ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 12 Modern Digital Communications
Midterm Solutions November 8, 2023

PROBLEM 1. 12 points (Paper and Pencil)

1.
$$s_{\mathcal{F}}(f) = \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft}dt.$$

2.
$$\begin{aligned} A_k &= \frac{1}{T_p}\int_0^{T_p} \tilde{s}(t)e^{-j\frac{2\pi}{T_p}kt}dt \\ &= \frac{1}{T_p}\int_0^{T_p} s(t)e^{-j\frac{2\pi}{T_p}kt}dt \\ &= \frac{1}{T_p}\int_{-\infty}^{\infty} s(t)e^{-j\frac{2\pi}{T_p}kt}dt \\ &= \frac{1}{T_p}s_{\mathcal{F}}\left(\frac{k}{T_p}\right). \end{aligned}$$

   The second equality follows from the fact that $\tilde{s}(t) = s(t)$ for $t \in [0, T_p]$ and the third one from the fact that $s(t) = 0$ for $t \notin [0, T_p]$.

3. Using the Fourier transform relationship seen in class, $AB1_{|t|\leq\frac{1}{2B}}(t) \overset{\mathcal{F}}{\longleftrightarrow} A\,\mathrm{sinc}(\frac{f}{B})$, we can quickly derive that $x_{\mathcal{F}}(f) = T_p\,\mathrm{sinc}(T_p f)$.

4.
$$\hat{x}_{\mathcal{F}}(f) = e^{-j2\pi f\frac{T_p}{2}}x_{\mathcal{F}}(f) = e^{-j\pi fT_p}T_p\,\mathrm{sinc}(T_p f).$$

5.
$$s(t) = \tilde{s}(t)\hat{x}(t).$$

6. $\tilde{s}(t) = \sum_{k\in\mathbb{Z}} A_k e^{j\frac{2\pi}{T_p}kt}$. Hence,
$$s(t) = \tilde{s}(t)\hat{x}(t) = \sum_{k\in\mathbb{Z}} A_k e^{j\frac{2\pi}{T_p}kt}\hat{x}(t).$$

   So $s_{\mathcal{F}}(f) = \sum_{k\in\mathbb{Z}} A_k\hat{x}_{\mathcal{F}}(f - \frac{k}{T_p}) = \sum_{k\in\mathbb{Z}} A_k T_p\,\mathrm{sinc}(T_p(f - \frac{k}{T_p}))e^{-j\pi T_p(f-\frac{k}{T_p})}$.