ICC SSV Theory Mid-Term Exam

Nov 2, 2024

ATTENTION: solutions are visible in the document
Version: November 2, 2024

Question 1. Binary System

A. 2^2

B. $2^2 - 1$

C. 2^{16}

D. $2^{16} - 1$

C. A byte has 8 bits, so 2^{16}.

Question 2. Positive Integer Representation

A. 10001100

B. 10011000

C. 01000110

D. 01100001

A. $2^2 + 2^3 + 2^7 = 4 + 8 + 128 = 140$

Question 3. Representation of Signed Numbers

A. 011111

B. 100000

C. 110000

D. 111111

B. 100000, which corresponds to -32 (in decimal).

Question 4. Binary Arithmetic with Signed Numbers

A. $10000 + 00010 = 10010$

B. $10001 - 01111 = 00010$

C. $00101 \cdot 00011 = 10111$

D. $01111 \div 00010 = 00111$

A. is correct
B. is correct
C. is incorrect
D. is correct.

Question 5. Fixed-Point Representation

A. incorrect: $1111.111_b = 8 + 4 + 2 + 1 + 1/2 + 1/4 + 1/8 = 15.875_d$
B. correct $6.25_d = 110.01_b = 4 + 2 + 1/4 = 6.25_d$
C. correct: the smallest distance if $0.001_b = 1/8_d$
D. incorrect: there are 7 digits, to we can represent $2^7 = 128$ numbers
E. correct: $8.874_d = 1000.1101.._b$, and $1000.110_b = 8.75_d$

Question 6. Floating Point Representation

Exponent range: -16 to +15
A. incorrect: only in the intervals with numbers smaller than 2 the distance is bound by 0.5, otherwise the distance is larger, e.g., in the interval [2,4], we can represent only the number 2 and 3 precisely, so the distance is 1.
B. correct. In any interval $[2^e, 2^{e+2})$ with e ranging from -16 to +15, we can represent only two values precisely.
C. incorrect: the smallest number is 2^{-16}
D. correct, 30.456 is in the interval [16,32), in which only the numbers 16 and 24 can be represented precisely.
E. correct: 001101 corresponds to an exponent of $00110 = 6_d$ and a mantissa of 1, so we get $1.1 * 2^6 = 1100000 = 2^6 + 2^5 = 96$.

Question 7. Signals

A. correct: every periodic signal can be represented by a sum of sinusoids.
B. correct: the signal repeats after 1s.
C. incorrect: there is at least on higher frequency in the signals that is responsible for the little wiggles.
D. incorrect: one needs to use a low-pass and not a high-pass filter.

Question 8. Sampling

B. we sample every $t_e = 1/f_e = 1/4 = 0.25s$
n=0: t= 0.0000 and y=0.0000
n=1: t= 0.2500 and y=0.6947
n=2: t= 0.5000 and y=1.5142
n=3: t= 0.7500 and y=1.7770

Question 9. Sampling and Reconstruction

A, C, und D. The signal $X(t)$ has frequencies 3, 4, and 6 Hz, so its bandwidth is 6 Hz. If $f_e > 12$, we can perfectly reconstruct it, so $X_I(t) = X(t)$ (A) and the bandwidth of the two signals $X(t)$ and $X_I(t)$ are the same (D). If $f_e = 10$, the frequency 6 Hz is subsampled and we get an apparent frequency of 4 Hz ($f_e - f = 10 - 6 = 4$). The amplitude doesn't change. Since we already have a sinusoid with 4 Hz with amplitude 4, the total amplitude is 4+3=7 for the 4 Hz sinsoid (C).

Question 10. Compression

A and C are correct.

Question 11. Entropy

$w_1 = \texttt{distance}$

$w_2 = \texttt{efficacy}$

$w_3 = \texttt{sentence}$

$w_4 = \texttt{mountain}$

The correct order is $H(w_3) < H(w_2) < H(w_4) < H(w_1)$.
The word w_1 has the highest entropy because all letters are different. The maximal entropy of a word with 8 letters is $\log_2(8) = 3 = H(w_1)$. The entropy of w_2 and w_4 are easy to compute.
$H(w_2) = 2 \cdot \frac{2}{8} \cdot log_2(\frac{8}{2}) + 4 \cdot \frac{1}{8} \cdot log_2(8) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 3 = 2.5$
$H(w_4) = \frac{2}{8} \cdot log_2(\frac{8}{2}) + 6 \cdot \frac{1}{8} \cdot log_2(8) = \frac{1}{4} \cdot 2 + \frac{3}{4} \cdot 3 = \frac{11}{4} = 2.75$
So, $H(w_2) < H(w_4) < H(w_1)$. Looking at the appearance profiles of w_2 and w_4, which are $2, 2, 1, 1, 1, 1$ and $2, 1, 1, 1, 1, 1, 1$, it is also easy to see that w_2 has a lower entropy. The word w_3 has the most similar letters, namely 3 appearances of "e", and 2 appearances of "n", so it has the lowest entropy. ($H(w_3) = \frac{3}{8} \cdot log_2(\frac{8}{3}) + \frac{2}{8} \cdot log_2(\frac{8}{2}) + 3 \cdot \frac{1}{8} \cdot log_2(8) = \frac{3}{8} \cdot (3 - \log_2(3)) + \frac{4}{8} + \frac{9}{8} \approx \frac{3}{8} \cdot \frac{9-5}{3} + \frac{13}{8} = \frac{17}{8} = 2.125$)

Question 12. Frequency

$sin(-2\pi ft) = -sin(2\pi ft) = sin(2\pi ft + \pi)$, so $f = 440$ Hz and $\delta = \pi$.

