

CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 8 Solution

Date: 31/10/2024

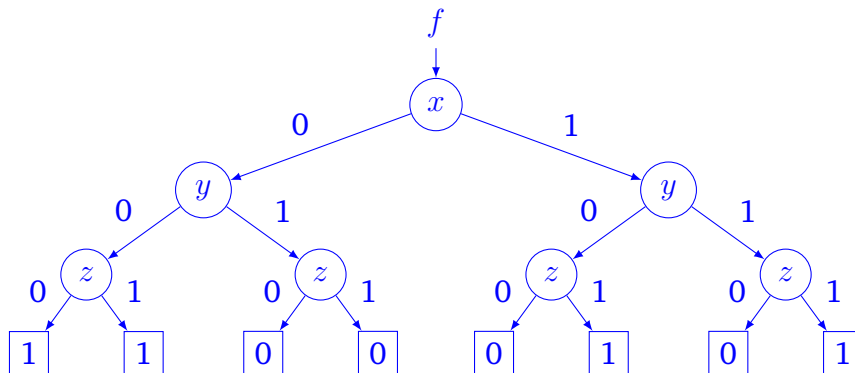
Topics: Binary Decision Diagrams (cf. slide set 10)

Problem 1

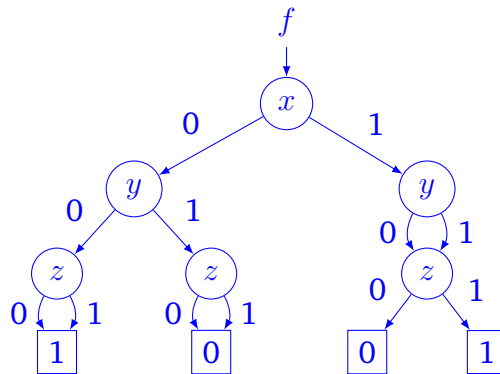
A Boolean function $f(x, y, z)$ is given as the following truth table.

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

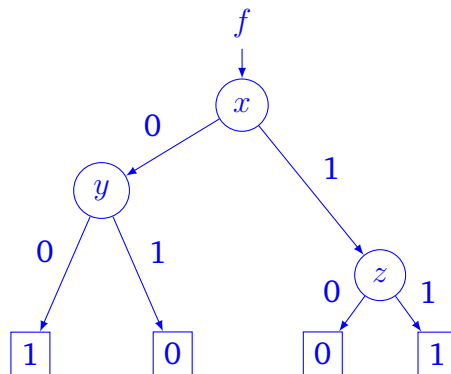
(a) Draw the complete, **non-reduced** BDD with the variable order $x < y < z$.



- (b) Apply the reduction rules.
1. Merge equivalent subtrees:



2. Remove node with identical children:



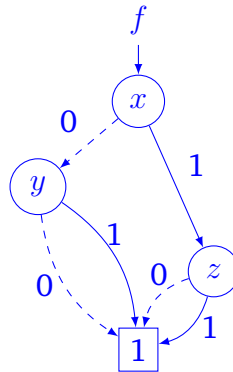
In software implementation, we usually also merge the constant 0 and 1 nodes, i.e., there are only one 0 node and one 1 node. Here, for clearer drawings, we keep them separated.

- (c) Derive a SOP from the BDD.
- Ans:* $f = \bar{x}\bar{y} + xz$. (Hint: enumerate all paths leading to 1.)

(d) Transform the BDD to use complemented edges.

Rules:

1. You have only the constant 1 node, no constant 0.
2. The 1-edge (THEN-child) is never complemented – use the transformation in slide pp.45 to avoid it.



