CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 3 Solution — Date: 03/10/2024

Topics: Control-flow expressions, finite state machines (cf. slide set 3); data flow optimization (cf. slide set 4)

Problem 1

Given the following three pieces of HDL pseudo-code:

```
while (a)
{
  while (b)
  {
    if (c)
      P1
    else
      P2
  }
  P3
}
```

```
wait (d)
{
  while (!e)
    P4
}
```

```
always
{
  if (f)
    P5
}
```

Write the control-flow expression that executes the three codes *in parallel*. Use parentheses properly for nested expressions to avoid ambiguity.

cf: Slide set 3, pp. 26.

Ans:

$$\left(\mathtt{a} : \left(\left(\mathtt{b} : (\mathtt{c} : \mathtt{P1} + \bar{\mathtt{c}} : \mathtt{P2})\right)^* \cdot \mathtt{P3}\right)\right)^* \parallel \left((\bar{\mathtt{d}} : 0)^* \cdot (\bar{\mathtt{e}} : \mathtt{P4})^*\right) \parallel \left(\mathtt{f} : \mathtt{P5}\right)^\omega$$

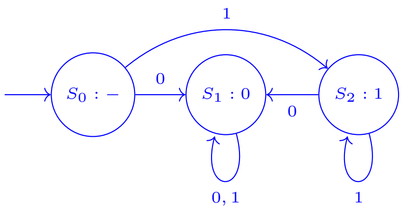
Problem 2

For the following specifications, draw a deterministic finite state machine that implements it. The machines receive a non-empty bit stream $(b_0, b_1, \ldots, b_{n-1})$ as input, where $b_i \in \{0,1\} \forall i$, and output $o \in \{0,1\}$ which may change upon receiving a new input bit. The length n of the stream can be any finite positive integer and is unknown to the machine. The machines need to be correct for all n seen during the process as if the input bits received so far is the entire bit stream. The output before receiving any input, if any (i.e. if it is a Moore machine), can be ignored.

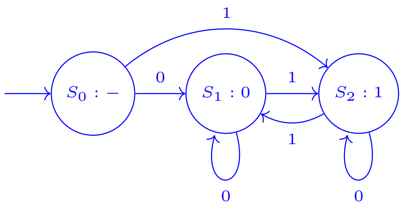
Let $\#0$ denote the number of 0 bits in the stream, and let $\#1$ denote the number of 1 bits in the stream. We say a FSM *accepts* a stream if it outputs $o = 1$ when the stream is completely fed into the machine.

Recall that a finite state machine must have a *finite* number of states, and the number of states cannot be an unknown number. Also note that for a FSM to be *deterministic*, the transition under every possible input at every reachable state must be specified and unique. Some machines may be impossible to construct; explain why if you think so.

(a) Construct a FSM which implements the AND operation, i.e., $o = b_0 \wedge b_1 \wedge \cdots \wedge b_n$.
Ans:

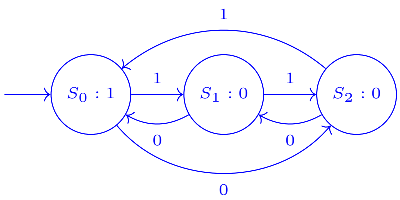


(b) Construct a FSM which implements the XOR operation, i.e., $o = b_0 \oplus b_1 \oplus \cdots \oplus b_n$.
Ans:



(c) Construct a FSM which accepts a stream where $\#0 = \#1$.
Ans: It is impossible to construct such a machine with finite states because we need to keep track of the difference between $\#0$ and $\#1$ of the current (sub-)stream at any time, which we do not have a known upper bound.

(d) Construct a FSM which accepts a stream where $(\#0 \bmod 3) = (\#1 \bmod 3)$.
Ans:



Problem 3

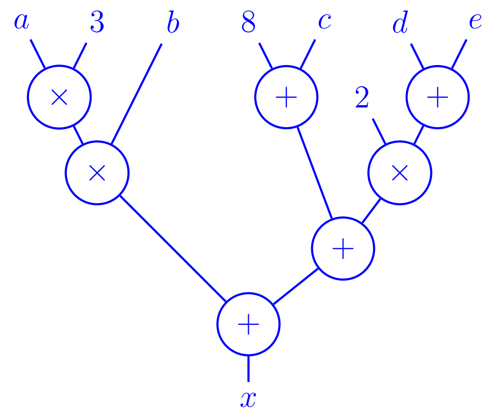
Given the following equations:

$$
\begin{aligned}
x_1 &= (a \times 3);\\
x_2 &= b;\\
x_3 &= (5 + c);\\
x_4 &= (d + e);\\
x &= (x_1 \times x_2) + 3 + x_3 + (x_4 \times 2);
\end{aligned}
$$

where inputs are a, b, c, d, e, while the output is x.

(a) Apply variable and constant propagation, and draw the data-flow graph. Assume all additions and multiplications have 2 inputs.

