Tools

Simple

Jupyter Notebooks with R kernel

- http://noto.epfl.ch
- Does not require any installation on your machine

More involved

R and Rstudio IDE

- https://rstudio.com/products/rstudio/download/#download
- Requires installation of the R language and Rstudio editor.

Alternatively, you can do the analyses in Python in NOTO or in your favourite computing environment, but I provide examples in R

References

Seltman, H. J. (2012). Experimental design and analysis.

http://www.stat.cmu.edu/~hseltman/309/Book/

- t-test: chapter 6
- ANOVA: chapter 7
- Regression: chapter 10
- Chi-square: chapter 16

Jose, P. E. (2013). Doing statistical mediation and moderation. Guilford Press. https://books.google.ch/books?id=aJFcO81Ro-0C&printsec=copyright&redir_esc=y#v=onepage&q&f=false

- Basic Mediation: chapter 3
- Basic Moderation: chapter 5

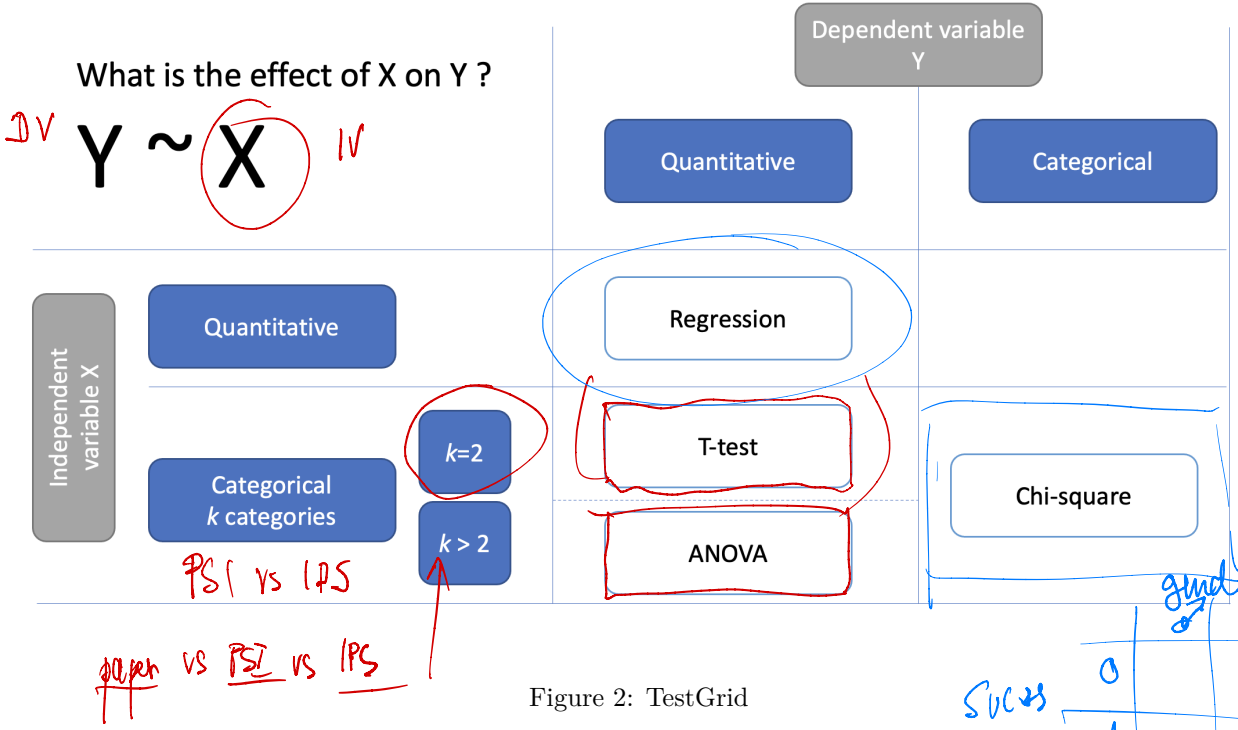


Figure 2: TestGrid

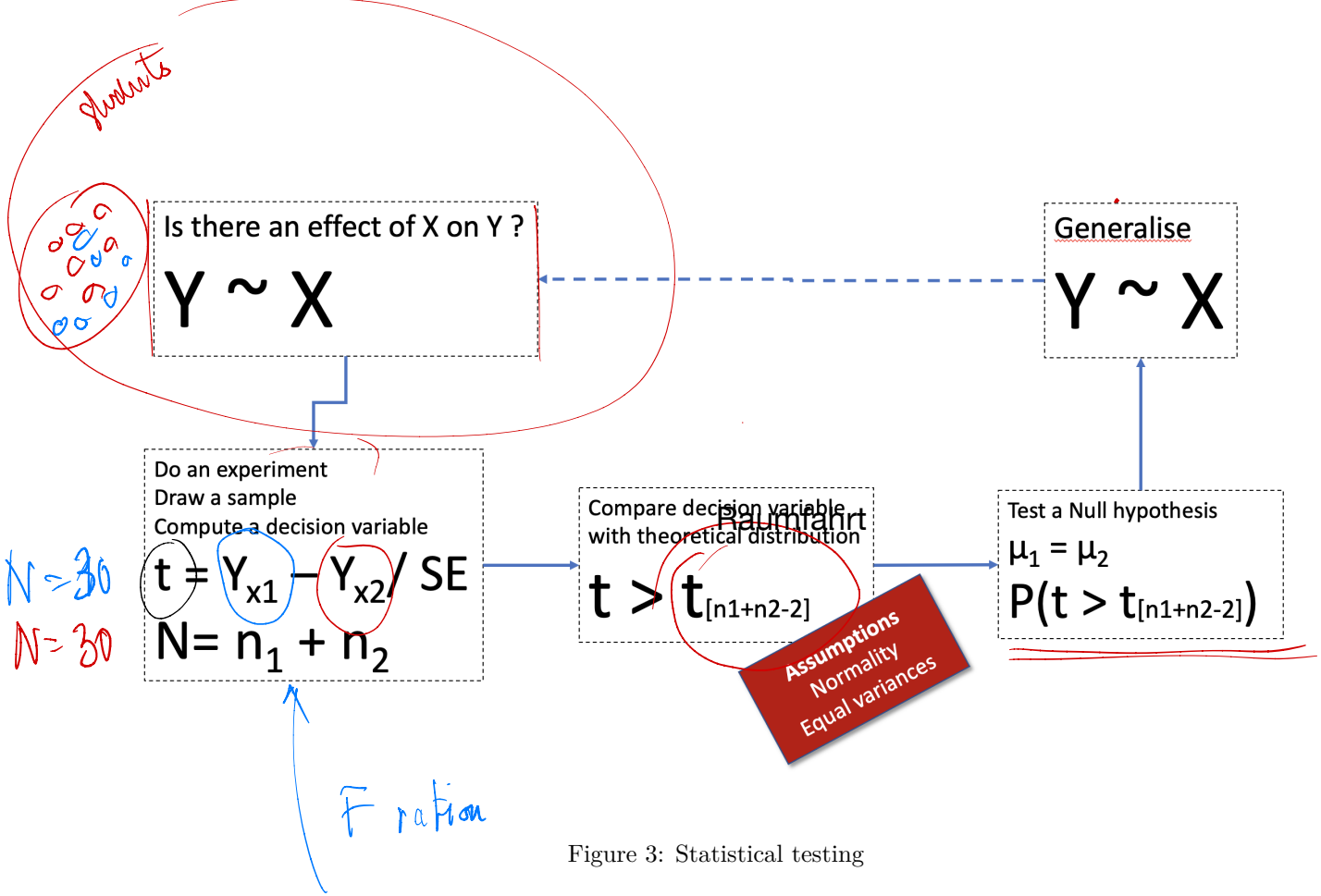


Figure 3: Statistical testing

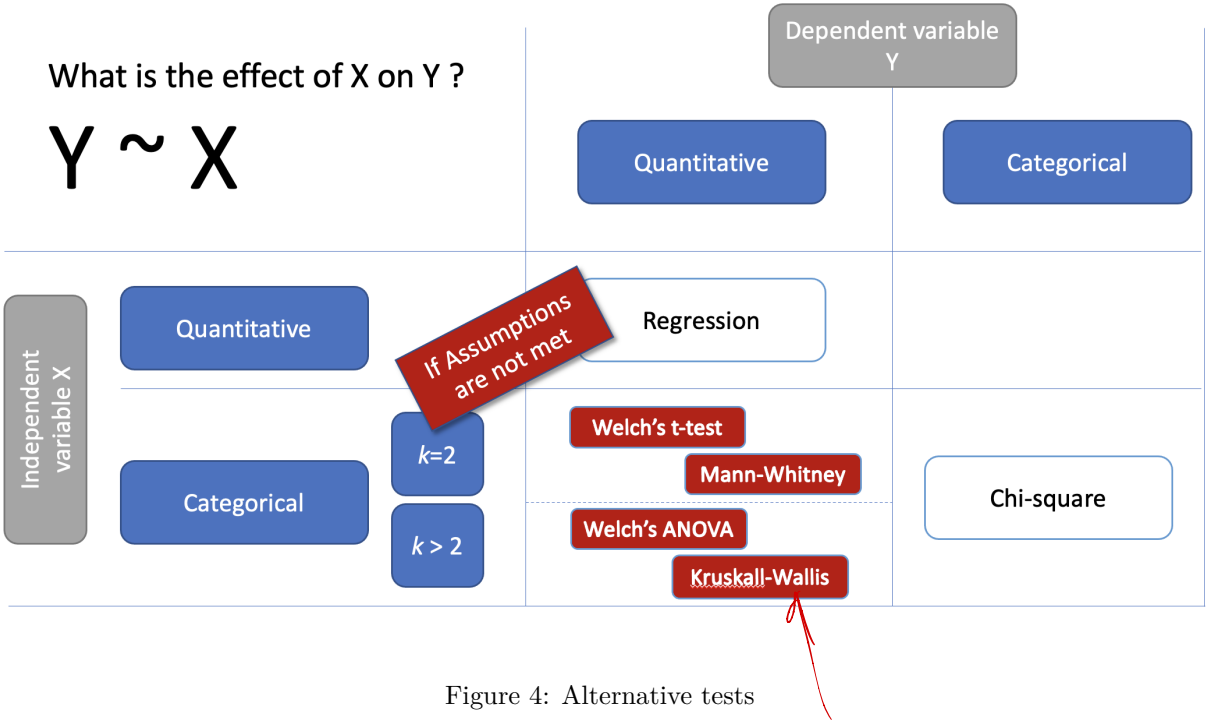


Figure 4: Alternative tests

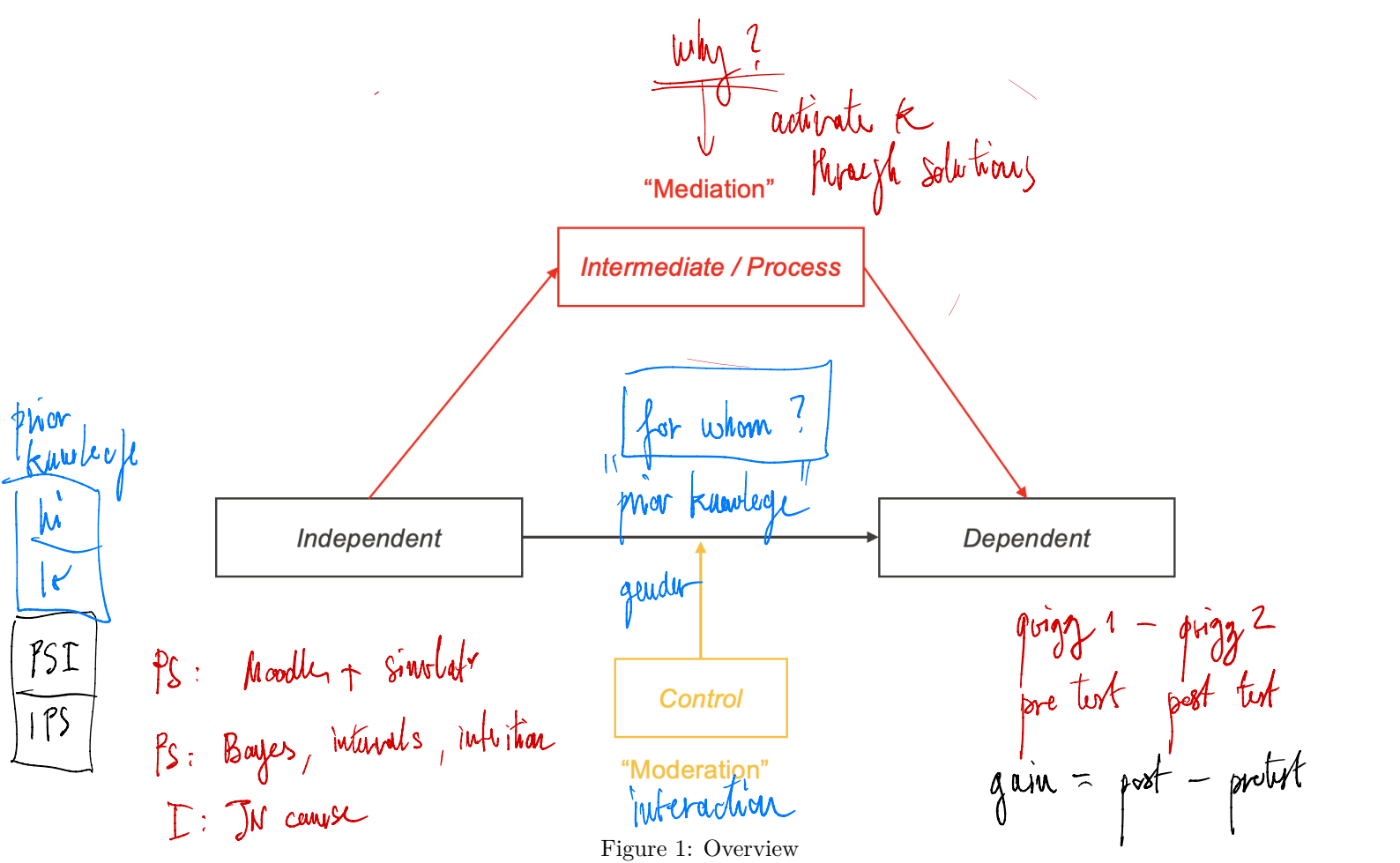


Figure 1: Overview

Mediation

- explains how or why an intervention works
- mediator explains all or part of the treatment's impact on an intended outcome
- is an intermediate outcome that is measured or observed after the onset of the intervention. E.g. fidelity of application, how many questions were asked ?

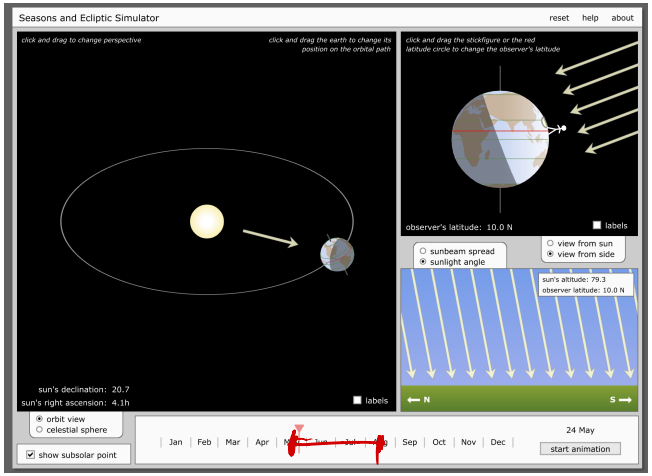
Moderation

- explains who the intervention benefits or what conditions must exist for the intervention to be effective.
- a factor that reflects who is most affected by the treatment
- a factor that exists prior to the introduction of an intervention Eg. student characteristics, such as special education status, gender, ...

Experiment (IPS vs PSI)

In this **imaginary** experiment, we are studying the effect of the order of instruction and problem-solving (independent variable) on learning (dependent variable) how the position of the earth relative to the sun influences seasons.

Participants used a simulation (https://astro.unl.edu/classaction/animations/coordsmotion/eclipticsimulator.html) during the problem-solving phase and watched a video during the instruction phase.



PS : simulation

I : youtube vidéo

Participants

The sample consisted of N=200 participants.

Independent variable

Order of instruction The independent variable has two modalities (also called conditions):

- I-PS : instruction followed by problem-solving
- PS-I : problem-solving followed by instruction

Participants were *randomly* assigned to one of the experimental conditions.

Dependent variable

Learning gain. Participants completed a 10 question *pre-test* before starting the experiment. The pre-test was a series of questions about their understanding of the sun-earth relative positions. After the experiment, particpants completed a 10 question *post-test* with similar questions as the pre-test. The learning gain was computed as :

$$learning.gain = post.test - pre.test$$

another possibilty would be the relative learning gain

$$rel.gain = \frac{post.test - pre.test}{max - pre.test}$$

Control variables

Age group. Participants were recruited among highschool students who are interested in following studies at EPFL (kids), students doing their bachelor as well as alumni who are active professionally (professionals).

