Intelligent Agents

Paper Exercise: Mechanism Design

Solutions

Question 1: in eBay, often a single seller several identical items of the same kind (such as iphones) in auctions that attract different bidders and end at different times. This makes it difficult for bidders because they have to choose which auction to bid in. Is this an advantage or a disadvantage for the seller? Assuming rational buyers with independent valuations, would it be better for a seller to sell all the items in a single multi-unit auction?

Answer: it's a disadvantage: it doesn't get as many bids for any individual item. Going to a multi-unit auction could be better because it corrects that problem, but on the other hand in single auctions some bidders can be made to pay higher prices, so it might not be better.

Question 2: Consider a double auction for the same identical item, with the following valuations:

Sell	4	7	9	14	10	8	13	16	11
Buy	14	12	15	8	10	5	9	17	14

Determine the clearing price! Is any of the agents better off by giving a non-truthful price?

Answer: The clearing price for an M-th price scheme would be 11 for an M+1st price scheme it would be 10.

Could you imagine a better matching of buy and sell offers that would allow more trades to take place? Could it be possible to satisfy every buyer? How could one make this work as an auction mechanism?

Answer: Sequential English auctions, but buyers must not know that more items are coming.

Question 3: Suppose there is a seller who wishes to sell two items, Item A and Item B. There are three bidders. B1 is interested in Item A and has value 1. B2 is interested in Item B and has value 1. B3 is interested in both the items but not individual item. She values bundle to be 1.25 If tbe auctioneer uses a VCG mechanism, what is allocation of these objects? How much does each bidder pay? Does the auctioneer have an incentive to collude with any of the bidders?

Answer: B1 gets Item A and B2 gets Item B. Both pay 0.25. Together they pay 0.5 which is less than how B3 values the bundle. Hence the auctioneer may be interested in forming a coalition with B3.

Question 4: The Smiths (father Bob, mother Alice and teenager Chris) need to decide where to spend their summer vacation. The two options they have are to go to the mountains (M) or to the seaside (S), and their individual preferences for the two locations are expressed as "utilities" described in Figure 1. The Smiths decide to use a VCG tax mechanism in order to give them the incentives to truthfully reveal their preferences. Compute the taxes that will be paid by A, B and C. Explain your answer.

	M	S
A	-1	10
B	5	-2
C	5	4

Figure 1. Individual utilities each person obtains in case that the family spends the summer vacation in the mountains or at the seaside. If Alicia, Bob and Chris truthfully reveal their preferences (i.e. utilities), the Smiths will go to the seaside, as (10+4-2) > (5+5-1).

Answer: A pays 8, B pays 0, C pays 0.

Question 5: What can be done with the tax they paid?

Answer: redistribute to the agents according to the Bailey/Cavallo mechanism.
If A is not there, then the tax is 0 -> no redistribution to A
If B is not there, then the tax is 1 for A and 0 for C -> 1/3 to B.
If C is not there, the tax is 7 for A and 0 for B -> 7/3 to C.
So 8/3 can be redistributed, the rest 16/3 must be wasted.

Question 6: How could B and C collude to force a Mountain outcome, yet pay no VCG taxes? What does this say about the resistance of the VCG mechanism to collusion and coalitions?

Answer: they both declare a utility of 15 (or more) for the mountain outcome.

Question 7: A group of students want to organize a block party. Depending on the size of the party, each of them has to work l hours, and will get a satisfaction p(l) from the party. They have to agree on the size, and thus on the hours they are willing to work.
What shape can be attributed to each individual utility function u(l)? Can you design an incentive-compatible mechanism for this problem? What additional constraints would have to be considered to ensure that each agent will indeed participate in the group activity?

Answer: single-peaked: each agent has some optimal l that maximizes the satisfaction-work balance. We could solve this using the median; however, we need to make sure that the mechanism is individually rational and so we cannot choose an l that would cause negative satisfaction to anyone. Thus, we need to choose the minimum of the peaks rather than the median. (See Algorithmic Game Theory book, p. 251/252).

Question 8: What are the Nash equilibrium strategies in an English auction?

Answer: Whenever they are not the highest bidder, agents increase their bids until their true valuation is reached. In fact this is a dominant strategy for each agent.