Question 13. Moving Average Filter

The solution is shown in Figure 1.

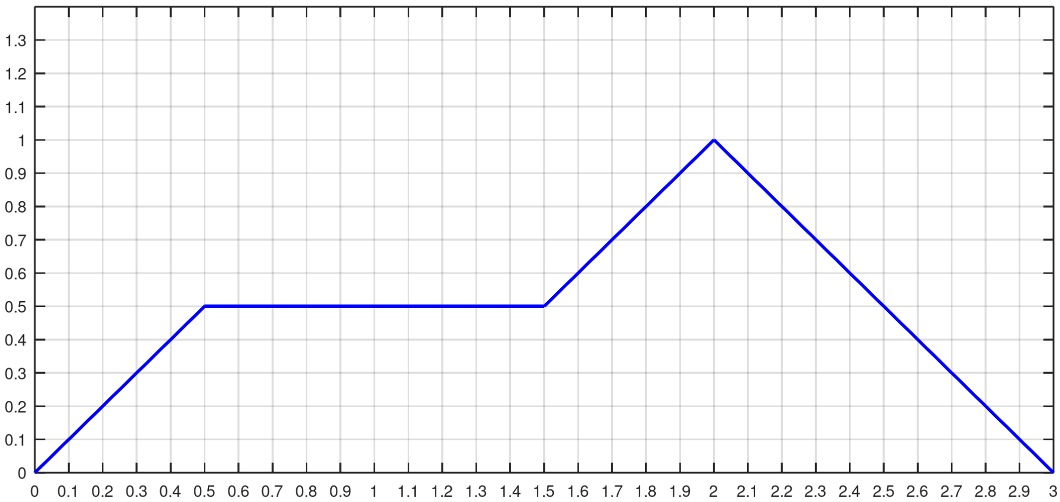

Figure 1: Output signal $\hat{X}(t)$ of moving average filter

Huffman Encoding

Question 14. Huffman Tree and Code

We create the huffman tree shown in Figure 2, which leads to the following code:

Letter	E	I	T	N	A	S	R	O
Appearances	8	8	4	4	4	2	1	1
Code word	11	10	011	010	001	0001	00001	00000
Code length	2	2	3	3	3	4	5	5

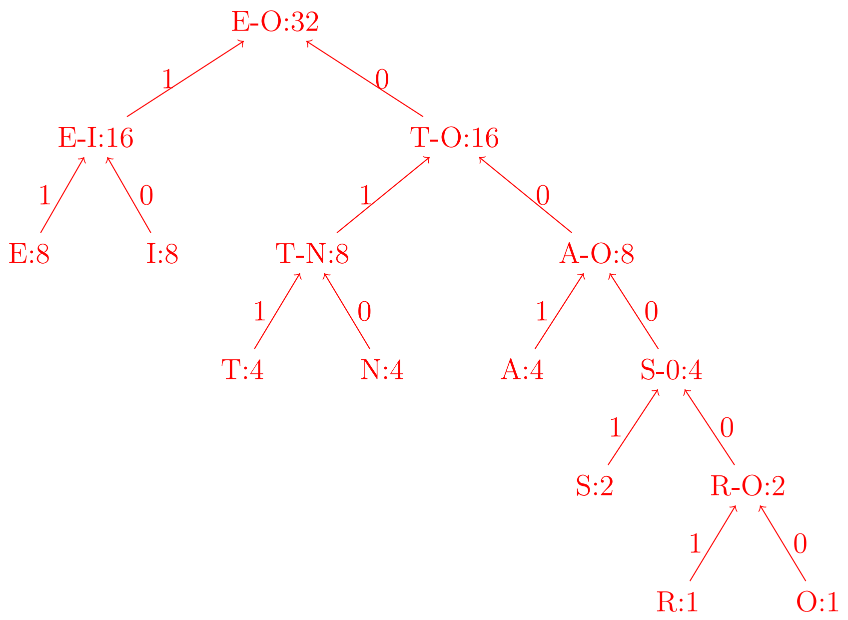

Figure 2: One correct Huffman tree. There are multiple correct trees.

Question 15. Message Length

The total number of bits is $16 \cdot 2$ bits $+ \; 12 \cdot 3$ bits $+ \; 2 \cdot 4$ bits $+ \; 2 \cdot 5$ bits $= 32 + 36 + 8 + 10 = 86$ bits.

Question 16. Optimality

We need to compute the entropy of the message and show that the average code length of our code is equal to the entropy. The average code length of our code is $L = \frac{86}{32} = \frac{43}{16}$. The entropy is

$$H = 2 \cdot \frac{8}{32} \cdot \log_2(\frac{32}{8}) + 3 \cdot \frac{4}{32} \cdot \log_2(\frac{32}{4}) + \frac{2}{32} \cdot \log_2(\frac{32}{2}) + 2 \cdot \frac{1}{32} \cdot \log_2(\frac{32}{1}) =$$
$$\frac{1}{2} \cdot \log_2(4) + \frac{6}{16} \cdot \log_2(8) + \frac{1}{16} \cdot \log_2(16) + \frac{1}{16} \cdot \log_2(32) =$$
$$\frac{1}{2} \cdot 2 + \frac{6}{16} \cdot 3 + \frac{1}{16} \cdot 4 + \frac{1}{16} \cdot 5 = \frac{16}{16} + \frac{18}{16} + \frac{4}{16} + \frac{5}{16} = \frac{43}{16}.$$

Since $H = L$ our code is optimal.