Young learners (e.g., second to fifth graders) may have insufficient prior knowledge about cognitive and metacognitive learning strategies to generate multiple solutions during initial problem solving

Gender. Experimenters also asked for the gender of the participants, either Male (M) or Female (M).

Self-regulation skills. Participants also filled in a questionnaire about their self.regulation skills by using the Learning Companion (https://companion.epfl.ch)

Intermediate / Process variables

Solutions. The simulation system logged every simulation run and counted how often students used the simulation to generate a potential solution.

Dataset

This dataset was generated to illustrate basic statistical techniques like ANOVA and regression as well lightly more advanced techniques like mediation and moderation. However, we tried as much as possible to implement variations compatible with insights found in the literature about Productive Failure:

Sinha, T., & Kapur, M. (2021). When Problem Solving Followed by Instruction Works: Evidence for Productive Failure. Review of Educational Research, 91(5), 761–798. https://doi.org/10.3102/00346543211019105

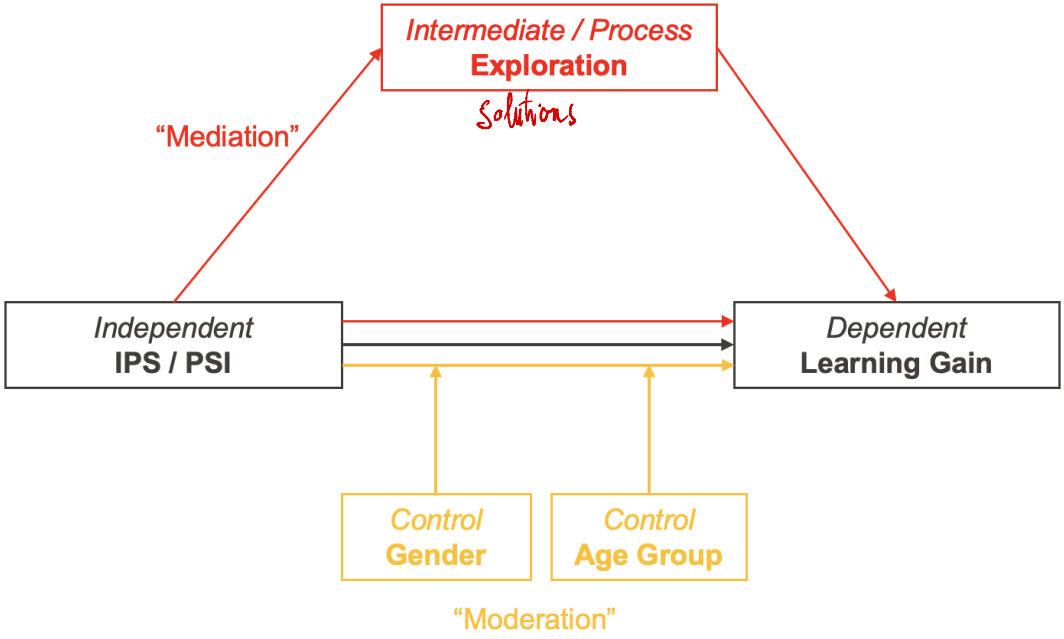


Figure 5: Overview

Loading data

```
library(tidyverse) # Give ggplot, read_delim, tidyr, etc.

df <- read_delim(file="dataset.csv", delim=",") %>%
  mutate(condition = factor(condition,labels=c("IPS","PSI")),
         gender = factor(gender, labels=c("M","F")),
         age.group= factor(age.group,
                           labels=c("kids","students","professionals")))

head(df)
```

```
## # A tibble: 6 x 9
##   condition gender age.group     age.group_0 age.group_1 age.group_2 solutions
##   <fct>     <fct>  <fct>               <dbl>       <dbl>       <dbl>     <dbl>
## 1 PSI       F      kids                    1           0           0        20
## 2 PSI       F      students                0           1           0        20
## 3 PSI       F      professionals           0           0           1        24
## 4 PSI       M      kids                    1           0           0        12
## 5 PSI       M      kids                    1           0           0         9
## 6 IPS       M      kids                    1           0           0         5
## # ... with 2 more variables: self.regulation <dbl>, learning <dbl>
```

Descriptives

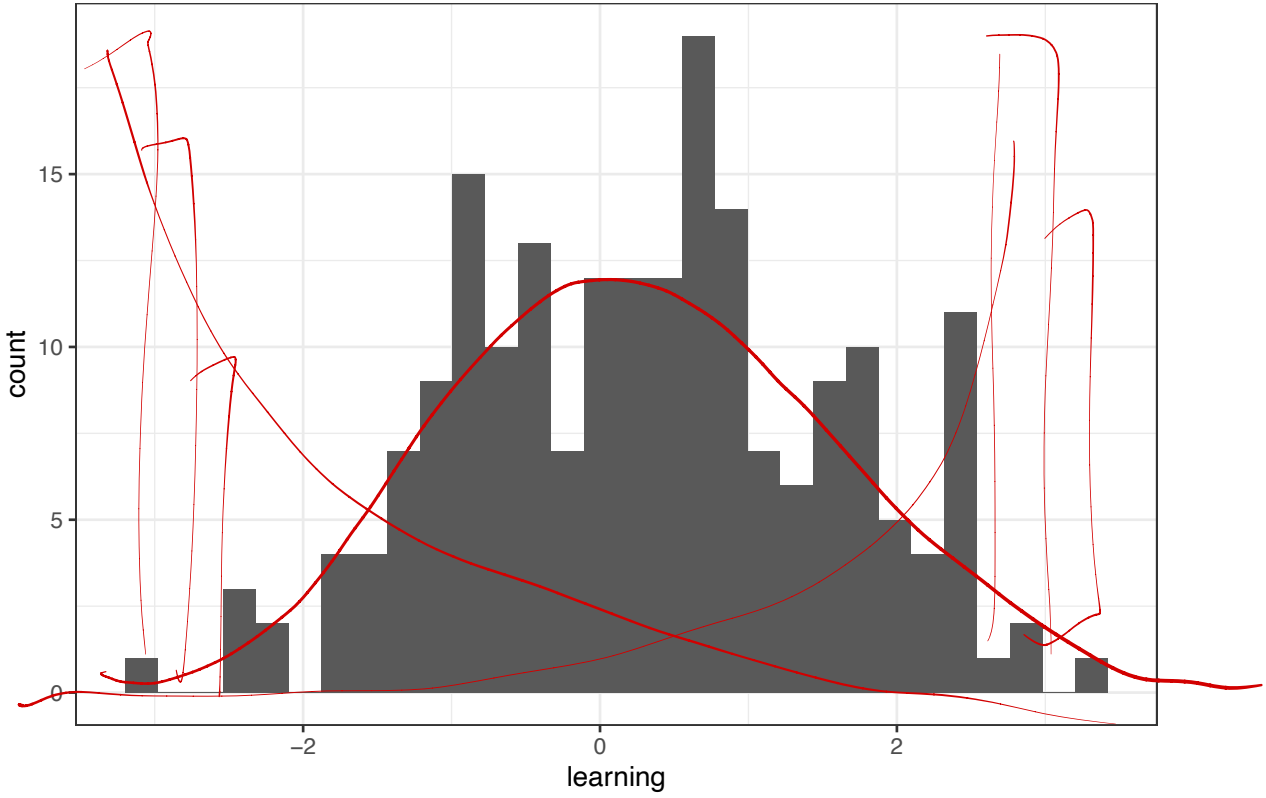
```
summary(df)
```

```
##  condition gender          age.group   age.group_0    age.group_1
##  IPS:102   M: 95   kids         :62   Min.   :0.00   Min.   :0.000
##  PSI: 98   F:105   students     :73   1st Qu.:0.00   1st Qu.:0.000
##                    professionals:65   Median :0.00   Median :0.000
##                                       Mean   :0.31   Mean   :0.365
##                                       3rd Qu.:1.00   3rd Qu.:1.000
##                                       Max.   :1.00   Max.   :1.000
##   age.group_2      solutions    self.regulation     learning
##  Min.   :0.000   Min.   :-1.0   Min.   :-5.984   Min.   :-3.0182
##  1st Qu.:0.000   1st Qu.:10.0   1st Qu.: 4.301   1st Qu.:-0.6820
##  Median :0.000   Median :15.0   Median : 8.151   Median : 0.3764
##  Mean   :0.325   Mean   :14.9   Mean   : 8.392   Mean   : 0.3159
##  3rd Qu.:1.000   3rd Qu.:19.0   3rd Qu.:12.184   3rd Qu.: 1.1734
##  Max.   :1.000   Max.   :31.0   Max.   :30.920   Max.   : 3.3847
```

```
#summary(df$learning)
```

