

Solutions to Exercise 12

Problem 1.

To prove that the $(n, \frac{n(n+1)}{2})$ -assignment object has consensus number at least n , we just have to devise a consensus algorithm for n processes. The $(n, \frac{n(n+1)}{2})$ -assignment object has $\frac{n(n+1)}{2}$ fields. For convenience we name the fields as follows. There are n fields $r_0, \dots, r_{n-1}$ where process i writes to register r_i , and $\frac{n(n-1)}{2}$ fields r_{ij} , where $i > j$, where processes i and j both write to field r_{ij} . All fields are initialized to *null*. Each process i atomically assigns its input value to n fields: its single-writer field r_i and its $n - 1$ multi-writer registers r_{ij} . For example, if $n = 3$, process 1 will write to single-writer register r_1 and to multi-writer registers r_{10} and r_{21} . The algorithm decides the first value to be assigned. After assigning to its fields, a thread determines the relative ordering of the assignments for every two processes i and j as follows:

- Read r_{ij} . If the value is *null*, then neither assignment has occurred.
- Otherwise, read r_i and r_j . If r_i 's value is *null*, then j precedes i , and similarly for r_j .
- If neither r_i nor r_j is *null*, reread r_{ij} . If its value is equal to the value read from r_i , then j precedes i , else vice versa.

Repeating this procedure, a process can determine which value was written by the earliest assignment.

This described algorithm is taken from the book “The Art of Multiprocessor Programming.” The interested student can also have a look at Section 3.6 of the “Wait-free Synchronization” paper on the consensus number of the (m, n) -assignment object.