ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 11 Modern Digital Communications
Midterm Exam November 8, 2023

Name:

Note:

- You have 2 hours to work on the exam.
- The exam is closed book, but you are allowed one sheet (one single-sided A4 page) of handwritten notes. Resources from the internet as well as code written outside this exam are not allowed (unless the code is written on the sheet of handwritten notes).
- The code will be evaluated according to the usual criteria, namely correctness, speed, form, and readability. Short comments that allow us to follow what you are doing will improve readability.
- The problems can be solved in any order.
- You will upload (to Moodle) your solution to the problems that require writing MATLAB or Python code. Do so in a single archive.

To get started with the exam, do the following:

1. Close all the windows and programs on your laptop.
2. Launch the MATLAB/Python editor and close all the tabs (previously written code).
3. From Moodle, download the file `mdc_midterm_2023.zip`. Unzip the file to create the directory `mdc_midterm_2023`. For the rest of the exam you are required to work inside that directory. The MATLAB/Python files for Problem n, are found in subfolder pn.
4. Turn your WiFi off until you are ready to upload your solutions.
5. Wait until you receive the go-ahead signal.

Problem 1. 12 points (Paper and Pencil)

Let $s(t)$ be a finite-duration signal, and $\tilde{s}(t) := \sum_{i\in\mathbb{Z}} s(t - iT_p)$ the periodic extension (period T_p) of $s(t)$. We assume that the support of $s(t)$ is contained in $[0, T_p]$ and $s(T_p) = 0$.

1. Write down the expression of $s_{\mathcal{F}}(f)$, the Fourier transform of $s(t)$.
2. Recall that the Fourier series coefficients A_k are expressed as:

$$A_k = \frac{1}{T_p}\int_0^{T_p} \tilde{s}(t)e^{-j\frac{2\pi}{T_p}kt}dt, \;\; k \in \mathbb{Z}.$$

   Derive the expression of A_k as a function of $s_{\mathcal{F}}()$.
3. Let

$$x(t) = \begin{cases} 1, & |t| \leq \frac{T_p}{2} \\ 0, & \text{otherwise.} \end{cases}$$

   Write down the expression of $x_{\mathcal{F}}(f)$, the Fourier transform of $x(t)$.
4. Derive the expression of $\hat{x}_{\mathcal{F}}(f)$, the Fourier transform of $\hat{x}(t) = x(t - \frac{T_p}{2})$.
5. Express $s(t)$ as a function of $\tilde{s}(t)$ and $\hat{x}(t)$.
6. Derive the expression of $s_{\mathcal{F}}(f)$ as a function of $\{A_k\}_{k\in\mathbb{Z}}$.

Problem 2. 14 points (MATLAB/Python)

The file `rx_samples.mat` contains the samples of a signal at the receiver. Somewhere in the corresponding transmitted signal there is inserted a preamble, followed by 1000 BPSK data symbols. The symbols of the preamble are stored into the file `preamble.mat`. The pulse shaping at the transmitter consists of a rectangular pulse of amplitude 1, and the duration of a symbol is equal to the duration of the rectangular pulse. The pulse shaping filter has used SPS = 5 samples per symbol. Due to a malfunction, at the transmitter, all the samples of the preamble were rotated by a certain phase ϕ_0 (constant over the duration of the preamble). The channel rotates the transmitted samples with a certain phase ϕ_1 (constant over the duration of the transmission) and then adds white Gaussian noise. There is no Doppler. We ask you to perform the following tasks:

- Find the start of the preamble within the received samples.
- Process the appropriate portion of the received samples in order to generate the sufficient statistics about the transmitted data symbols.
- Perform differential decoding in order to estimate the transmitted symbols.
- Compute the symbol error rate with the help of the file `tx_data_symbols.mat` which contains the transmitted BPSK data symbols.

Please follow the instructions in the script `p2.m[py]`.

Problem 3. 14 points (MATLAB/Python)

The file rx_samples.mat contains the samples of a signal at the receiver. We have transmitted 100 BPSK data symbols by modulating a certain pulse with the values of the symbols (± 1). The pulses are formed by repeating 10 times the pa_sequence contained in the file pa_sequence.mat. So the duration of a transmitted symbol is 10 times the duration of the pa_sequence. The channel rotates the transmitted samples with a certain phase ϕ_2 (constant over the duration of the transmission) and then adds white Gaussian noise. The channel also adds some extra samples at the beginning and at the end of the transmitted data samples. There is no Doppler. We ask you to perform the following tasks:

- Detect the beginning of the first full pa_sequence in the received samples. You should use the minimum amount of received samples while performing this task.
- Detect the beginning of the first full symbol in the received samples. There are various ways to perform this task, but we specifically ask you to do this by means of detecting a phase change of roughly π between two inner products of appropriately chosen consecutive chunks of received samples with the pa_sequence. You have seen this procedure in class.
- Extract the portion of the received samples corresponding to full data symbols and process them in order to generate the sufficient statistics about the transmitted data symbols.
- Using the fact that the first transmitted symbol was a $+1$, estimate the phase ϕ_2 introduced by the channel.
- Correct for the phase rotation introduced by the channel, estimate the transmitted data symbols and compute the symbol error rate (the file tx_data_symbols.mat contains the transmitted BPSK data symbols).

Please follow the instructions in the script p3.m[py].