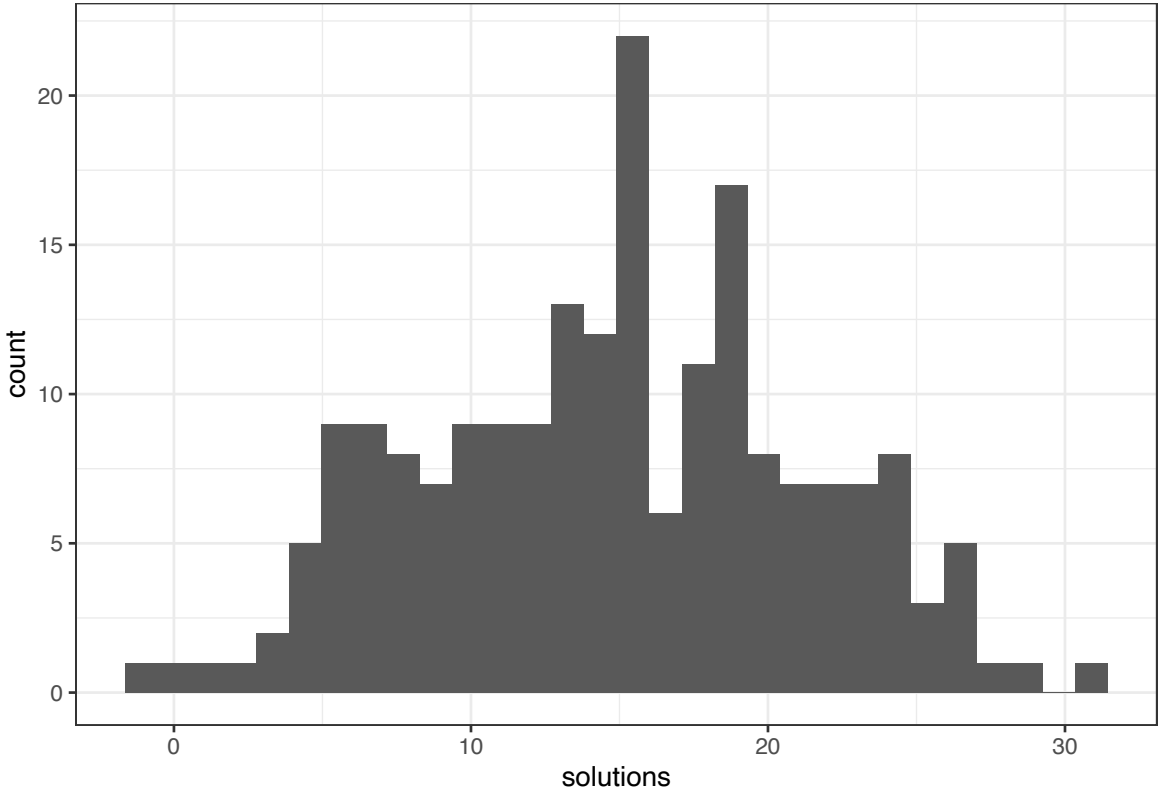
Dependent variable: learning gain

```
df %>%
  ggplot(aes(x=learning)) +
  geom_histogram() +
  theme_bw()
```


count
15
10
5
0
−2
0
2
learning

Intermediate variable: solutions

```
df %>%
  ggplot(aes(x=solutions)) +
  geom_histogram() +
  theme_bw()
```


count
20
15
10
5
0
0
10
20
30
solutions

Control variables

Gender and Condition

```
library(janitor) # Gives tabyl

df %>% tabyl(condition, gender)
```

```
##  condition  M  F
##        IPS 51 51
##        PSI 44 54
```

```
df %>% tabyl(condition, gender) %>% chisq.test()
```

```
##
##  Pearson's Chi-squared test with Yates' continuity correction
##
## data:  .
## X-squared = 0.33718, df = 1, p-value = 0.5615
```

Age Group and Condition

```
df %>% tabyl(condition, age.group)
```

```
##  condition kids students professionals
##        IPS   35       35            32
##        PSI   27       38            33
```

```
df %>% tabyl(condition, age.group) %>% chisq.test()
```

```
##
##  Pearson's Chi-squared test
##
## data:  .
## X-squared = 1.0914, df = 2, p-value = 0.5794
```

Age Group and Gender

```
df %>% tabyl(gender, age.group)
```

```
##  gender kids students professionals
##       M   30       37            28
##       F   32       36            37
```

```
df %>% tabyl(gender, age.group) %>% chisq.test()
```

```
##
##  Pearson's Chi-squared test
##
## data:  .
## X-squared = 0.82643, df = 2, p-value = 0.6615
```

Question 1: Does the experimental treatment affect learning ?

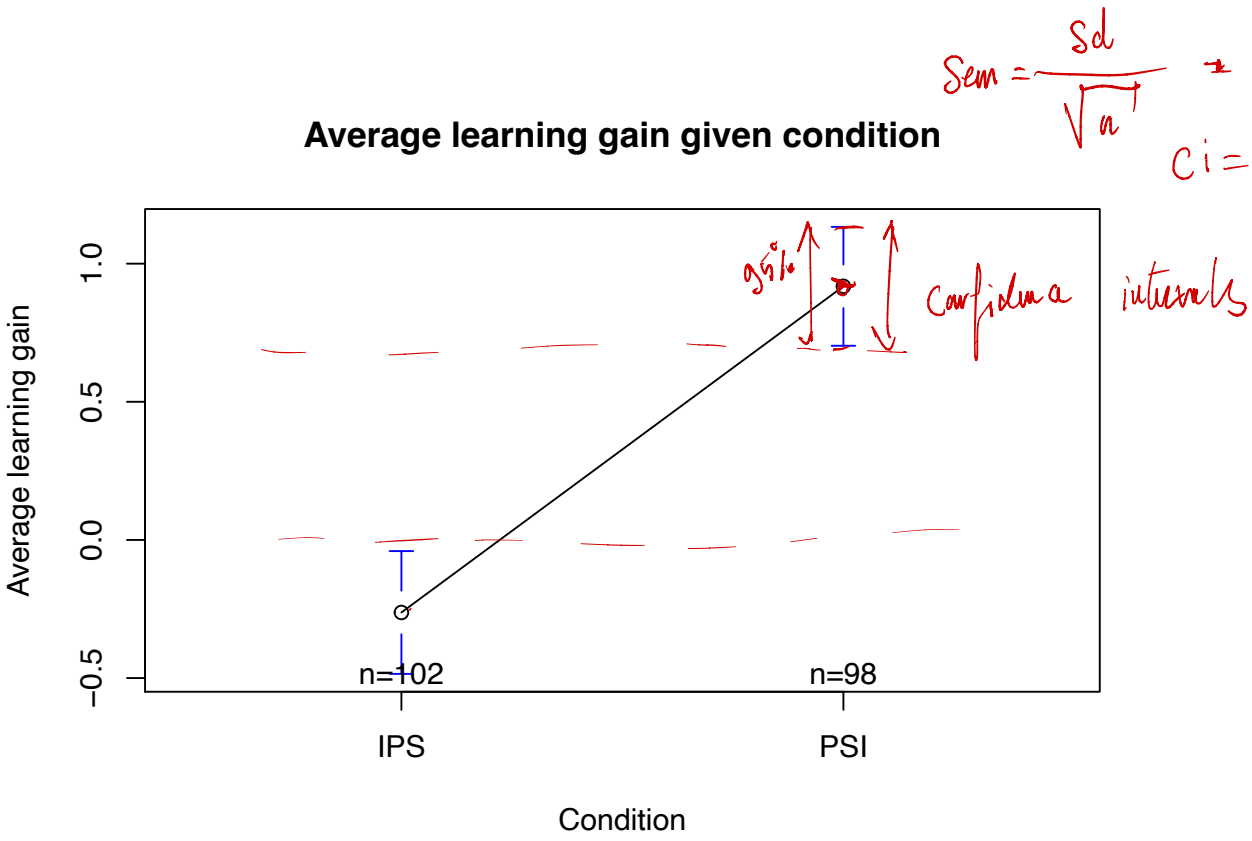
In other terms, does the manipulation of the IV affect the DV ?

Plotting

A straightforward was to look at the ifference in learning gains given the experimental condition consists of looking at the means and confidence intervals of the dependent variable given the condition. This is done using `plotmeans`.

A visual way to "see" if there is a statistically significant difference consists of comparing the confidence intervals. If they overlap, there is no statistically significant difference.

```
library(gplots) # Gives plotmeans
plotmeans(learning ~ condition,
          main="Average learning gain given condition",
          ylab="Average learning gain",
          xlab = "Condition",
          data=df)
```


Average learning gain given condition
Average learning gain
1.0
0.5
0.0
−0.5
n=102
n=98
IPS
PSI
Condition

Analysis of Variance

ANOVA with one factor

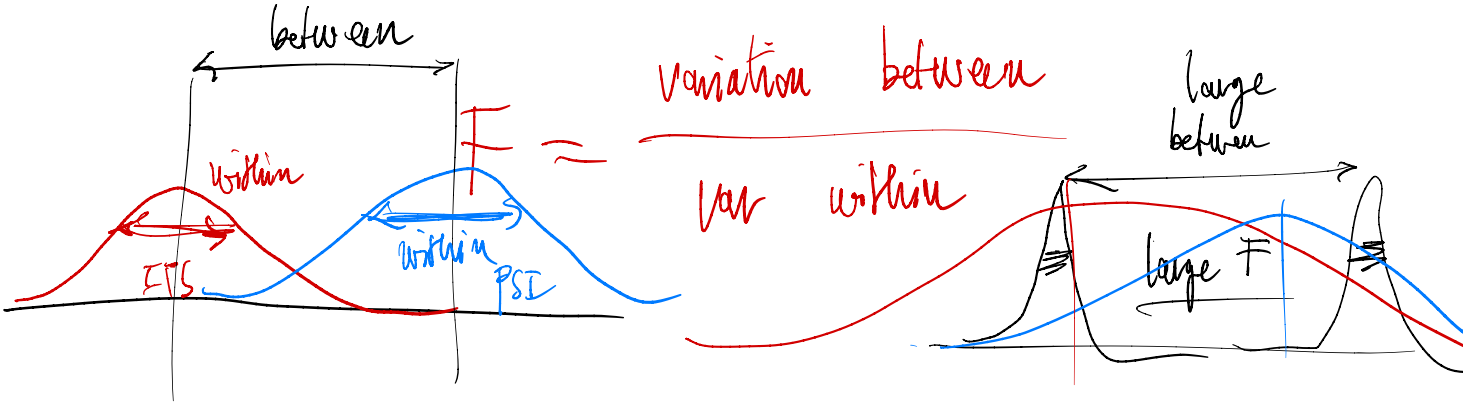
ANOVA compares the variation "between" the groups and "within" the groups based on their ratio. It assumes that the measured variable is normally distributed in each group and that the variance is the same in each group.

