

PROBLEM 1. (Paper and Pencil)

1.

$$\mathbf{s}_{\mathcal{F}}[k] = \begin{cases} \frac{1}{T_s} s_{\mathcal{F}}\left(\frac{k}{T_p}\right), & k = 0, 1, \dots, \frac{N}{2} - 1 \\ \frac{1}{T_s} s_{\mathcal{F}}\left(\frac{k-N}{T_p}\right), & k = \frac{N}{2}, \dots, N-1. \end{cases}$$

2.

$$\mathbf{u}_{\mathcal{F}}[k] = \begin{cases} \frac{2}{T_s} s_{\mathcal{F}}\left(\frac{k}{T_p}\right), & k = 0, 1, \dots, N-1 \\ \frac{2}{T_s} s_{\mathcal{F}}\left(\frac{k-2N}{T_p}\right), & k = N, \dots, 2N-1. \end{cases}$$

Notice that  $s_{\mathcal{F}}\left(\frac{k}{T_p}\right)$  vanishes when  $k$  is bigger than  $N/2$  or smaller than  $-N/2$ . Comparing the expressions of  $\mathbf{u}_{\mathcal{F}}[k]$  and  $\mathbf{s}_{\mathcal{F}}[k]$  we see that

$$\mathbf{u}_{\mathcal{F}}[k] = \begin{cases} 2\mathbf{s}_{\mathcal{F}}[k], & k = 0, 1, \dots, \frac{N}{2} - 1 \\ 0, & k = \frac{N}{2}, \dots, N-1 \\ 0, & k = N, \dots, \frac{3N}{2} - 1 \\ 2\mathbf{s}_{\mathcal{F}}[k-N], & k = \frac{3N}{2}, \dots, 2N-1. \end{cases}$$

To say in words, the expression above tells us that we can obtain  $\mathbf{u}_{\mathcal{F}}$  by simply inserting  $N$  zeros in the middle of  $2\mathbf{s}_{\mathcal{F}}$ .

3. 

```
function uf = upsampleBy2(sf)
    N = length(sf);
    uf = [sf(1:N/2), zeros(1,N), sf(N/2+1:N)];
end
```