

PROBLEM 3. (Paper and Pencil Part) (6 points)

1. (1 point) The total distance traveled by the signal at time T_d is cT_d . At time $\frac{T_d}{2}$ it hits the object. At this time the object is at distance $c\frac{T_d}{2}$. So, the answer is $\frac{T_d}{2}$.
2. (3 points) We are following the hint: The carrier frequency is f_c . For a receiver on the moving object, the center frequency is $f_r = f_c(1 - \nu)$, where $\nu = \frac{v}{c}$ (see GPS notes). If the moving object sends a signal at center frequency f_r , you receive it at center frequency

$$f_r(1 - \nu) = f_c(1 - \nu)^2 = f_c(1 - 2\nu + \nu^2) \approx f_c(1 - 2\nu).$$

So, the Doppler frequency is $f_d \approx -2\nu f_c$.

3. (2 points)

$$x(t) = v \left(t - \frac{T_d}{2} \right) + x \left(\frac{T_d}{2} \right) = v \left(t - \frac{T_d}{2} \right) + c \frac{T_d}{2},$$

where $v \approx -\frac{cf_d}{2f_c}$.