ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 22 | Modern Digital Communications
Final Exam | December 19, 2018

Name:

Note:

- You have 2 hours to work on the exam.
- The exam is closed book, but your are allowed one double-sided A4 page of handwritten notes. Resources from the internet as well as code written outside this exam are not allowed (unless the code is written on the sheet of handwritten notes).
- The code will be evaluated according to the usual criteria, namely correctness, speed, form, and readability. Short comments that allow us to follow what you are doing will improve readability.
- The problems can be solved in any order.
- You will upload (to Moodle) your solution to the problems that require writing MATLAB code. Do so in a single archive.

To get started with the exam, do the following:

1. Close all the windows and programs on your laptop.
2. Launch MATLAB and close all the tabs (previously written code).
3. From Moodle, download the file `mdc_final_2018.zip`. Unzip the file to create the directory `mdc_final_2018`. For the rest of the exam you are required to work inside that directory. The MATLAB files for Problem 3 are found in subfolder p3.
4. Turn your WiFi off until you are ready to upload your solutions.
5. Wait until you receive the go-ahead signal.

Problem 1. 20 points (Paper and Pencil)

Let $b(t)$ be a baseband signal and suppose that the support of its Fourier transform $b_{\mathcal{F}}(f)$ is contained in the interval $(-\frac{B}{2}, \frac{B}{2})$.

1. What condition must the sampling interval T_s fulfill in order to be able to reconstruct $b(t)$ from its samples taken every T_s seconds?
2. Write down the interpolation formula that recovers $b(t)$ from the samples $b(iT_s)$, $i \in \mathbb{Z}$, where the interpolation is done using

$$h_0(t) = \operatorname{sinc}\left(\frac{t}{T_s}\right).$$

3. Let

$$p(t) = b(t)e^{j2\pi f_c t}.$$

   How does $p_{\mathcal{F}}(f)$ (the Fourier transform of $p(t)$), relate to $b_{\mathcal{F}}(f)$? Make a qualitative plot of $|p_{\mathcal{F}}|$ and $|b_{\mathcal{F}}|$.
4. Find the necessary and sufficient condition on f_c, so that, for every integer n we have

$$p(nT_s) = b(nT_s).$$

5. For a (T_s, f_c) pair that fulfills the above conditions, what signal do you obtain if you interpolate the samples of $b(t)$ using the interpolation filter that has the following impulse response?

$$h_c(t) = e^{j2\pi f_c t} \operatorname{sinc}\left(\frac{t}{T_s}\right).$$

6. Bonus question: Suppose that f_d is an arbitrary center frequency larger than B. Write $f_d = f_c + \Delta$, where the pair (T_s, f_c) is as in (5). Find the sequence q_n, so that when you interpolate the sequence

$$b(nT_s)q_n, \quad n \text{ integer},$$

   using the interpolation filter

$$h_d(t) = e^{j2\pi f_d t} \operatorname{sinc}\left(\frac{t}{T_s}\right)$$

   leads to

$$q(t) = b(t)e^{j2\pi f_d t}.$$

PROBLEM 2. 20 points (Paper and Pencil)

Let $\boldsymbol{x} = (x_0, \ldots, x_{N-1})^T$ be a symbol sequence transmitted over a symbol-level channel and let

$$y_n = \sum_{i=0}^{L-1} h_i x_{n-i} + z_n, \quad n = 0, \ldots, N+L-2,$$

be the noisy channel output (where $x_n = 0$ if $n \notin \{0, \ldots, N-1\}$). All quantities are complex-valued and $N \geq L$.

1. Let $\boldsymbol{y} = (y_0, \ldots, y_{N+L-2})^T$, $\boldsymbol{h} = (h_0, \ldots, h_{L-1})^T$ and $\boldsymbol{z} = (z_0, \ldots, z_{N+L-2})^T$. Specify the matrix C such that
$$\boldsymbol{y} = C\boldsymbol{h} + \boldsymbol{z}.$$
2. Write down the objective function which is being minimized by the least squares (LS) estimate $\hat{\boldsymbol{h}}$ of $\boldsymbol{h}$ based on $\boldsymbol{y}$.
3. The vector $C\hat{\boldsymbol{h}}$ is an element of an inner-product space $\mathcal{V}$. What is $\mathcal{V}$?
4. The projection theorem specifies conditions that need to be satisfied by the LS estimate $\hat{\boldsymbol{h}}$? What are those conditions? Express them in terms of a matrix equation.
5. Find the expression for $\hat{\boldsymbol{h}}$ as a function of C and $\boldsymbol{y}$.

Problem 3. 20 points (MATLAB)

In this problem we look at the power spectral density (PSD) of an OFDM signal. Consider an OFDM system with $N = 400$ carriers. Let $T = 10^{-4}$ [s] be the time that it takes to transmit an OFDM block. We would like to transmit data on 4 carriers only, namely those at the frequencies ± 0.5 [MHz] and ± 1.5 [MHz].

Since we expect that the channel's frequency response at ± 0.5 [MHz] is worse than at ± 1.5 [MHz], we decide to do the following:

- use symbols from a 4-QAM constellation for the carriers at ± 0.5 [MHz],
- use symbols from a 16-QAM constellation for the carriers at ± 1.5 [MHz].

1. Generate the bits necessary to create the data symbols for 10^4 OFDM blocks. (Recall that only 4 carriers will carry data, as specified above).
2. Create the 4-QAM and 16-QAM symbols and insert them on the right carriers, as specified. To convert bits to symbols, use the provided `my_bi2de` and `my_qamMap` functions.
3. Create the OFDM time-domain signal using a cyclic prefix of length $L = 50$. The time-domain signal is a *vector* (not a matrix) $\boldsymbol{v}$ of length 450×10^4.
4. Take the FFT of $\boldsymbol{v}$ (one FFT for the entire time-domain signal $\boldsymbol{v}$) and plot its absolute value, making sure that you label the frequency axis in Hz. You should get a picture similar to the following one, with appropriately labeled frequency axis.

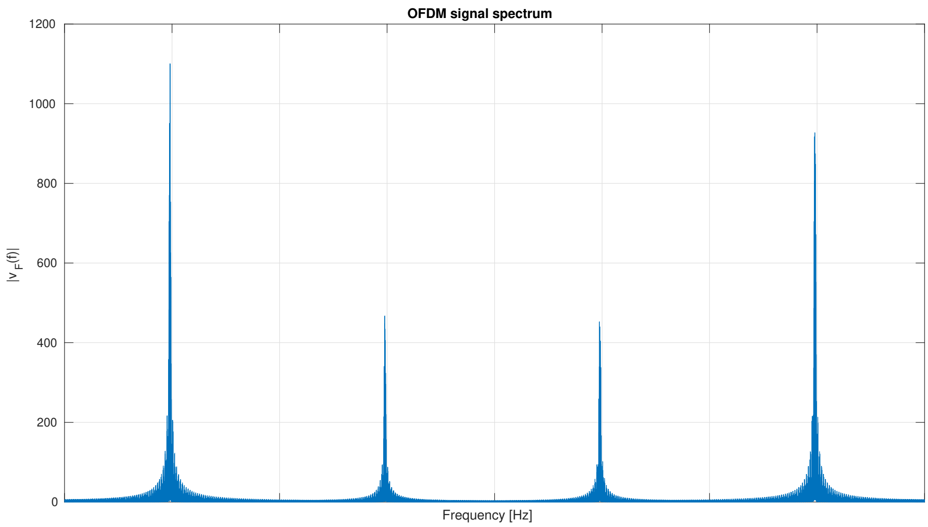

Figure 1: OFDM Spectrum.