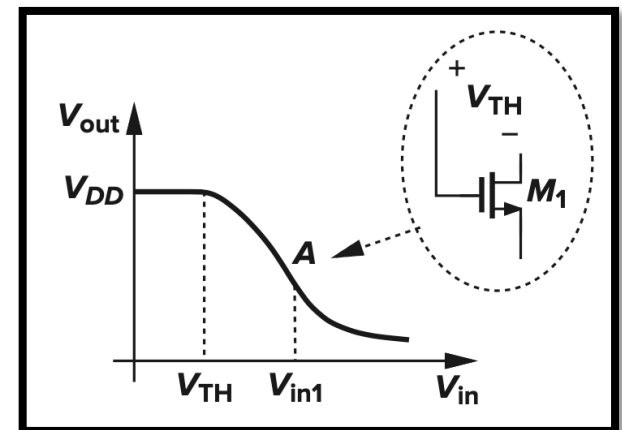
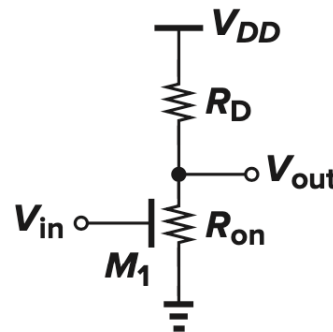
$$V_{in1} - V_{TH} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2$$

↓ (triode)

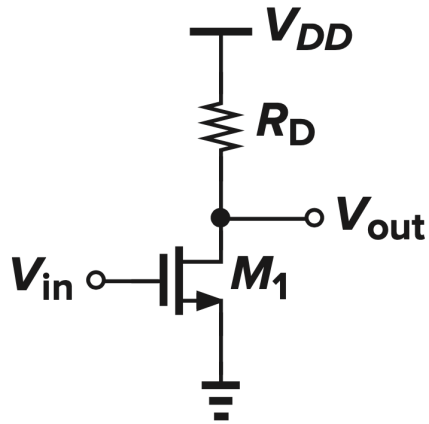
$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{in} - V_{TH})V_{out} - V_{out}^2]$$

$$V_{out} \ll 2(V_{in} - V_{TH})$$

$$\begin{aligned} V_{out} &= V_{DD} \frac{R_{on}}{R_{on} + R_D} \\ &= \frac{V_{DD}}{1 + \mu_n C_{ox} \frac{W}{L} R_D (V_{in} - V_{TH})} \end{aligned}$$



Common-Source Stage - with Resistive Load



$$V_{out} > V_{in} - V_{TH}$$

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}}$$

$$= -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$$

$$= -g_m R_D$$

small-signal model:

