ICC SV Training

18.10.2024

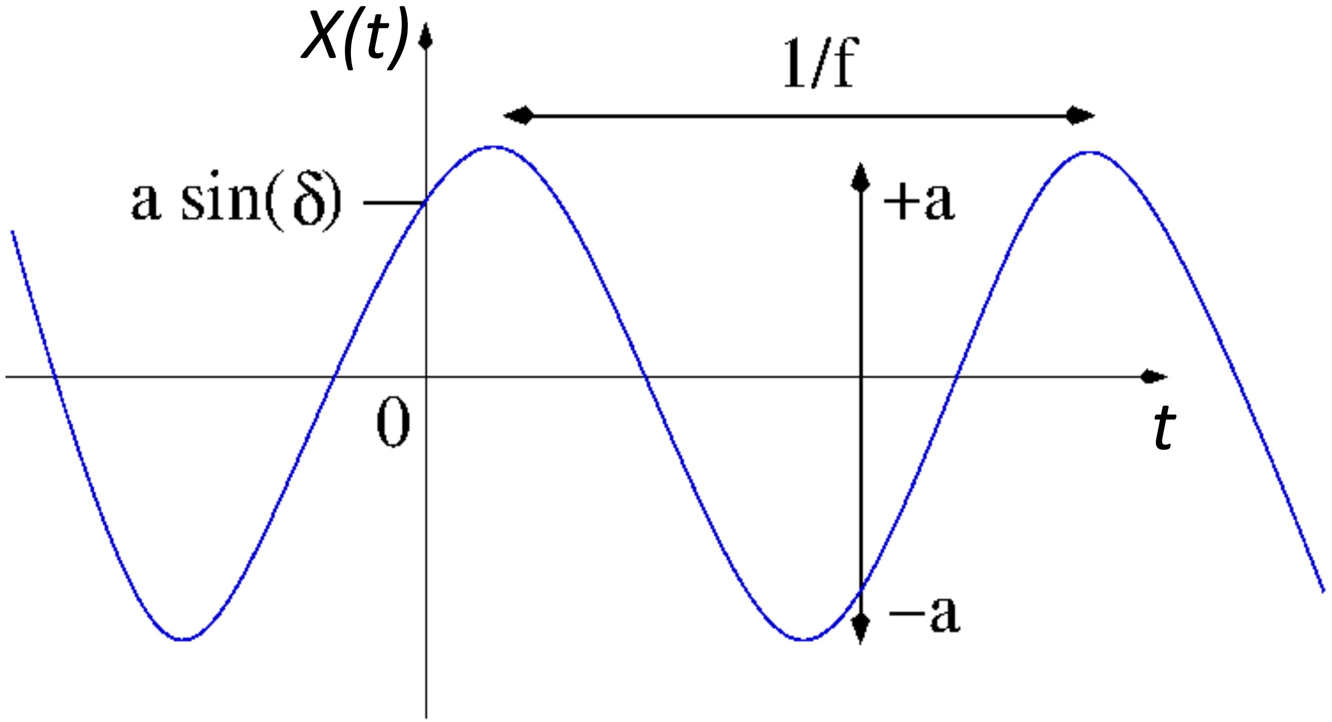
Outline

- Signal
- Filter
- Subsampling/Apparent Frequency
- Entropy
- Huffman

Recall: Sinusoid

$$X(t) = a \sin(2\pi f\, t + \delta), \qquad t \in \mathrm{R}$$

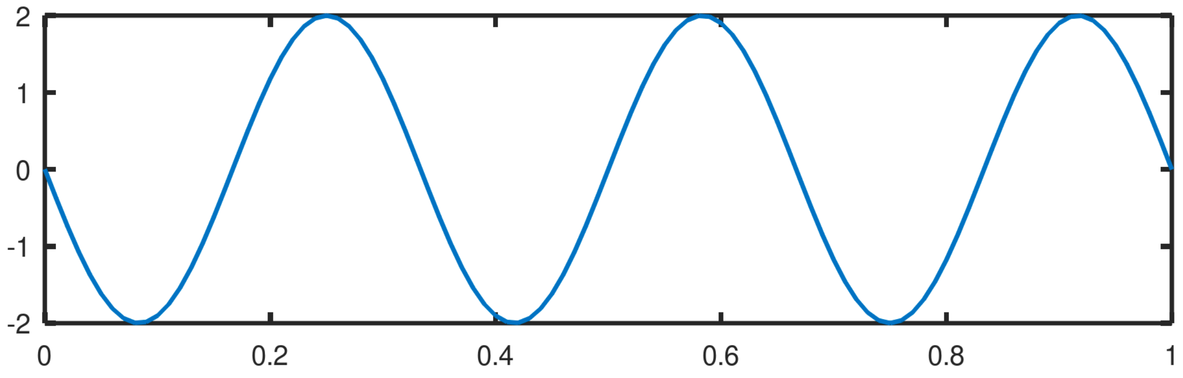
$a =$ amplitude, $f =$ frequency, T = period = 1/f , $\delta =$ phase (shift)



δ	$a \cdot \sin(\delta)$
0	0
π/2	a
π	0
3π/2	-a
2π	0

Exercises

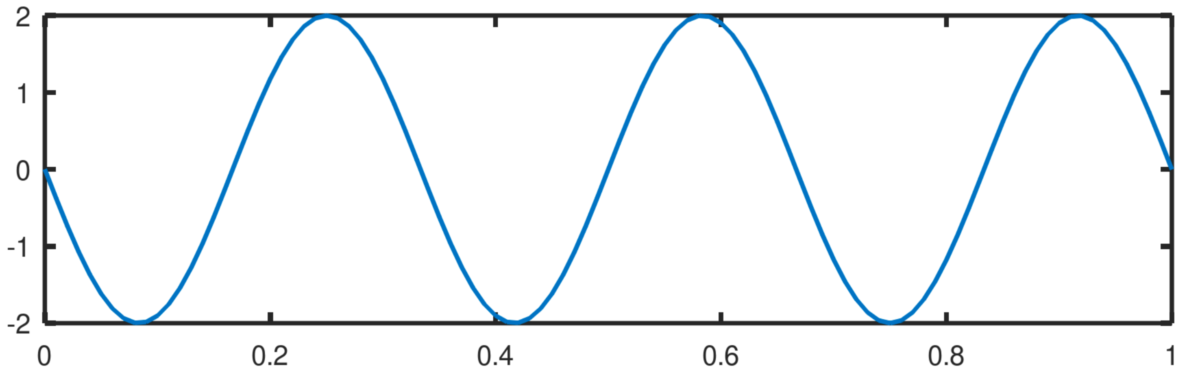
1. Write the mathematical description of this signal:



2. Draw the following signals:
$X(t)=3 \cdot \sin(4 \cdot \pi \cdot t + \pi/2)$

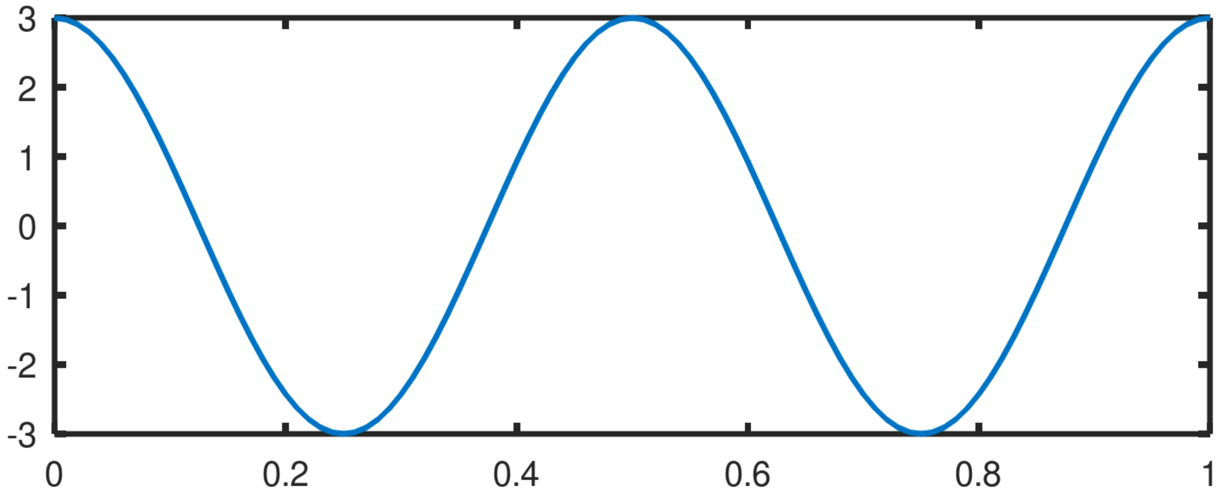
Exercises

1. Write the mathematical description of this signal:



X(t)=2 · sin(6 · π · t + π)

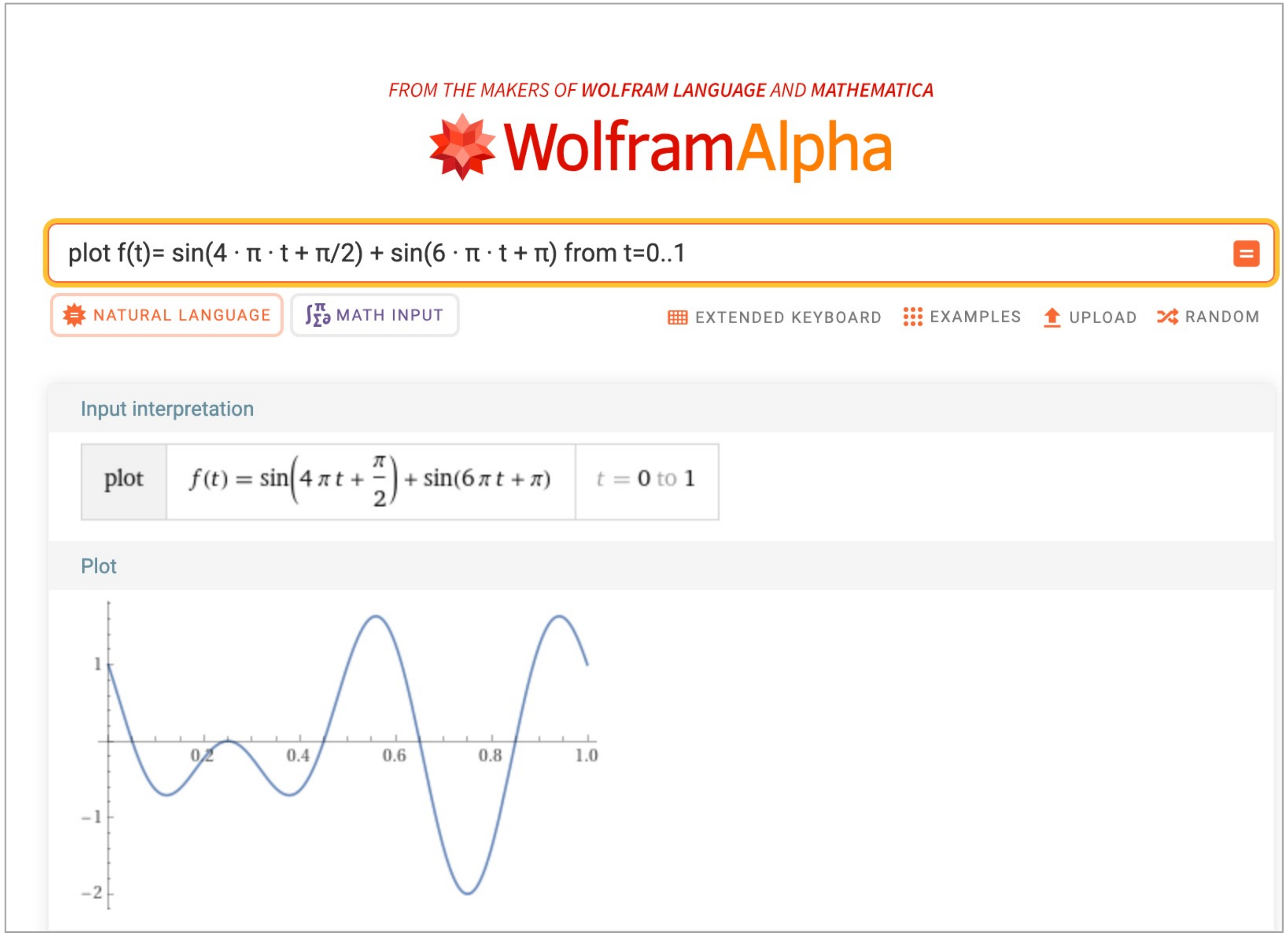
2. Draw the following signals:
X(t)=3 · sin(4 · π · t + π/2)



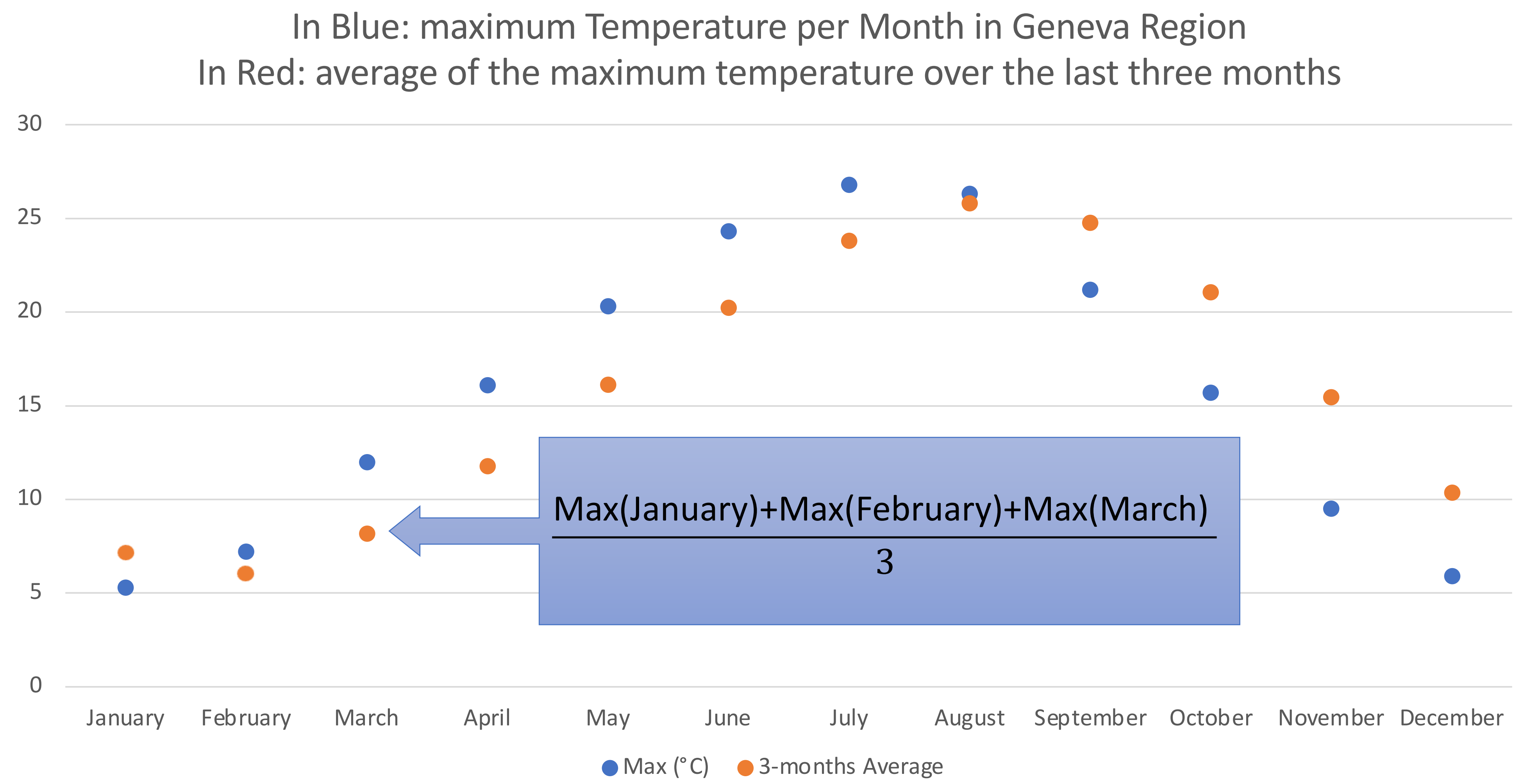
Make your own Exercises with https://www.wolframalpha.com/

- Draw the following signal:

$$X(t) = \sin(4 \cdot \pi \cdot t + \pi/2) + \sin(6 \cdot \pi \cdot t + \pi)$$



Moving Average Filter: Discrete Signal

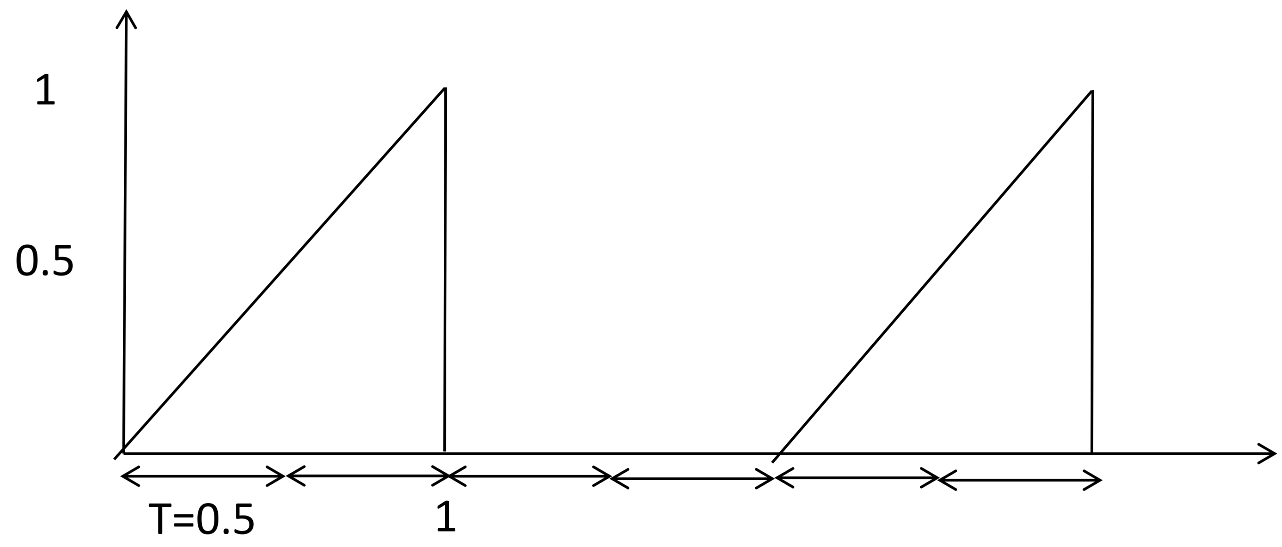


$$\frac{\text{Max(January)+Max(February)+Max(March)}}{3}$$

Moving Average Filter

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal X(t) with period 4T. Into what signal is X(t) transformed if it passes through a moving average filter with Tc = T?



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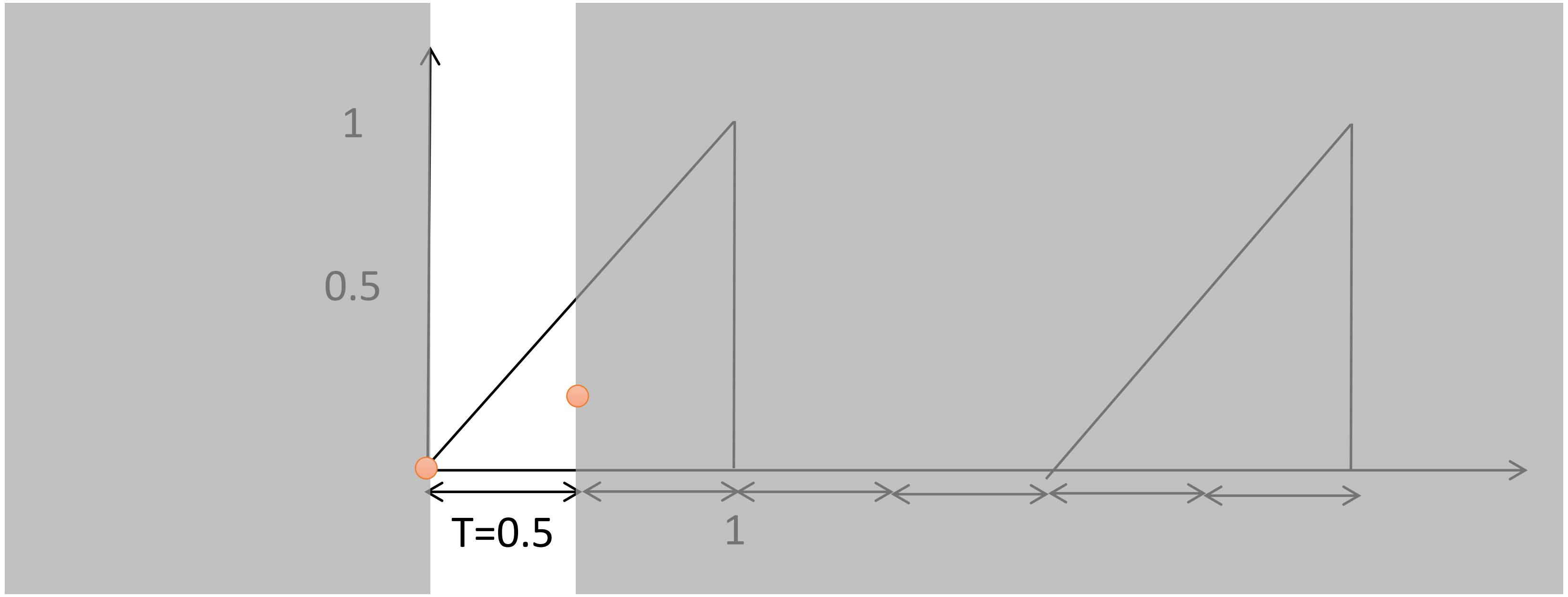

$$\hat{X}(0.5) = \frac{1}{0.5}\cdot\frac{0.5\cdot 0.5}{2} = 0.25$$

Moving Average Filter

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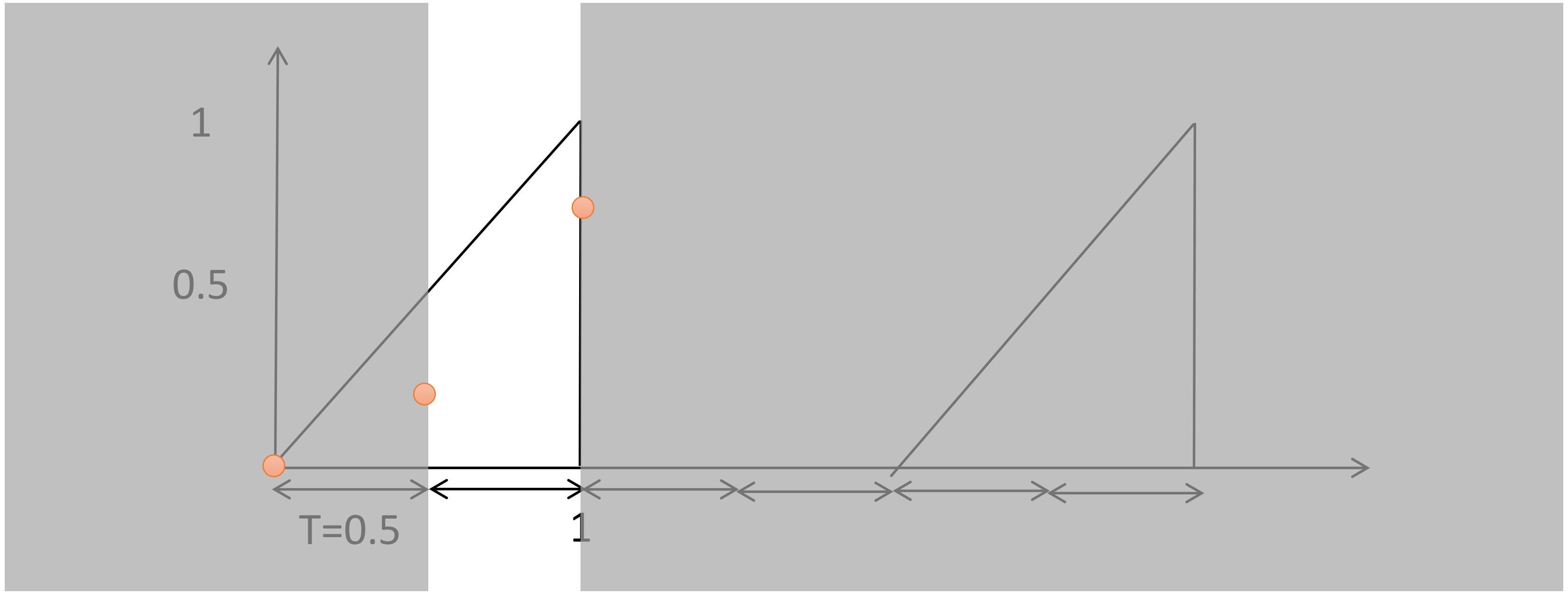

$$\hat{X}(1) = \frac{1}{0.5}(0.5 \cdot 0.5 + \frac{0.5 \cdot 0.5}{2}) = 0.75$$

Moving Average Filter

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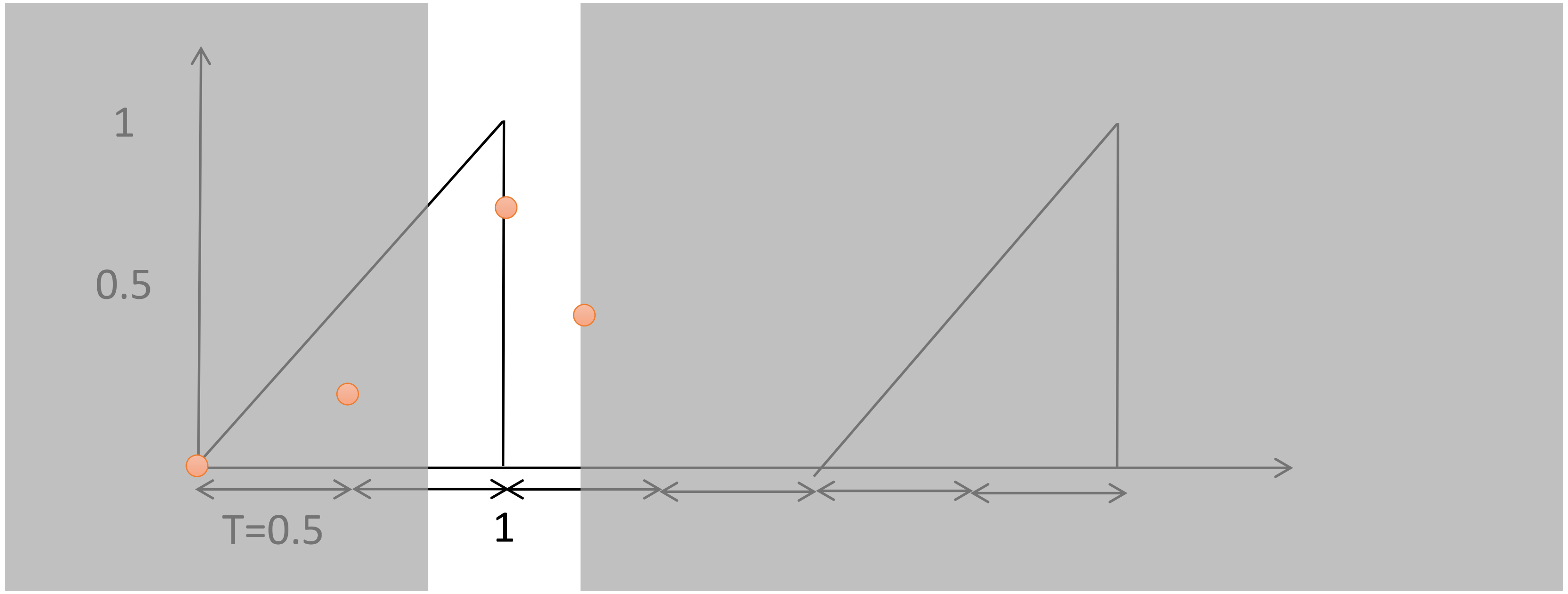
- Consider the following signal X(t) with period 4T. Into what signal is X(t) transformed if it passes through a moving average filter with Tc = T?



$$\hat{X}(1.25) = \frac{1}{0.5}(0.25 \cdot 0.75 + \frac{0.25 \cdot 0.25}{2}) = 0.\ 0.4375$$

Moving Average Filter

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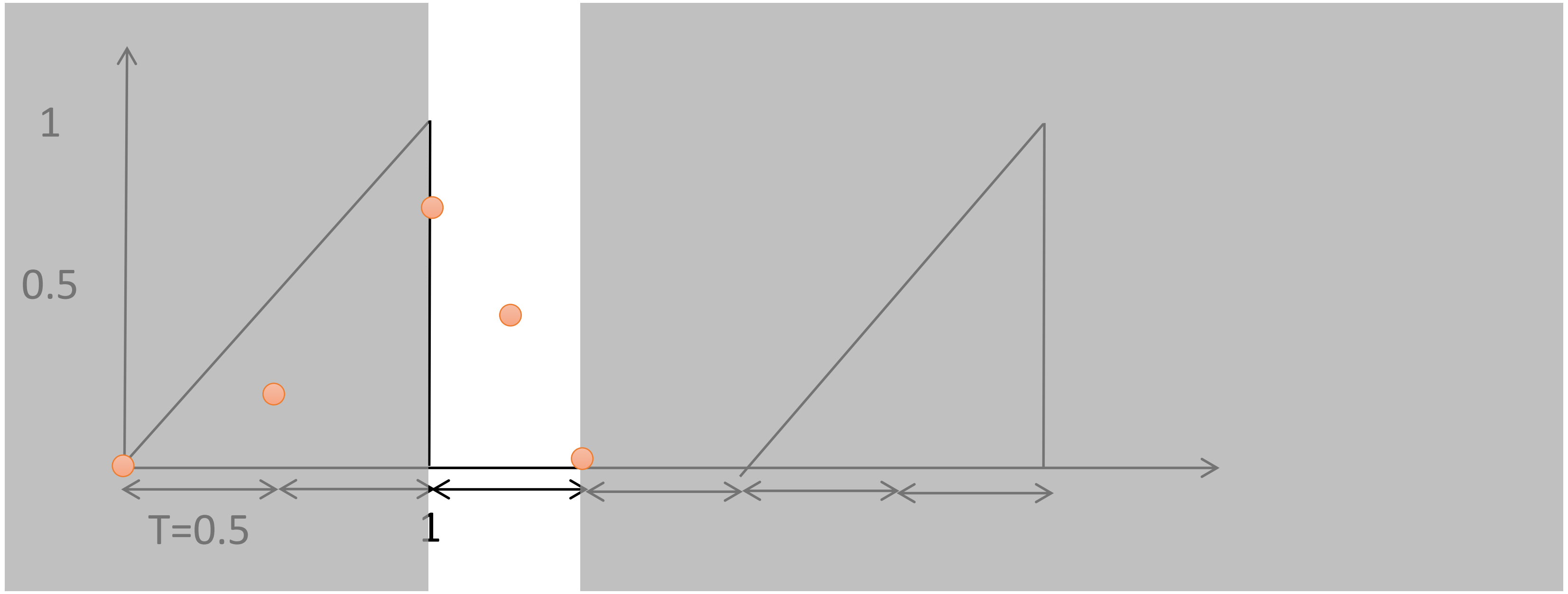

$\hat{X}(1.5)$ =0

Moving Average Filter

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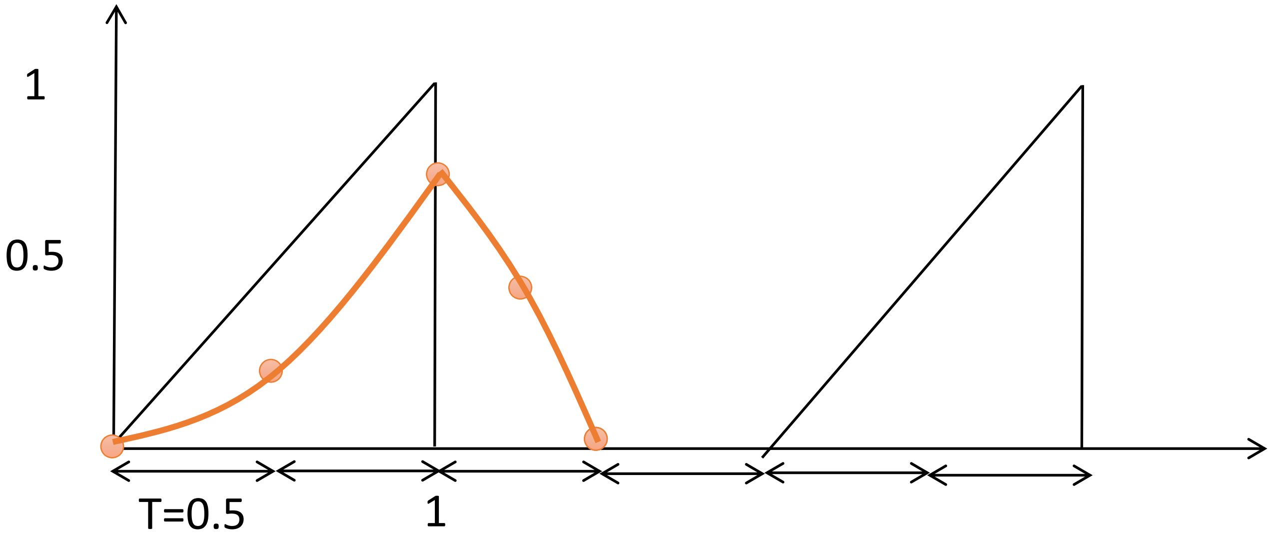

Moving Average Filter

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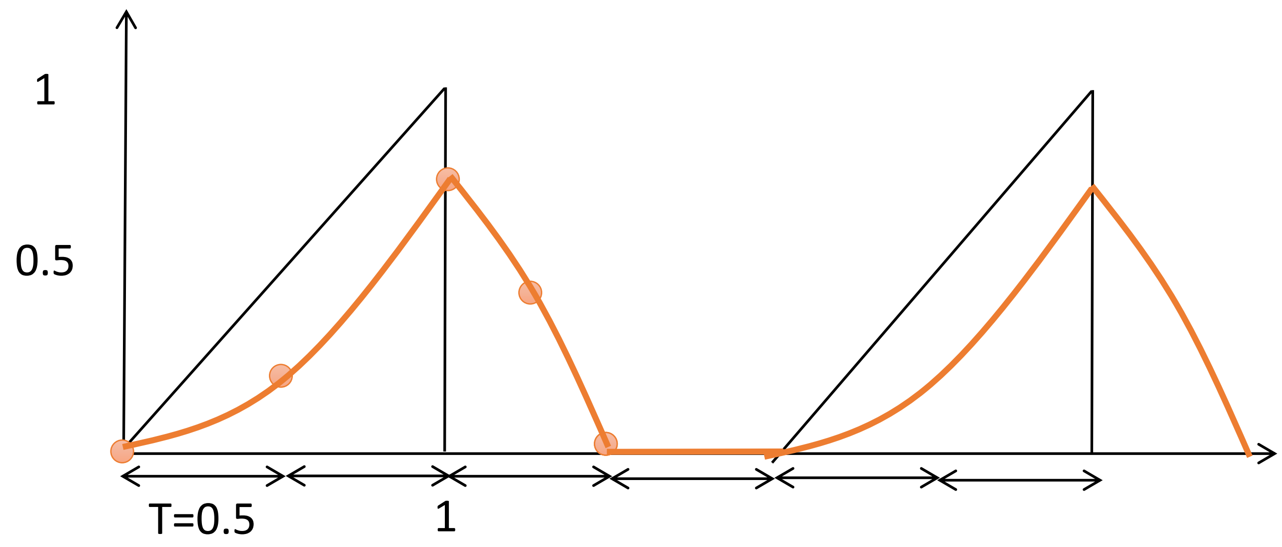
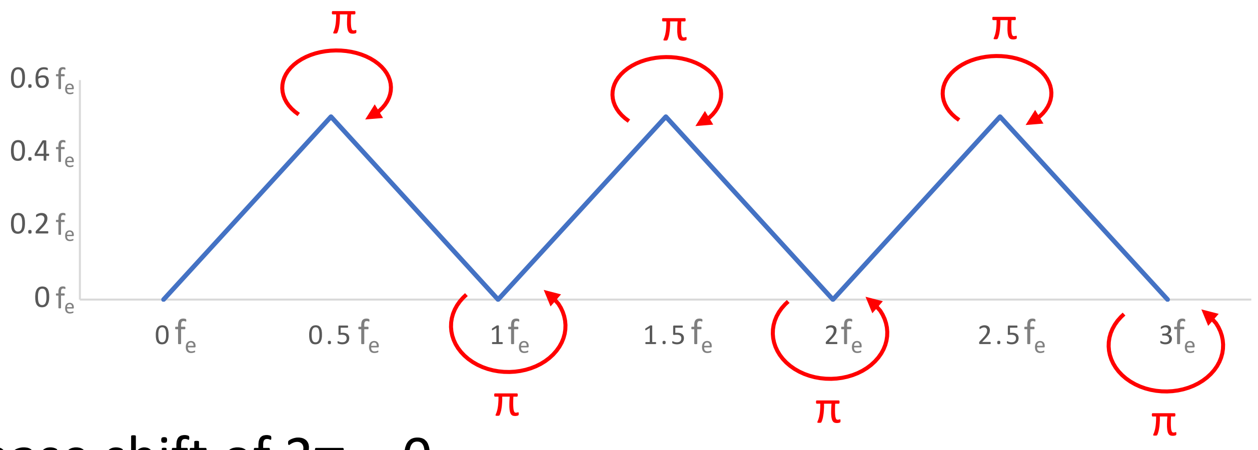


Computation of Apparent Frequency

- Inputs:
  - input frequency f
  - sampling frequency fe
- Output: apparent frequency fa



- Note: If we increase f by fe, we get a phase shift of 2π = 0.
- Algorithm for computing the apparent frequency:
  - Choose an n and compute $f' = f - n \cdot fe$ s.t. $0 \leq f' < fe$
  - If $f' < 0.5fe$, then $fa = f'$ and the phase shift is 0
  - If $f' > 0.5fe$, then $fa = fe - f'$ and the phase shift is π
- Example: fe = 1 Hz, f = 2.6 Hz
  - n = f / fe = 2.6 / 1 = 2 times (where / is an integer division)
  - $f' = 2.6 – 2 \cdot 1 = 0.6$ Hz
  - Since $f' > 0.5$ Hz, fa = 0.4 Hz and delta = π

$$H(X) = \sum_{i=1}^{n} p_i \cdot \log_2 \frac{1}{p_i} = p_1 \cdot \log_2 \frac{1}{p_1} + \cdots + p_n \cdot \log_2 \frac{1}{p_n}$$

Comparing the Entropy of Words

Question 10 (Mid-term exam 2022): $\log_2(10) = 3.3219$, $\log_2(5) = 2.3219$, and $\log_2(2.5) = 1.3219$.

Words: motivation, appreciate, kinikkinik, whizzbangs (10 letters each)

1. motivation: appearance profile 2,2,2,1,1,1,1
   $p_1=p_2=p_3= 2/10$
   $p_4=p_5=p_6=p_7= 1/10$
2. appreciate: 2,2,2,1,1,1,1 => same entropy as motivation
3. kinikkinik: 4,4,2 => $p_1=p_2= 4/10$, $p_3= 2/10$
4. whizzbangs: 2,1,1,1,1,1,1,1,1 => $p_1= 2/10$, $p_2=p_3=p_4=p_5=p_6=p_7=p_8= 1/10$

$$H(X3) = 2 \cdot \frac{4}{10} \cdot \log_2 \frac{10}{4} + \frac{2}{10} \cdot \log_2 \frac{10}{2}$$

$<$ or $>$ or $=$

$$H(X4) = \frac{2}{10} \cdot \log_2 \frac{10}{2} + 8 \cdot \frac{1}{10} \cdot \log_2 \frac{10}{1}$$

$$\cancel{0.8} \cdot \log_2 \frac{5}{2} + \cancel{0.2 \cdot \log_2 5} < \cancel{0.2 \cdot \log_2 5} + \cancel{0.8} \cdot \log_2 10$$

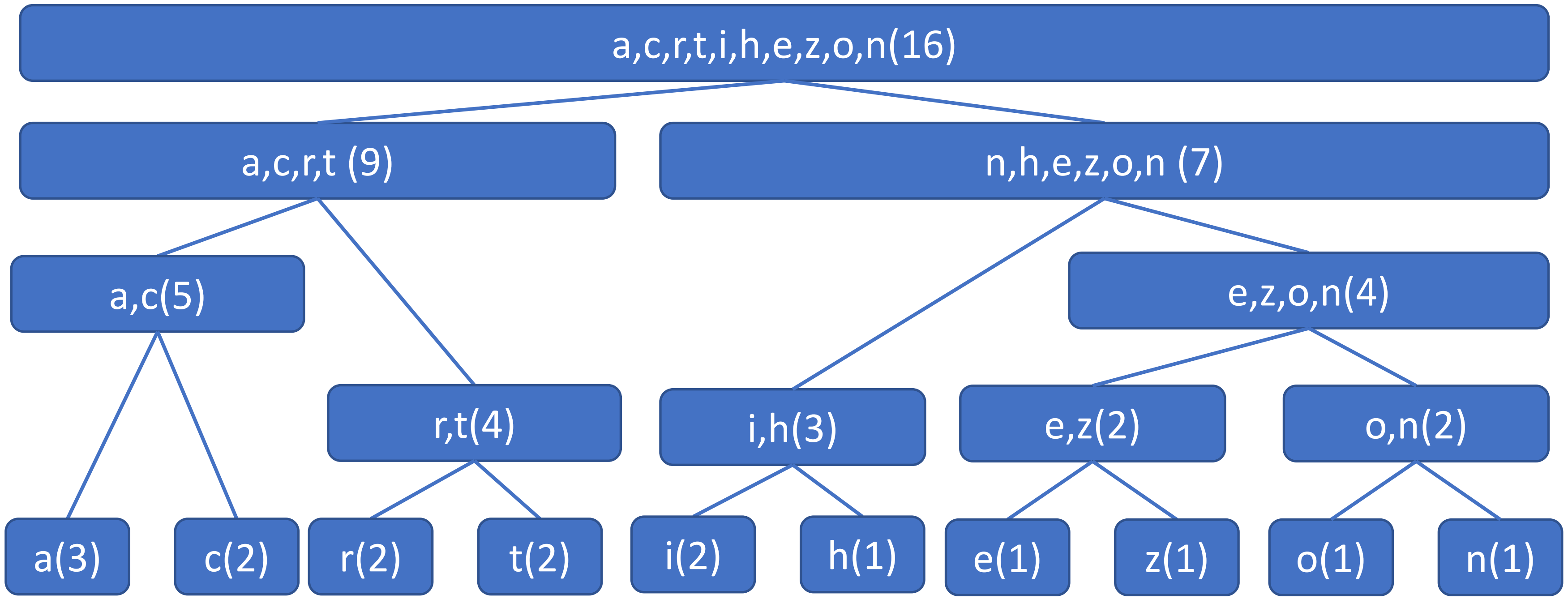
$-0.2 \cdot \log_2 5$ on both sides
$/0.8$ on both sides

$$\log_2 \frac{5}{2} < \log_2 10$$

Huffman Algorithm

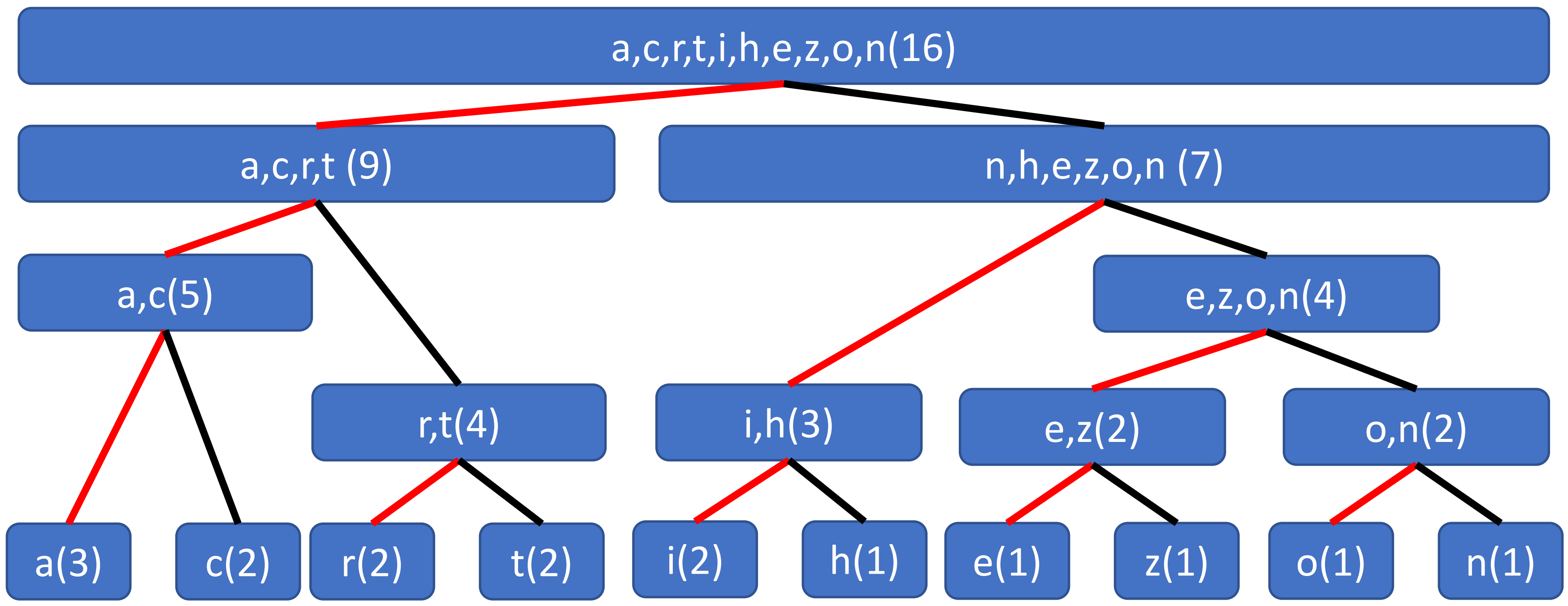
- Example: Characterization (16 letter)
- Appearance profile:

a	c	r	t	i	h	e	z	o	n
3	2	2	2	2	1	1	1	1	1



Generate Prefix-free Code

- Given a tree for each node assign a 1 to one edge and a 0 to the other (which edge is 0 and which is 1 does not matter).
- In example: red edges are 1, black edges are 0
- Read code from top to bottom!



Letter	Code
a	111
c	110
r	101
t	100
i	011
h	010
e	0011
z	0010
o	0001
n	0000