$F = \frac{MeanSquares_{between}}{MeanSqares_{within}}$

The sample variance (Mean Sum of Squares) is computed as the Sums of Squares divided by the Degrees of freedom.

$F = \frac{SS_{between}/df_{between}}{SS_{within}/df_{within}}$

If F is larger than 1, the differences between the groups are more important than the differences inside the groups.



Total Variance: between and within groups

Variance is a measure of "spread" based on the average squared deviation from the mean.

$SS_{total} = \sum_{i=1,j=1}^{n_i,k}(Y_{ij} - \bar{Y})^2$

Mean Sums of Square between groups

$SS_{between} = \sum_{i=1}^{k} n_i(\bar{Y_i} - \bar{Y})^2$

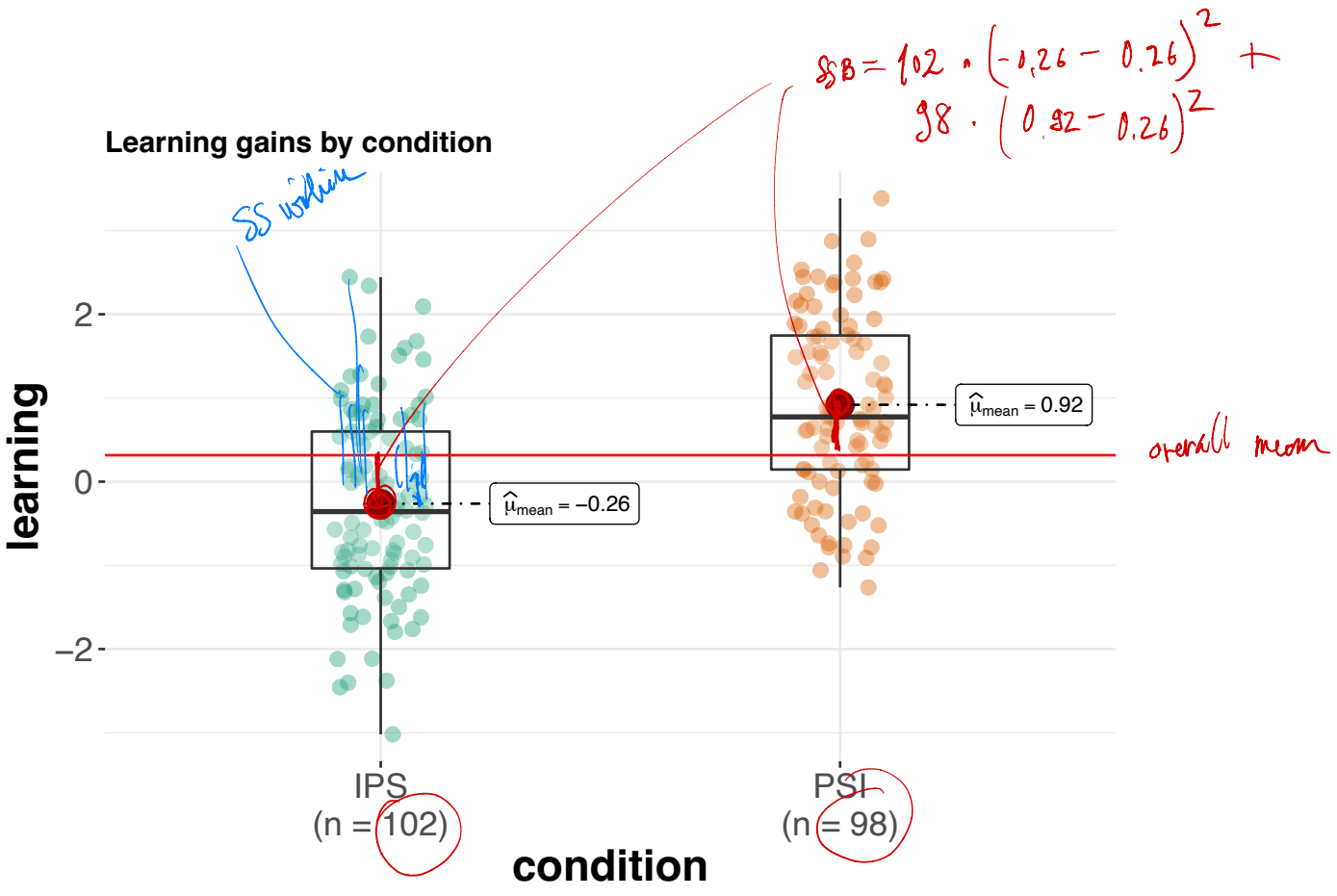
Where k is the number of groups $\bar{Y_i}$ is the mean for group i and $\bar{Y}$ is the grand mean. We multiply by n_i because we account for the difference between the group mean and the global mean for each observation.

Finally, we divide by the degrees of freedom: $MS_{Between} = SS_{between}/df_{within}$ where the df_{within} is $k - 1$.

Mean Sums of Square within groups

For each group i we have $SS_i = \sum_{j=1}^{n_i}(Y_{ij} - \bar{Y_i})^2$ which is essentially the difference between each observation j of the group and the mean for that group.

To obtain the Mean sums of squares, we add up the SS_i for each group: $SS_{within} = \sum_{i=1}^{k} SS_i$, we divide by the degrees of freedom : $MS_{within} = SS_{within}/df_{within}$ where the degrees of freedom are is the sum of the degrees of freedom for each subgroup. $df_{within} = \sum_{i=1}^{k} df_i = \sum_{i=1}^{k}(n_i - 1) = N - k$


Learning gains by condition
learning
2
0
−2
$\hat{\mu}_{mean} = -0.26$
$\hat{\mu}_{mean} = 0.92$
IPS
(n = 102)
PSI
(n = 98)
condition
overall mean

Computing Variances by hand in R

```
Y_bar = mean(df$learning) # The mean of learning for all subjects
Y_s = sd(df$learning) # The standard deviation of the learning for all subjects
```

Computing SStotal

```
ss_total = sum((df$learning - Y_bar)^2)
ss_total
```

```
## [1] 310.5642
```

Computing SSB

```
between_ss = function(x) {
    sum(length(x)*(mean(x) - Y_bar)^2)
}

ss_between = sum(tapply(df$learning, df$condition, between_ss))
ss_between
```

```
## [1] 69.66193
```

```
mss_between = ss_between / 1 # (k groups - 1)
mss_between
```

```
## [1] 69.66193
```

Computing SSW

```
within_ss = function(x) {
    sum((x - mean(x))^2)
}
ss_within = sum(tapply(df$learning, df$condition, within_ss))
ss_within
```

```
## [1] 240.9023
```

```
mss_within = ss_within / (length(df$learning) - 2) # N - k groups
mss_within
```

```
## [1] 1.216678
```

The F-Ratio

$F = \frac{MeanSquares_{between}}{MeanSqares_{within}} = \frac{69.66193}{1.216678} = 57.25584$

```
F = mss_between / mss_within
F
```

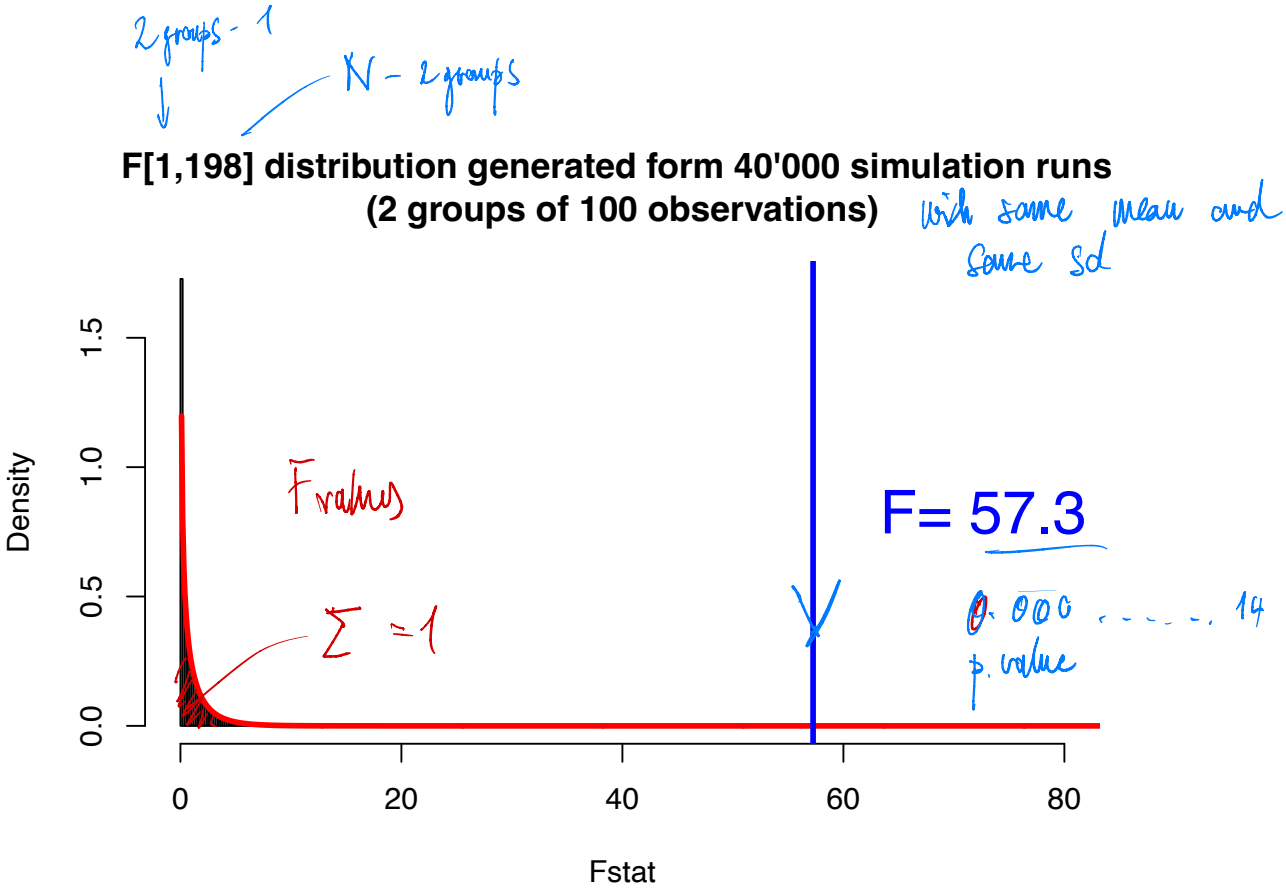
```
## [1] 57.25584
```

This F-ratio (computed from our experimental groups) is to be compared with a F distribution parametrised with df=1 (2 groups - 1) and df= 198 (200 subjects - 2 groups).

The theoretical F distribution corresponds to the F-ratios that would be obtained when:

- two samples are drawn from two populations with means μ_1 and μ_2
- two populations have the same mean: $\mu_1 = \mu_2$. This corresponds to our *null hypothesis*.
- three populations have the same variance: σ^2

We generated 40000 runs of a simulation that draws two samples of 100 observations from a normal population with the same mean and variance. For each randomly generated example, we computed the F-ratio. Here is the distribution of these 40000 F-ratios and the corresponding F[1,198] distribution.


2 groups - 1
N - 2 groups
F[1,198] distribution generated form 40'000 simulation runs
(2 groups of 100 observations)
with same mean and same sd
Density
0.0
0.5
1.0
1.5
Fratios
Σ = 1
F= 57.3
0.000 14
p. value
0
20
40
60
80
Fstat

The ANOVA test

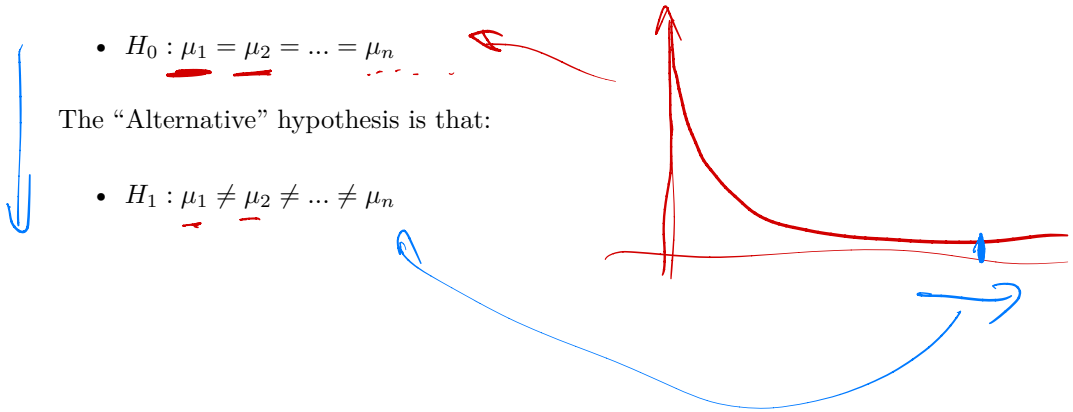
Given our observed F-ratio and the theoretical F-distribution for 2 groups (df1 = 2 groups - 1 = 1) of 100 observations (df2 = 200 observations - 2 groups = 198), we now can perform our test.

Under the "Null" hypothesis for the ANOVA, the F-ratio for 2 groups and a sample size of 200, which have the **same mean** and **same variance**, follows a F-distribution with [1,198] degrees of freedom.

- $H_0 : \mu_1 = \mu_2 = ... = \mu_n$

The "Alternative" hypothesis is that:

- $H_1 : \mu_1 \neq \mu_2 \neq ... \neq \mu_n$



How to decide whether our F-ratio is "following" the F-distribution ?

p-value

Our experiment produced a F-ratio which is rather extreme: there are only 0.0000000000014% of the theoretical F-ratios for such experiments that would be larger than the value we observed. This proportion is called the *p-value*: what is the probability to have drawn samples for our experiment which would produce a F-ratio larger than 57.3 It corresponds to the area under the curve to the right of F = 57.3

```
p.value = pf(F, 1, 198, lower.tail = FALSE)
p.value
```

```
## [1] 1.411514e-12
```

In social sciences, it is commonly accepted that to reject the null hypothesis, i.e. to say that our F-ratio does probably not stem from the theoretical F-distribition, it has to come from the 5% most extreme values. *This is called the alpha level*, written $\alpha = 0.05$.

In our example $p = 1.411514e - 12 < < < \alpha = 0.05$ and hence we **reject the Null hypothesis**. Therefore we conclude that the two samples do not belong to two populations with the same means.

Critical value

What is the F-ratio above which we can reject the Null hypothesis ? This value is called the *critical value* and corresponds to the F value for a probablity of $1 - \alpha$, i.e. 0.95.

```
alpha = 0.05
F.critical = qf(1-alpha, df1=1, df2=198)
F.critical
```

```
## [1] 3.888853
```

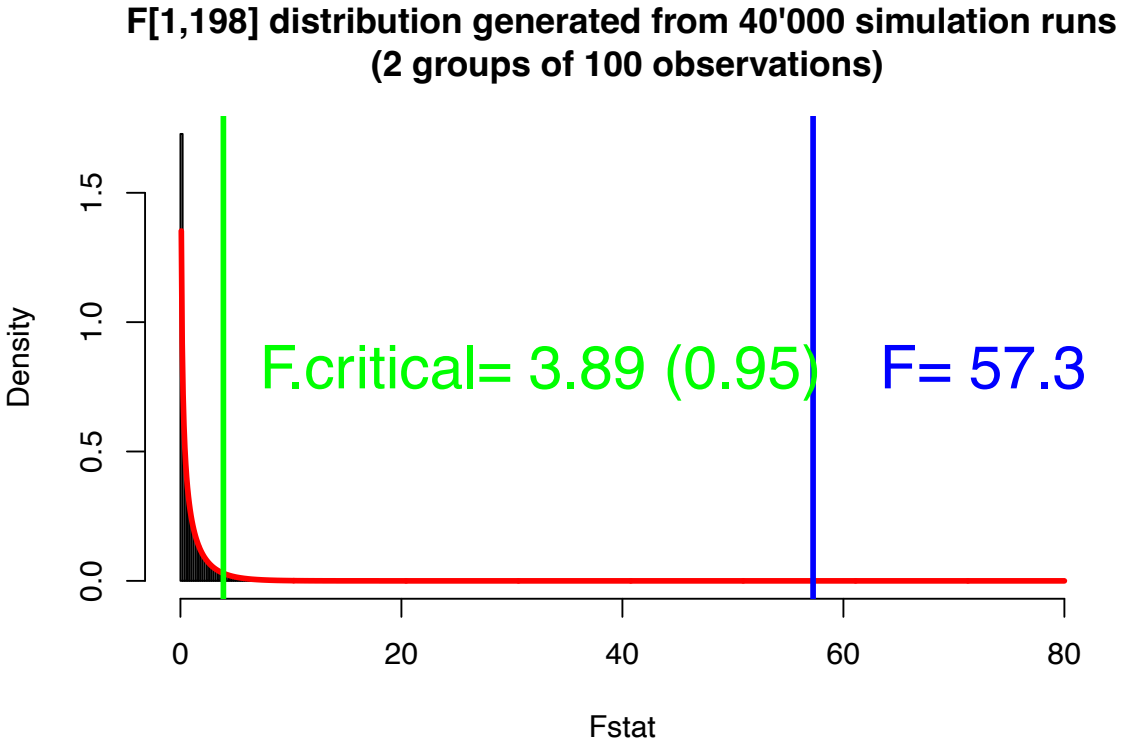

F[1,198] distribution generated from 40'000 simulation runs
(2 groups of 100 observations)
Density
0.0
0.5
1.0
1.5
F.critical= 3.89 (0.95)
F= 57.3
0
20
40
60
80
Fstat

Image taken from Hartmann, K., Krois, J., Waske, B. (2018): E-Learning Project SOGA: Statistics and Geospatial Data Analysis. Department of Earth Sciences, Freie Universitaet Berlin: https://www.geo.fu-berlin.de/en/v/soga/Basics-of-statistics/Continous-Random-Variables/F-Distribution/index.html

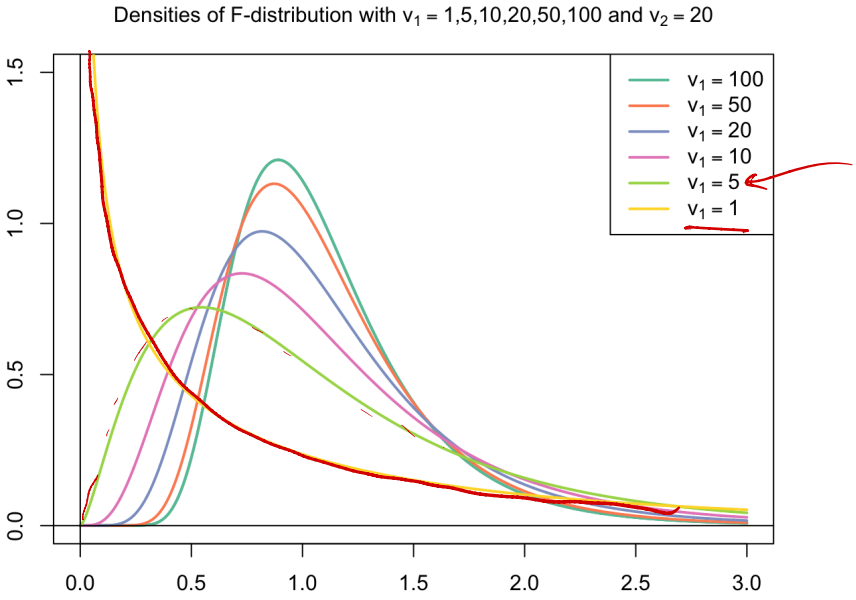
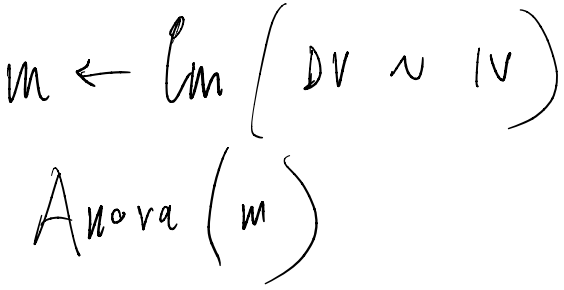


Figure 6: F-distributions

ANOVA in R

Steps:

- Step 1: Build a linear model with DV ~ IV `model = lm(DV ~IV)`:
- Step 2: Calculates type-II or type-III analysis-of-variance tables `Anova(model)`
- Step 3: Check assumptions
  - Normality
  - Homoscedasticity



Step 1 : build a linear model

NB: specify the contrasts that are used for the linear model as "contr.sum", which is not the default in R.

```
model.0 <- lm(learning ~ condition,
              contrasts=list(condition=contr.sum),
              data=df)
summary(model.0)
```

```
##
## Call:
## lm(formula = learning ~ condition, data = df, contrasts = list(condition = contr.sum))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.7555 -0.7754 -0.1243  0.8605  2.7054
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.32767    0.07801   4.200 4.03e-05 ***
## condition1  -0.59030    0.07801  -7.567 1.41e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.103 on 198 degrees of freedom
```

```
## Multiple R-squared:  0.2243, Adjusted R-squared:  0.2204
## F-statistic: 57.26 on 1 and 198 DF,  p-value: 1.412e-12
```

Step 2: Look at the Anova "interpretation" of the model

```
library(car) # load library car first.
Anova(model.0, type="II")
```

```
## Anova Table (Type II tests)
##
## Response: learning
##             Sum Sq  Df F value    Pr(>F)
## condition   69.662   1  57.256 1.412e-12 ***
## Residuals 240.902 198
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Step 3: Check Assumptions

Checking Normality assumptions

I present three methods to check the normality of the residuals for our linear model.

- The Shapiro Wilks test (available as `shapiro.test()`)
- The Kolmogorov-Smirnov test (available as `ks.test()`)
- a visual inspection test.

Shapiro.test : Testing Normality of residuals in each group The Shapiro-Wilks test allows to test whether a variable is normally distributed.

H_0: The sample is normally distributed.

H_1: The sample *is not* normally distributed.

```
shapiro.test(model.0$residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model.0$residuals
## W = 0.9902, p-value = 0.1909
```

The p-value is larger than 0.05 and therefore we cannot reject the Null hypothesis. According to this test, the residuals from our model are normally distributed.

The `shapiro.test()` is very sensitive to deviations from normality, especially if the sample size is large. Textbooks usually recommend checking the normality assumption visually (with qq plots) rather than through tests.

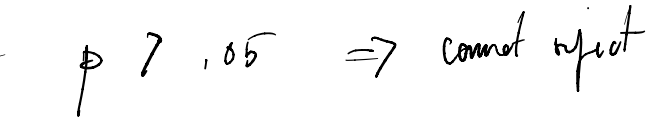
Kolmogorov-Smirnov test: Testing normality of residuals The Kolmogorov-Smirnov test allows to test whether two samples were drawn from the same distribution. This allows to compare our observations with a sample that follows a normal distibution with the same mean and standard deviation. This test is preferred to the Shapiro Wilks test for large samples.

H_0 : The two samples stem from the same distribution

H_1 : The two samples *do not* stem from the same distribution

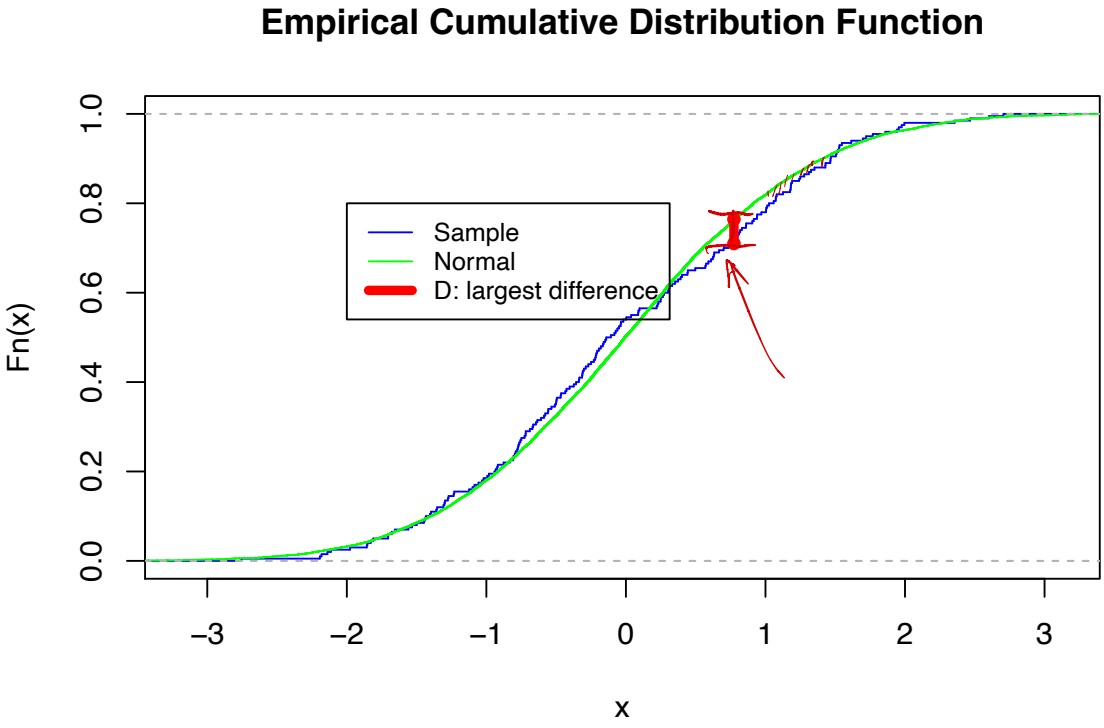
```
x <- model.0$residuals
ks.test(x, "pnorm", mean(x, na.rm = T), sd(x, na.rm = T))
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  x
## D = 0.050941, p-value = 0.677
## alternative hypothesis: two-sided
```



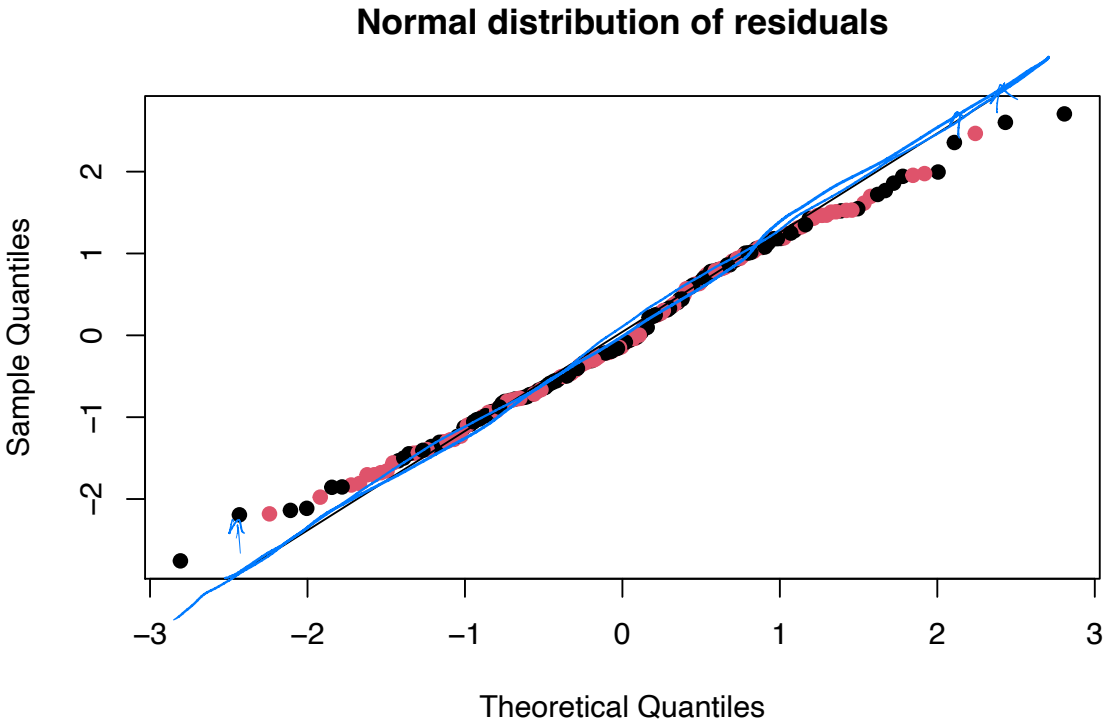
The p-value is larger than 0.05, we therefore cannot reject H_0 and hence conclude that it is likely that the residuals follow a normal distribution.

```
plot.normal.ks(model.0$residuals)
```


Empirical Cumulative Distribution Function
Fn(x)
0.0
0.2
0.4
0.6
0.8
1.0
−3
−2
−1
0
1
2
3
x
Sample
Normal
D: largest difference

Visually checking normality of residuals

```
qqnorm(model.0$residuals, col=df$condition, pch=19,
       main="Normal distribution of residuals")
qqline(model.0$residuals)
```


Normal distribution of residuals
Sample Quantiles
Theoretical Quantiles
−3
−2
−1
0
1
2
3
−2
−1
0
1
2

Checking Homoscedasticity assumptions

We would like to have the same variance of residuals across groups. This means that the model explains similarly well observations from both groups. If this was not the case, we'd have for example very similar errors for all observations in the IPS group and a larger variation of errors in the PSI group. This would indicate that there is something "wrong" in the measured data, e.g. all individuals from IPS have the same learning gain, whereas individuals from the PSI group have a spread of learning gains.

Equality of variances can be tested with the `bartlett.test()` in R.

H_0: The variances are the same in the groups

H_1: The variances *are not* the same in the groups

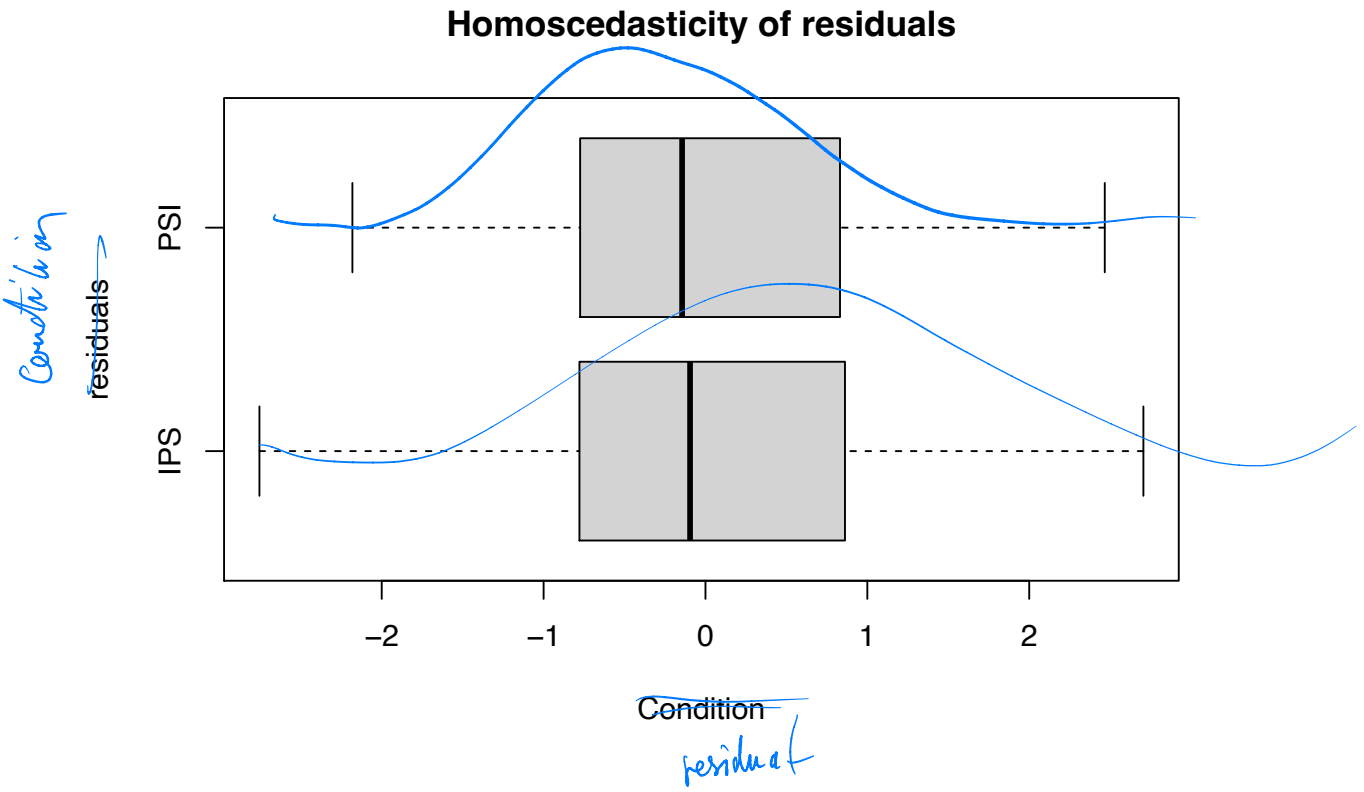
```
bartlett.test(residuals(model.0) ~ df$condition)
```

