ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 22 Modern Digital Communications

Final Exam December 18, 2019

Name:

Note:

- You have 1h45 min to work on the exam.
- The exam is closed book, but you are allowed one double-sided A4 page of handwritten notes and the printed reference card. Resources from the internet as well as code written outside this exam are not allowed (unless the code is written on the sheet of handwritten notes).
- The code will be evaluated according to the usual criteria, namely correctness, speed, form, and readability. Short comments that allow us to follow what you are doing will improve readability.
- The problems can be solved in any order.
- You will upload (to Moodle) your solution to the problems that require writing MATLAB code. Do so in a single archive.

The following might be useful: the inverse of a 2×2 matrix is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}. \tag{1}$$

To get started with the exam, do the following:

1. Close all the windows and programs on your laptop.
2. Launch MATLAB and close all the tabs (previously written code).
3. From Moodle, download the file `mdc_final_2019.zip`. Unzip the file to create the directory `mdc_final_2019`. For the rest of the exam you are required to work inside that directory. The MATLAB files for Problem n, are found in subfolder pn.
4. Turn your WiFi off until you are ready to upload your solutions.
5. Wait until you receive the go-ahead signal.

Problem 1. 25 points (MATLAB)

This problem is about constructing a GPS page (matrix of size 300×5), starting from an initial matrix (also of size 300×5) filled with random bits. You find the step-by-step procedure in the file `pageConstruction.m` of directory `p1`. In that directory you also find our final result (named `pageOurSolution`), that you can use to check the correctness of your solution (see the end of the script).

The following table shows how to compute the bits of a word, including the parity bits. For the actual computation of the parity bits, we provide the matrix `H` (see `pageConstruction.m`).

Table 20-XIV. Parity Encoding Equations

D_1	=	$d_1 \oplus D_{30}^\star$
D_2	=	$d_2 \oplus D_{30}^\star$
D_3	=	$d_3 \oplus D_{30}^\star$
•		•
•		•
•		•
•		•
D_{24}	=	$d_{24} \oplus D_{30}^\star$
D_{25}	=	$D_{29}^\star \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_{10} \oplus d_{11} \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{17} \oplus d_{18} \oplus d_{20} \oplus d_{23}$
D_{26}	=	$D_{30}^\star \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_6 \oplus d_7 \oplus d_{11} \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{18} \oplus d_{19} \oplus d_{21} \oplus d_{24}$
D_{27}	=	$D_{29}^\star \oplus d_1 \oplus d_3 \oplus d_4 \oplus d_5 \oplus d_7 \oplus d_8 \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{19} \oplus d_{20} \oplus d_{22}$
D_{28}	=	$D_{30}^\star \oplus d_2 \oplus d_4 \oplus d_5 \oplus d_6 \oplus d_8 \oplus d_9 \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{17} \oplus d_{20} \oplus d_{21} \oplus d_{23}$
D_{29}	=	$D_{30}^\star \oplus d_1 \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_7 \oplus d_9 \oplus d_{10} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{17} \oplus d_{18} \oplus d_{21} \oplus d_{22} \oplus d_{24}$
D_{30}	=	$D_{29}^\star \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_8 \oplus d_9 \oplus d_{10} \oplus d_{11} \oplus d_{13} \oplus d_{15} \oplus d_{19} \oplus d_{22} \oplus d_{23} \oplus d_{24}$

Where

$d_1, d_2, \ldots, d_{24}$ are the source data bits;

the symbol $\star$ is used to identify the last 2 bits of the previous word of the subframe;

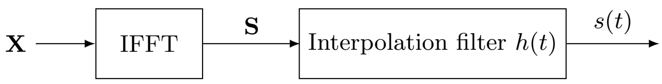
$D_{25}, D_{26}, \ldots, D_{30}$ are the computed parity bits;

$D_1, D_2, \ldots, D_{29}, D_{30}$ are the bits transmitted by the SV;

$\oplus$ is the "modulo-2" or "exclusive-or" operation.

PROBLEM 2. 20 points (Paper and Pencil / MATLAB)

Suppose that we want to use an OFDM-type of transmitter (see Figure)



to generate a BPSK (binary phase shift keying) signal of the form

$$s(t) = \sum_{i=0}^{9} b_i p(t - iT) \ ,$$

where $b_i \in \{\pm 1\}$ and

$$p(t) = \begin{cases} e^{j2\pi f_c t}, & \text{if } t \in [0, T) \\ 0, & \text{otherwise.} \end{cases}$$

The IFFT block size is $N = 1024$, the symbol time is $T = 10^{-6}$ [s], and the carrier frequency is $f_c = 200$ [MHz].

The input and output to the IFFT are $N \times 10$ matrices and the IFFT is taken column-wise. The elements of $\mathbf{S}$ are then serialized (column-wise as usual) and sent to the interpolation filter.

1. Determine the sampling time T_s.
2. Give the exact expression for the impulse response $h(t)$ of the interpolation filter.
3. Precisely specify the entries of the matrix $\mathbf{X}$ so as to obtain the desired $s(t)$.
4. In the OFDM setup, after the IFFT operation, we insert the cyclic prefix. Why is it not needed here?
5. Suppose you are given a standard OFDM transmitter (that comes with the insertion of the cyclic prefix). Is it possible to generate a BPSK signal with such a system? Explain.
6. Complete the template script `p2.m` to generate the matrices $\mathbf{X}$ and $\mathbf{S}$, then use the serialized entries of $\mathbf{S}$ to plot the absolute value of the Fourier transform of $s(t)$. No need to label the vertical axis, but do label the frequency axis appropriately.

Problem 3. 15 points (Paper and Pencil)

Consider a 2×2 MIMO system. Let $\mathbf{X} = (X_1, X_2)^T \in \mathbb{R}^2$ and $\mathbf{Y} = (Y_1, Y_2)^T \in \mathbb{R}^2$ be the transmitted and the received signal, respectively, and let the channel be modeled by

$$\begin{aligned} Y_1 &= X_1 + Z_1, \\ Y_2 &= X_1 + X_2 + Z_2, \end{aligned}$$

where $\mathbf{Z} = (Z_1, Z_2)^T \in \mathbb{R}^2$ is a zero-mean Gaussian random vector of covariance matrix $K_{\mathbf{Z}} = I$. (I is the 2×2 identity matrix.)

Suppose that $\mathbf{X}$ is zero-mean and Gaussian with covariance matrix $K_{\mathbf{X}} = I$ and is independent of $\mathbf{Z}$. Hence $\mathbf{X}$ and $\mathbf{Y}$ are independent, identically distributed, jointly Gaussian random vectors.

1. Derive the formula for the zero-forcing estimator (ZF) $\hat{\mathbf{X}}_{\mathrm{ZF}}(\mathbf{y}) = B\mathbf{y}$. Here B is the matrix for which

$$B\mathbf{Y} = \mathbf{X} + B\mathbf{Z}.$$

2. Derive an explicit expression for the minimum mean square error estimator $\hat{\mathbf{X}}_{\mathrm{MMSE}}(\mathbf{y})$.

3. Do the same for the linear minimum mean square error estimator $\hat{\mathbf{X}}_{\mathrm{LMMSE}}(\mathbf{y})$.

4. Derive the least squares (LS) approximation $\hat{\mathbf{X}}_{\mathrm{LS}}(\mathbf{y})$.