Intelligent Agents

Paper Exercise: Introduction to Game Theory

Ungraded

Question 1: Robert and Charlotte like each other and are thinking of what to do on Saturday evening. Robert would like to attend a Basketball game, while Charlotte would like to attend a Ballet performance. But most of all, they would like to do something together. Suppose that each gets a utility of 1 for attending his/her most preferred activity, and another utility of 1 for being at the same place as the other person. Model this situation as a game, both in extensive and normal form.

Question 2: Consider the game in Figure 1. Does this game have a dominant strategy equilibrium? What is it? Explain your answer.

Player A	Player B: B1	Player B: B2
A1	-1, 1	0, 4
A2	2, 2	3, 3
A3	0, 1	2, 2

Figure 1.

Question 3: Can a game have multiple dominant equilibria? Motivate.

Question 4: Consider the game in Figure 2. Does it have a dominant strategy equilibrium? Do the players have pure minimax strategies? What are these strategies? Motivate your answer.

Player A	Player B: B1	Player B: B2
A1	-1, 1	0, 0
A2	3, -3	2, -2
A3	4, -4	-1, 1

Figure 2.

Question 5: Consider the game in Figure 3. What are the minimax strategies (pure or mixed) of the two players? Motivate your answer.

		Player B	
		Head	Tail
Player A	Head	1, -1	-1, 1
	Tail	-1, 1	1, -1

Figure 3. *The Matching Pennies Game*. Each of two players chooses either Head or Tail. If the choices differ, player A pays 1 Franc to player B. If they are the same, player B pays 1 Franc to player A.

Question 6: We would like to characterize an agent's preferences among the following 4 events by a utility function that assigns a numerical utility to each of them, where the utility of the least preferred event should be equal to 1:

1. it obtains a low quality image of the Cervin.
2. it obtains a low quality image of the Mont Blanc.
3. it obtains a high quality image of the Cervin.
4. It obtains a high quality image of the Mont Blanc.

Given that we know that the following are equally good to the agent:

a) a lottery that gives it 2 or 4 with 50% probability each vs. outcome 1 with certainty.
b) a lottery that gives it 1 or 3 with 50% probability each vs. outcome 4 with 60% and 2 with 40%.
c) 3 vs. 4 with 80% probability.

Question 7: Do the games in Figures 1, 2 and 3 have a Nash Equilibrium? What is it? Motivate. Is it true that any dominant equilibrium is also a Nash equilibrium?

Question 8: Find all Nash equilibria of the game in Figure 5 using the Algorithm given in class.

		Player B			
		B0	B1	B2	B3
Player A	A0	1, 2	1,2	0,3	1,0
	A1	2,1	0,0	2,1	4,2
	A2	1,1	1,2	3,0	1,1
	A3	2,1	2,4	2,1	2,2

Figure 5.

Question 9: We have seen that finding Nash equilibria in zero-sum games is significantly easier than in general games. Now consider the problem of finding Nash equilibria in a zero-sum game with 3 (not 2) players. Show how to reduce the problem of finding Nash equilibria in general 2 player games to Nash equilibria of 3 player zero sum games, and thus prove the hardness of this problem.