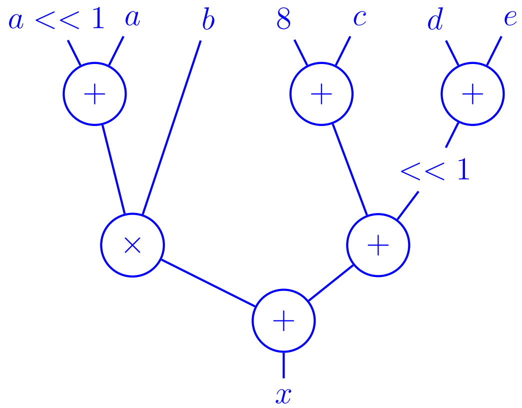
Ans:



(b) Apply *operator strength reduction* on the data-flow graph from point (a). Assume that shifting takes no latency. Draw the resulting data-flow graph.

cf: Slide set 4, pp. 20.

Ans:



Problem 4

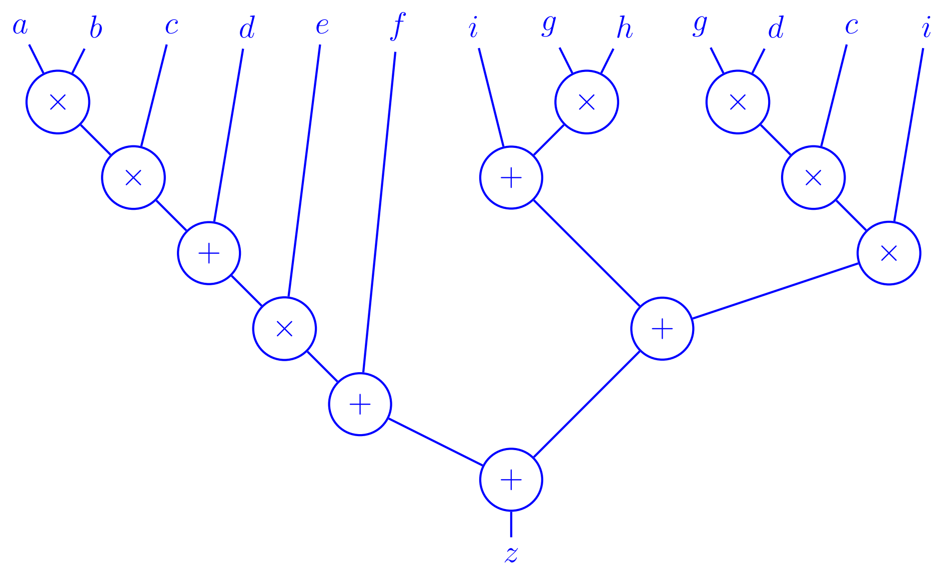
Consider the following equations:

$$
\begin{aligned}
x &= (a \times b \times c + d) \times e \\
w &= x + f \\
y &= i + g \times h + (g \times d \times c \times i) \\
z &= w + y
\end{aligned}
$$

where inputs are $a, b, c, d, e, f, g, h, i$, while the output is z.

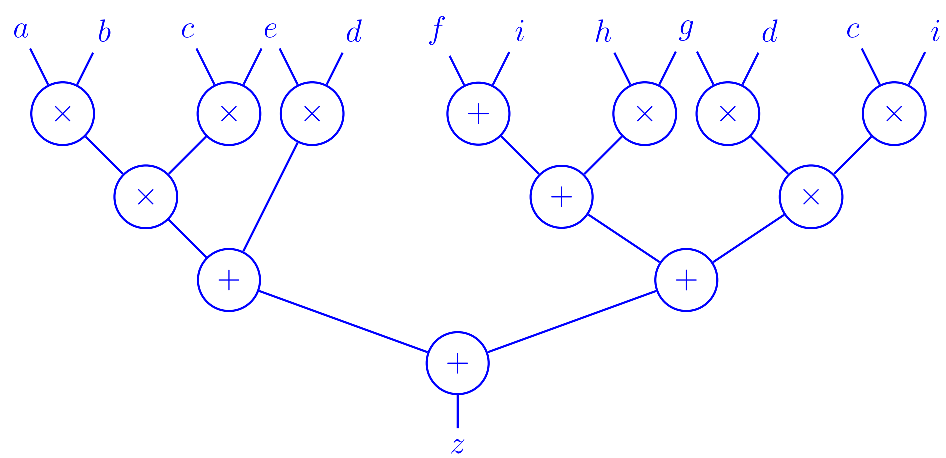
(a) Draw the data-flow graph using the operations as they appear in the expression, without any optimization. Assume additions and multiplications have 2 inputs.

Ans:



(b) Apply *tree-height reduction* to the data-flow graph drawn in (a).

cf: Slide set 4, pp. 15-17. *Ans:*



(c) Discuss on the different resources usage between the graph in (a) and the graph in (b).

Ans: The data-flow in point (b) needs more resources than the one in point (a) since by optimizing the height more operations are executed in parallel.

(d) Assume that $a = 4$, $b = 2$, $c = 3$, $h = 2$, and $i = 4$ are constant. Optimize the data-flow graph from (a) using the behavioral optimization techniques seen during the lecture. Draw the resulting data-flow graph.

Ans: