

CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 2

Date: 19/09/2024

Problem 1

Given the following function: $f(a, b, c) = a\bar{b}c + bc + \bar{a}b\bar{c}$

- (a) Write the truth table.
- (b) Write the function in terms of its minterms.
- (c) Write the function using only NAND-2 gates.

Problem 2

A truth table is a complete listing of all points in a Boolean input space and the corresponding output values. For example, the two-input AND function has the following truth table:

x_1	x_0	x_0x_1
0	0	0
0	1	0
1	0	0
1	1	1

To compactly represent a truth table, we can use a bitstring $b_{2^n-1} \dots b_1 b_0$ where $b_x = f(w_1, \dots, w_n)$ when $w = (x_n \dots x_1)_2$. Each bit in the bitstring corresponds to the output of f evaluated at some assignment to its variables. The most-significant bit b_{2^n-1} corresponds to all variables assigned to 1, $f(1, \dots, 1)$ and the least-significant bit b_0 correspond to $f(0, \dots, 0)$. Using this compact representation, the two-input AND function is 1000.

- (a) Suppose that on a remote country logicians use the symbol 1 for “false” and 0 for “true”. How do our logical operations associate to theirs?

For example, they have a binary operation “or”

$$1 \text{ or } 1 = 1, \quad 1 \text{ or } 0 = 0, \quad 0 \text{ or } 1 = 0, \quad 0 \text{ or } 0 = 0$$

which we associate with AND (1000).

Table 1: The sixteen logical operations on two variables, $f(x, y)$.

Truth table	Notation(s)	Name
0000	$\perp$	Contradiction; antilogy; constant 0
0001	$x \bar{\vee} y, x \downarrow y$	Nondisjunction; NOR
0010		Converse nonimplication
0011	$x', \neg x, \bar{x}$	Left complementation
0100		Nonimplication
0101	$y', \neg y, \bar{y}$	Right complementation
0110	$x \oplus y$	Exclusive disjunction; nonequivalence; XOR
0111	$x \bar{\wedge} y, \uparrow y$	Nonconjunction; NAND
1000	$x \cdot y, xy, x \wedge y$	Conjunction; AND
1001	$x \equiv y, x \Leftrightarrow y$	Equivalence
1010	y	Right projection
1011	$x \Rightarrow y$	Implication
1100	x	Left projection
1101	$x \Leftarrow y$	Converse implication
1110	$x + y, x \vee y$	Disjunction; OR
1111	$\top$	Affirmation; tautology; constant 1

- (b) All operations in Table 1 can be expressed in terms of NAND. For each of the 16 operations in the table find a formula equivalent to the function that uses only two-input NAND. The formula should be as short as possible and not contain any constant.
- (c) Similarly, find 16 short formulas when constants are allowed.
- (d) For each of the 16 operations in the table try to find a formula equivalent to the function using only the two-input XOR operator ($\oplus$) and without the use of constants 0 or 1. Is it possible? Why?
- (e) Consider the previous exercise using the material implication ($\Rightarrow$) as basis operator instead of $\oplus$.
- (f) If the use of constants is allowed, how would it affect the answer for (e)?