```
##
##  Bartlett test of homogeneity of variances
##
## data:  residuals(model.0) by df$condition
## Bartlett's K-squared = 0.27535, df = 1, p-value = 0.5998
```

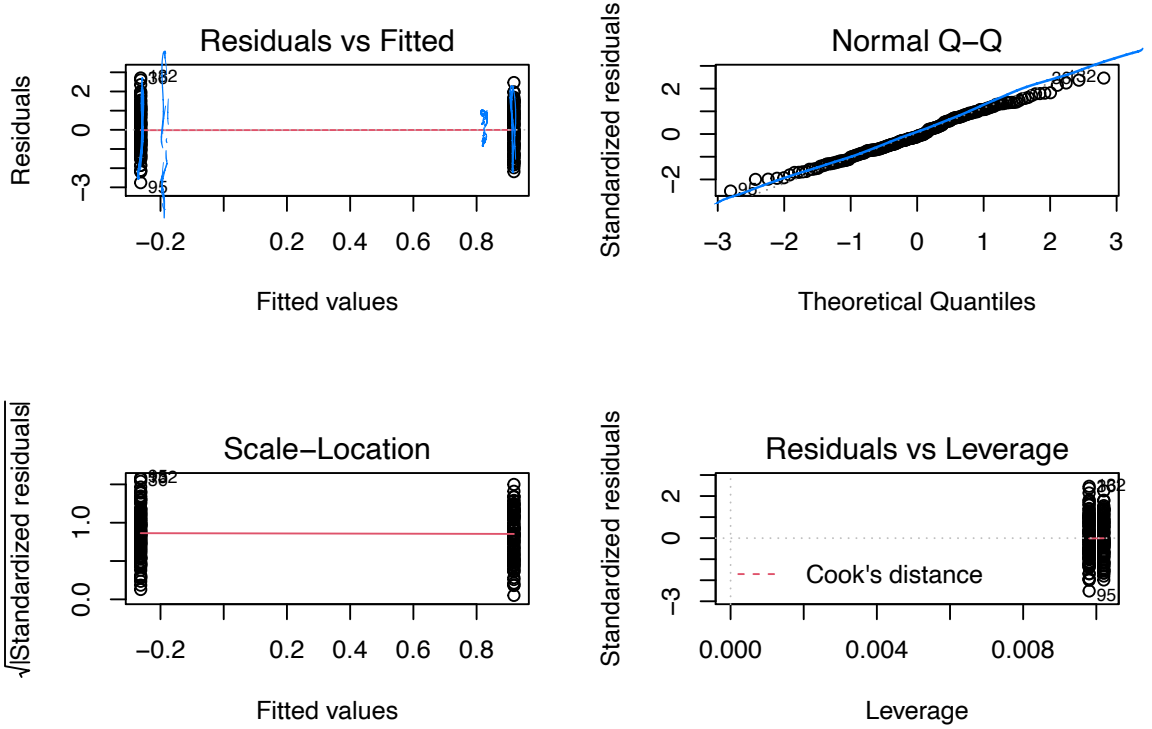
In our case, the p-value is much larger than .05 which does not allow us to reject the null hypothesis H_0. Hence we conclude that the variances are equal in both groups.

Visual inspection of equal variances An alternative was to check for equality of variances conists of plotting boxplots of the residuals. If the shape of the boxplots is more or less the same, the variances are more or less equivalent.

```
boxplot(model.0$residuals ~ df$condition,
        main="Homoscedasticity of residuals",
        ylab="residuals",
        xlab="Condition",
        horizontal=TRUE)
```


Homoscedasticity of residuals
PSI
IPS
residuals
−2
−1
0
1
2
Condition

```
par(mfrow=c(2,2))
plot(model.0)
par(mfrow=c(1,1))
```


Residuals vs Fitted
Residuals
Fitted values
Normal Q–Q
Standardized residuals
Theoretical Quantiles
Scale–Location
√|Standardized residuals|
Fitted values
Residuals vs Leverage
Standardized residuals
Cook's distance
Leverage

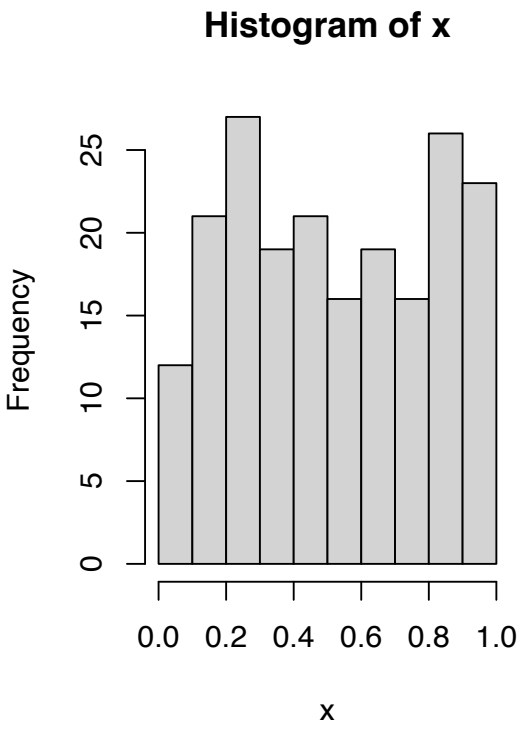
How would these graphs look if they were not normally distributed ?

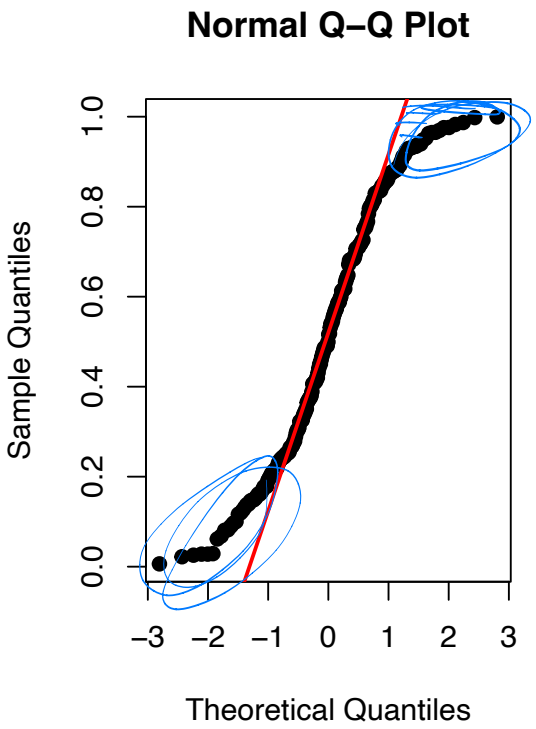
```
x <- runif(200)

shapiro.test(x)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  x
## W = 0.94566, p-value = 7.36e-07
```

```
par(mfrow=c(1,2))
hist(x)
qqnorm(x, pch=19)
qqline(x, col="red", lwd=2)
par(mfrow=c(1,1))
```


Histogram of x
Frequency
0
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0
x


Normal Q–Q Plot
Sample Quantiles
0.0
0.2
0.4
0.6
0.8
1.0
–3
–2
–1
0
1
2
3
Theoretical Quantiles

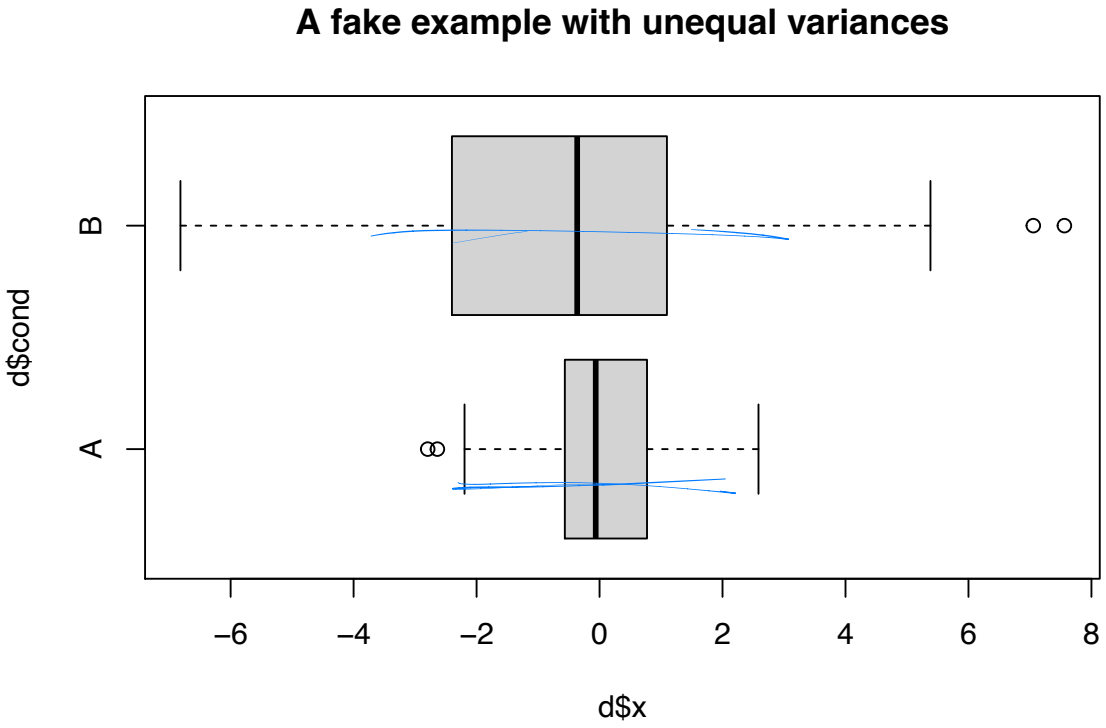
How would these graphs look if they did not have equal variances ?

```
x1 <- rnorm(100,mean = 0, sd = 1)
x2 <- rnorm(100,mean = 0, sd = 3)
x <- c(x1, x2)
cond <- c(rep("A",100), rep("B",100))
d <- data.frame(cond, x)

bartlett.test(d$x ~ d$cond)
```

```
##
##  Bartlett test of homogeneity of variances
##
## data:  d$x by d$cond
## Bartlett's K-squared = 81.861, df = 1, p-value < 2.2e-16
```

```
boxplot(d$x ~ d$cond, horizontal=TRUE, main="A fake example with unequal variances")
```


A fake example with unequal variances
d$cond
B
A
−6
−4
−2
0
2
4
6
8
d$x

What if assumptions are not met ?

Normality: ANOVA is said to be pretty robust against deviations of normality, which means that the validity of p-values are not too much affected by skew (the distribution is asymetric) or kurtosis (the distribution is too heavy or too light tailed).

=> Data Transformation. Trying to transform the dependent variable so that the distribution approaches normality, by taking 1/x, log(x) or sqrt(x).

=> Using a non-parametric equivalent for ANOVA: Kruskall-Wallis rank test.

Equality of variance: Deviations for the equality of variance have most impact on the result of the ANOVA if the group sizes are unequal.

=> Using the Welch correction for `oneway.test()` by specifying `var.equal=FALSE`.

Running a non-parametric Kruskall-Wallis as an alternative

The principle for the Kurskall Wallis test is very similar to the idea behind ANOVA. The difference is that rather than using the raw scores, the Kruskall-Wallis test relies on **ranks**. This test does **not make assumptions** about the distribution of the residuals, nor about the variances.

H_0: The mean ranks of the groups are the same.

H_1: The mean ranks of the groups *are not* the same.

The decision variable:

$$H = (N-1)\frac{\sum_{i=1}^{g} n_i(\bar{r_{i\cdot}}-\bar{r})^2}{\sum_{i=1}^{g}\sum_{j=1}^{n_i}(\bar{r_{ij}}-\bar{r})^2} \sim \chi^2_{[g-1]}$$

```
kruskal.test(learning ~ condition, data=df)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  learning by condition
## Kruskal-Wallis chi-squared = 43.443, df = 1, p-value = 4.365e-11
```

From the results of the test we see that we can reject the Null hypothesis ($p < .05$) and therefore conclude that the mean ranks are different among the two groups.

ANOVA with 2 factors

We now add a control variable (`age.group`) as a new factor to the ANOVA. This introduces the possibility for interaction between variables.

In order to test for a potential moderation effect (the effect of the condition varies depending on another variable), we include interaction effects in the linear model.

The total variance is now decomposed into:

$SSTotal = SSFactor1 + SSFactor2 + SSInteraction + SSWithin$

The degrees of freedom for an interaction effect between 2 variables with k and m levels are $(k-1)(m-1)$, with condition and gender: $(2-1)(2-1) = 1$ and with condition and age group $(2-1)(3-1) = 2$.

In the specification of the model, the interaction between 2 factors is written with a column as in `condition:age.group`.

f1 : f2

