ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 12 Modern Digital Communications
Midterm Solutions November 10, 2021

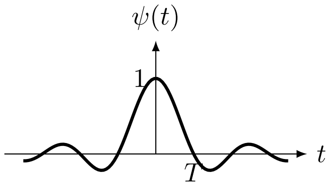
PROBLEM 1. 13 points (Paper and Pencil)

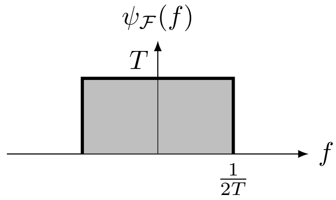
1. If we choose $\psi(t) = \operatorname{sinc}\left(\frac{t}{T}\right)$, we obtain

$$s(t) = \sum_k s_k \operatorname{sinc}\left(\frac{t-kT}{T}\right).$$

According to the sampling theorem, we can reconstruct $s(t)$ from the values $s_k = s(kT)$. One can choose as well $\psi(t) = \frac{1}{\sqrt{T}} \operatorname{sinc}\left(\frac{t}{T}\right)$ such that $\|\psi(t)\|^2 = 1$. In this case, the samples are scaled accordingly.

2. Using the Fourier transform relationship $A \operatorname{sinc}(\frac{t}{B}) \stackrel{\mathcal{F}}{\longleftrightarrow} AB1_{|f| \le \frac{1}{2B}}(f)$, we get $\psi_{\mathcal{F}}(f) = T1_{|f|\le\frac{1}{2T}}(f)$.





3. $s_{\mathcal{F}}(f) = 0$ for $f \notin \left[-\frac{1}{2T}, \frac{1}{2T}\right]$. So $f_{min} = -\frac{1}{2T} = -f_{max}$.

4. $$\operatorname{l.i.m.} \sum_{k=-\infty}^{\infty} \left|\psi_{\mathcal{F}}\Big(f - \frac{k}{T}\Big)\right|^2 = T^2, \qquad f \in \mathbb{R}.$$

So the Nyquist criterion is fulfilled up to a scaling factor (of magnitude T).

5. The main advantage is that the pulse $\psi(t)$ is the minimum-bandwidth Nyquist pulse (for a given symbol period T). The main drawback comes from the fact that $\psi(t)$ has infinite-length tails, which decay very slowly. In practice, one has to truncate these tails, but this results into an increased ISI and sensitivity to the sampling-time offset.

6. $h_{\mathrm{MF}}(t) = \psi^*(-t) = \operatorname{sinc}\left(\frac{t}{T}\right)$. One can choose as well $h_{\mathrm{MF}}(t) = \frac{1}{T} \operatorname{sinc}\left(\frac{t}{T}\right)$ such that $Y_k = s_k$ (in the absence of noise); or $h_{\mathrm{MF}}(t) = \frac{1}{\sqrt{T}} \operatorname{sinc}\left(\frac{t}{T}\right)$ such that $\|h_{\mathrm{MF}}\|^2 = 1$.

7. The noise variance at the output of the matched filter is $\|h_{\mathrm{MF}}\|^2 N_0$. So it is TN_0 if we choose $h_{\mathrm{MF}}(t) = \operatorname{sinc}\left(\frac{t}{T}\right)$, and $\frac{N_0}{T}$ if we choose $h_{\mathrm{MF}}(t) = \frac{1}{T} \operatorname{sinc}\left(\frac{t}{T}\right)$. If $h_{\mathrm{MF}}(t) = \frac{1}{\sqrt{T}} \operatorname{sinc}\left(\frac{t}{T}\right)$, the noise variance remains unchanged.

8. If we use `fftshift`, the frequency axis labelling is

$$\left[-\frac{1}{2T_s}, -\frac{1}{2T_s} + \frac{1}{100T}, \ldots, -\frac{1}{100T}, 0, \frac{1}{100T}, \ldots, \frac{1}{2T_s} - \frac{1}{100T}\right].$$

Otherwise, the first and second halfs of the vector above are swapped.