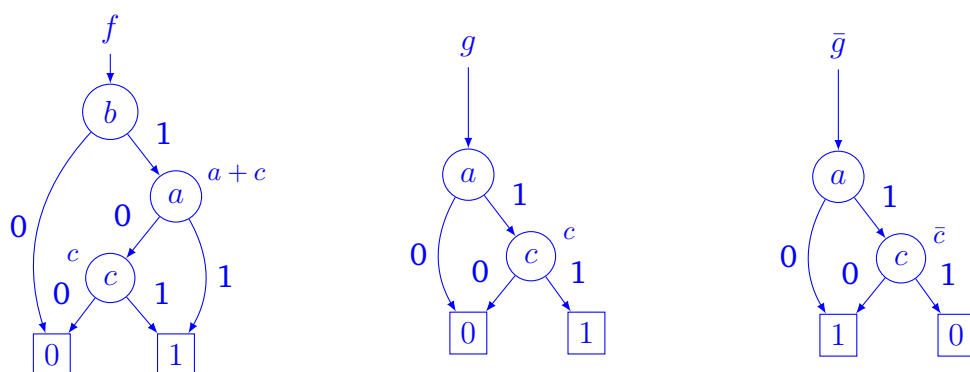
Note: We use dashed lines to represent complemented edges.

Problem 2

Consider the functions $f = ab + bc$ and $g = ac$.

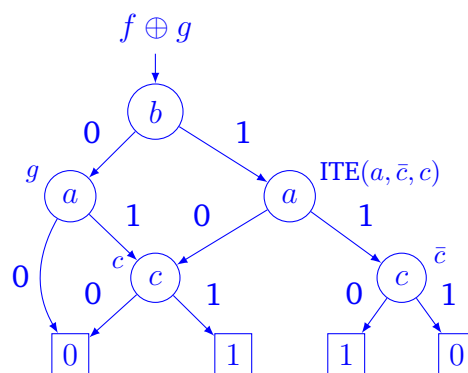
(a) Draw the BDDs (reduced and ordered) for f , g and $\bar{g}$ (select a variable order that minimizes the BDDs).

Observe that $f = b(a + c)$. — b is more “important” than a and c , so we put b on top.



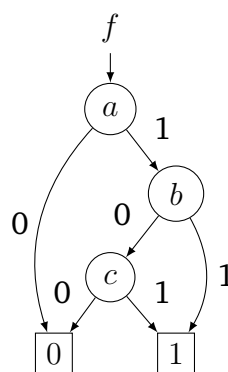
- (b) Use the ITE operator to compute $f \oplus g$ and draw the BDD of $f \oplus g$.
Rewrite $f \oplus g$ using ITE and follow the algorithm on slides pp. 31:

$$\begin{aligned}
 f \oplus g &= \text{ITE}(f, \bar{g}, g) \\
 &= \text{ITE}(b, \text{ITE}(f_b, \bar{g}_b, g_b), \text{ITE}(f_{\bar{b}}, \bar{g}_{\bar{b}}, g_{\bar{b}})) \\
 &= \text{ITE}(b, \text{ITE}(a + c, \bar{g}, g), \text{ITE}(0, \bar{g}, g)) \\
 &= \text{ITE}(b, \text{ITE}(a, \text{ITE}((a + c)_a, \bar{g}_a, g_a), \text{ITE}((a + c)_{\bar{a}}, \bar{g}_{\bar{a}}, g_{\bar{a}})), g) \\
 &= \text{ITE}(b, \text{ITE}(a, \text{ITE}(1, \bar{c}, c), \text{ITE}(c, 1, 0)), g) \\
 &= \text{ITE}(b, \text{ITE}(a, \bar{c}, c), g)
 \end{aligned}$$



Problem 3

Given the following zero-suppressed BDD (ZDD): cf: Slide pp.47-49.



- (a) Write down the item set represented by the ZDD.

Hint: Each path leading to 1 is an item.

Ans: $\{ab, ac\}$, or some people prefer to write as a set of sets: $\{\{a, b\}, \{a, c\}\}$

- (b) Give an SOP of the characteristic function f . (Verify f with the item set!)

Hint: Each minterm of the characteristic function is an item in the item set.

Ans: $f = ab\bar{c} + a\bar{b}c$