```
model.2 <- lm(learning ~
                condition +
                age.group +
                condition:age.group,
              contrasts=list(condition=contr.sum, age.group=contr.sum),
              data=df)
summary(model.2)
```

```
##
## Call:
## lm(formula = learning ~ condition + age.group + condition:age.group,
##     data = df, contrasts = list(condition = contr.sum, age.group = contr.sum))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.55457 -0.74262 -0.03032  0.87658  2.24990
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               0.31422    0.07583   4.144 5.10e-05 ***
## condition1               -0.56384    0.07583  -7.435 3.27e-12 ***
## age.group1               -0.33434    0.10940  -3.056  0.00256 **
## age.group2                0.16601    0.10465   1.586  0.11428
## condition1:age.group1    -0.02690    0.10940  -0.246  0.80599
## condition1:age.group2    -0.24722    0.10465  -2.362  0.01915 *
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.066 on 194 degrees of freedom
## Multiple R-squared:  0.2897, Adjusted R-squared:  0.2714
## F-statistic: 15.82 on 5 and 194 DF,  p-value: 4.628e-13
```

```
Anova(model.2, type="III")
```

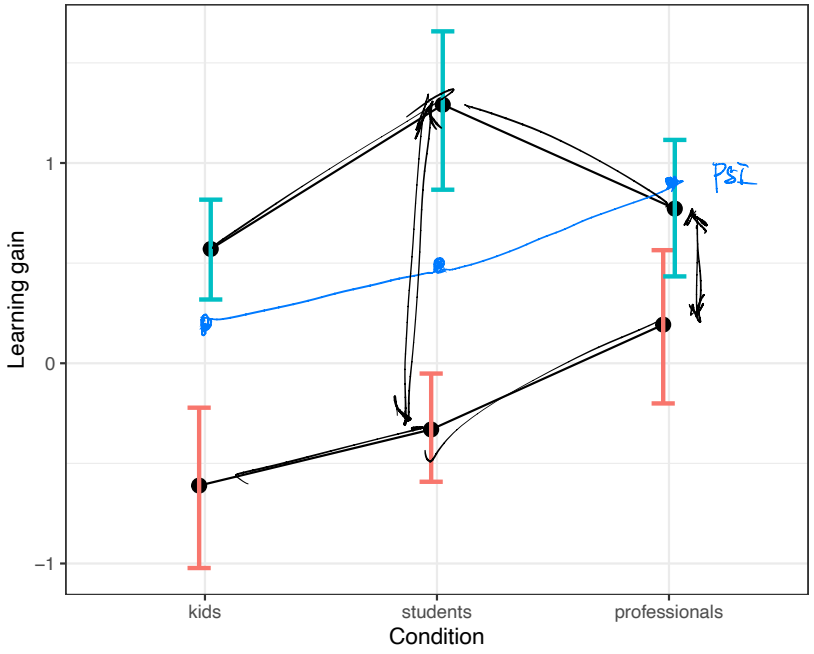
```
## Anova Table (Type III tests)
##
## Response: learning
##                       Sum Sq  Df F value    Pr(>F)
## (Intercept)           19.525   1 17.1706 5.099e-05 ***
## condition             62.867   1 55.2856 3.267e-12 ***
## age.group             10.630   2  4.6739   0.01041 *
## condition:age.group    9.365   2  4.1177   0.01773 *
## Residuals            220.602 194
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

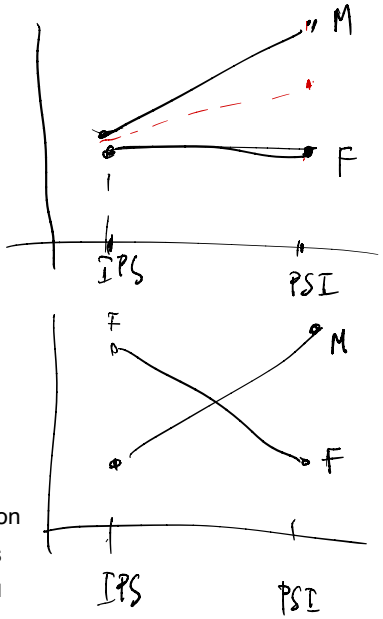
Is there an interaction ?

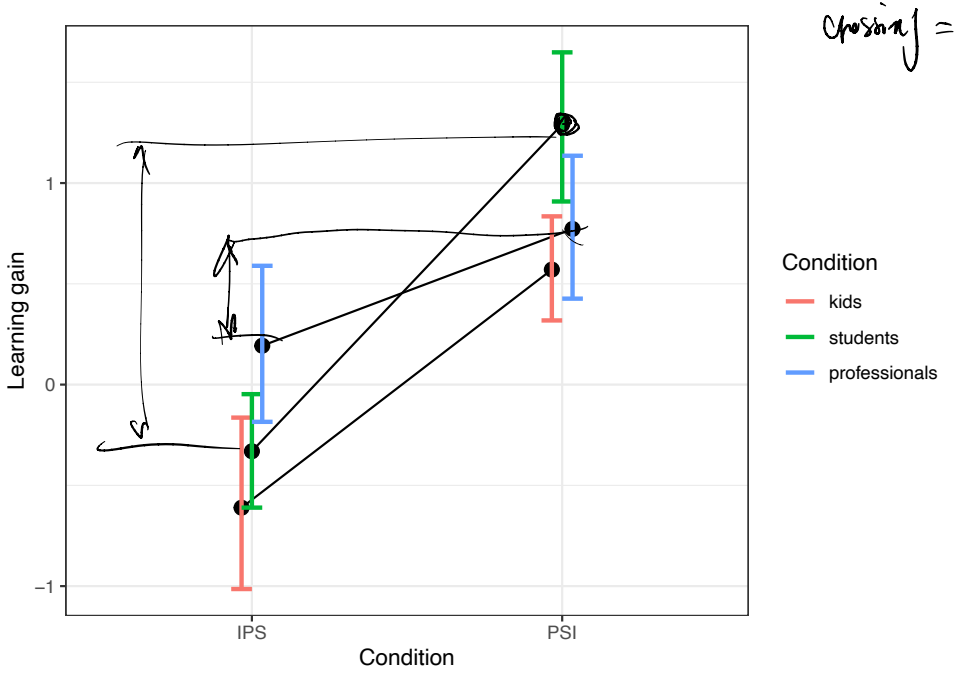
- When there is an interaction, we use **type III sums of squares** and don't interpret main effects.
- If there are no interactions, switch to a model that only includes main effects and use **type II sums of squares**.

Interpreting interactions

Age Group






crossing = interaction
Learning gain
1
0
−1
IPS
PSI
Condition
Condition
kids
students
professionals

Reporting the ANOVA with interaction

A two way ANOVA was conducted with the experimental condition, and the control variable age group. We tested for interactions between the condition and the control variables. There was a significant interaction effect between condition and age group (F[2,194]=4.1177, p =.0104). Inspection of the graphical patterns of the means indicates that the PSI condition worked especially well for students in comparison with kids and professionals.

Gender

Let's do the same analysis with the control variable `gender`. We start with a model that contains the interaction term (type III). Since there is no interactions between the factors, we re-run the model without interaction and use type II sums of squares.

```
model.with.interaction <- lm(learning ~ condition + gender + condition:gender,
              contrasts=list(condition=contr.sum, gender=contr.sum),
              data=df)

Anova(model.with.interaction, type="III")
```

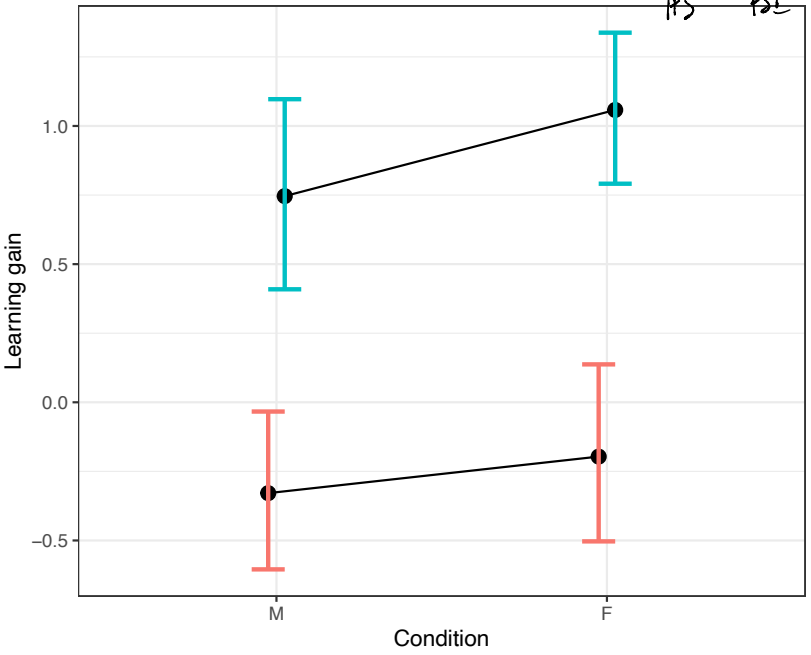
```
## Anova Table (Type III tests)
##
## Response: learning
##                   Sum Sq  Df F value    Pr(>F)
## (Intercept)       20.326   1 16.7324 6.282e-05 ***
## condition         67.435   1 55.5118 2.899e-12 ***
## gender             2.451   1  2.0176    0.1571
## condition:gender   0.401   1  0.3301    0.5663
## Residuals        238.099 196
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

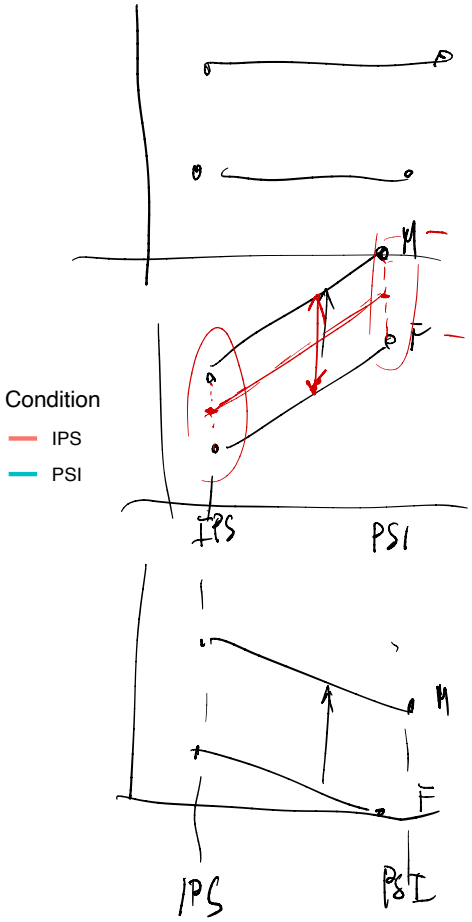
```r
model.without.interaction <- lm(learning ~ condition + gender,
              contrasts=list(condition=contr.sum, gender=contr.sum),
              data=df)

Anova(model.without.interaction, type="II")
```

```
## Anova Table (Type II tests)
##
## Response: learning
##             Sum Sq  Df F value    Pr(>F)
## condition  68.167   1 56.3055 2.086e-12 ***
## gender      2.403   1  1.9846    0.1605
## Residuals 238.500 197
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Gender





Reporting the ANOVA without interaction

```
tapply(df$learning, df$condition, mean)
```

```
##        IPS        PSI
## -0.2626251  0.9179664
```

```
tapply(df$learning, df$condition, sd)
```

```
##      IPS      PSI
## 1.131207 1.072908
```

A two way ANOVA was conducted with the experimental condition, and the control variable gender. There was no interaction effect between condition and gender. There is a main effect of the experimental condition (F[1,197]=56.306, p<.000). The subjects in the PSI group had a larger learning gain (M=0.918, sd=1.07) than the subjects in the IPS group (M=-0.262, sd=1.13). There was no main effect of gender (F[1,197]=1.98, p > .05).