ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 12 Modern Digital Communications
Midterm Solutions November 8, 2017

PROBLEM 1. (Paper and Pencil)

1. `TBD1 = up`, `TBD2 = up`.
2. In the absence of channel noise, the alphabet of `suffStat` is $\pm\sqrt{E_s}\|p\|^2 = \pm\sqrt{E_s}\cdot$`up`.
3. `TBD3`$\cdot\|p\|^2 = 1$, so `TBD3` $= \frac{1}{\|p\|^2} =$ `1/up`.

PROBLEM 4. (Paper and Pencil / MATLAB)

1. Since the conditions of the sampling theorem are fulfilled, we can write $p(t)$ as

$$p(t) = \sum_{i=0}^{L-1} p(iT_s)\,\mathrm{sinc}\left(\frac{t}{T_s} - i\right).$$

It follows that

$$\begin{aligned} p\left(n\frac{T_s}{k}\right) &= \sum_{i=0}^{L-1} p(iT_s)\,\mathrm{sinc}\left(\frac{nT_s}{kT_s} - i\right) \\ &= \sum_{i=0}^{L-1} p(iT_s)\,\mathrm{sinc}\left(\frac{n-ki}{k}\right). \end{aligned}$$

2.

$$\begin{aligned} p\left(n\frac{T_s}{k}\right) &= \sum_{i=0}^{kL-1} \hat{p}[i]\,\mathrm{sinc}\left(\frac{n-i}{k}\right) \\ &= (\hat{p} \star b)[n] \end{aligned} \tag{1}$$

where $\hat{p}[n]$ is the upsampled version of the initial sequence p with upsampling factor k, i.e.,

$$\hat{p}[n] \coloneqq \begin{cases} p[n/k] & \text{if } n/k \text{ is an integer,} \\ 0 & \text{otherwise,} \end{cases}$$

and $b[n] = \mathrm{sinc}\left(\frac{n}{k}\right)$.

3. `a` $= \hat{p}$ and `b` as above truncated to some length, and with the peak of the sinc in the middle of `b` so as to contain both tails of the sinc.

4. If one tail of the sinc has length l, then the first l components of `y` should be discarded. Hence, `ind` should be $l + 1$.

5. Consider the formula given in Equation 1. To find where we can truncate the left tail of the sinc, we set $n = 0$ and choose the largest value of i for which $\hat{p}[i]$ is non-zero, namely $i = k(L-1)$. But $\mathrm{sinc}\left(-\frac{k(L-1)}{k}\right) = \mathrm{sinc}\,(-L+1) = 0$, so we need only $k(L-1)-1$ points of the left tail. The length of the right tail is obtained by setting $i = 0$ and by taking the maximum value of n, which is $kL-1$. All together we need the two tails and the center of the sinc, which amounts to a total of $2kL - k - 1$ positions.