ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Software-Defined Radio:	Final Exam
A Hands-On Course	December 19, 2012

Name:

Note:

- You have 2 h 45 min to work at the exam.
- There are five problems that can be solved in any order.
- The exam is closed book (no notes allowed). You are only allowed to use the workstations in the laboratory (not your own laptops). Resources from the internet as well as code written outside this exam are not allowed.
- The code will be evaluated according to the usual criteria, namely correctness, speed, form, and readability. Short comments that allow us to follow what you are doing will improve readability.
- The first two problems require writing Matlab code that you will upload on Moodle (as a single archive). The rest requires handwritten solutions that we will take at the end of the exam.

Start by downloading from Moodle and unzipping the file with all the data required for the different problems.

Problem 1. (15 p.) The file `ofdm_ex.mat` contains the output of a finite-response AWGN channel over which we have transmitted two OFDM symbols. The first of the two OFDM symbols corresponds to the preamble. All the OFDM parameters (number of carriers, number of zero carriers, length of the cyclic prefix, etc.) are provided in the file `ofdm_ex.m`. Complete this file as instructed so as to recover the transmitted BPSK symbols.

Problem 2. (12 p.) The file preamble_ex.mat contains a sequence of 1s and -1s representing data received from a GPS satellite. Complete the script preamble_ex.m that reads this data and removes the bits that are not part of complete subframes. Each subframe contains 300 bits and begins with a fixed preamble (possibly flipped). The preamble is also given in preamble_ex.mat.

Problem 3. (5 p.) The MATLAB code below estimates the Doppler shift for a received GPS signal. It was written by an inexperienced user and, although it provides the right result, it is not efficiently implemented. Show how it can be optimized.

```
% CA: the satellite C/A code
% data: the received data of the length of one C/A code.
% We assume that it starts with the beginning of a code
% t: the corresponding time vector

% initialize the vector containing max values
m=[];
% Considered Doppler frequencies (Take this as is)
Dshift=-100:100;

for i=1:length(Dshift)
     data=data.*exp(-1j*2*pi*Dshift(i)*t);
     c=xcorr(data, CA);
     m(i)=max(abs(c));
end

[maximum, pos]=max(m);
result=Dshift(pos);
```

Now, the programmer wants to find the values of `m` from the above code for several Doppler shift values. Improve the code written below.

```
% determine the values of m for Dshift = -100, -50, 0, 50, 100
v=[];
for i=1:length(Dshift)
     if ( (Dshift(i)==-100) || (Dshift(i)==50) || (Dshift(i)==0) || ...
          ... (Dshift(i)==50) || (Dshift(i)==100)   )
          v=[v,m(i)];
     end
end
```

Problem 4. (13 p.) Let

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

be the parity check matrix of a binary (toy) code $\mathcal{C}$. We assume that all codewords are used with the same probability.

(a) (2 p.) Find a codeword of $\mathcal{C}$.

(b) (3 p.) Draw the factor graph for the MAP decoding rule, assuming that codewords of $\mathcal{C}$ are transmitted over a discrete memoryless channel.

(c) (3 p.) Label the sockets with indices from $\{1, 2, 3, \cdots, N\}$, where N is the number of sockets, and specify the corresponding socket matrix.

(d) (3 p.) Write down the $N \times N$ matrix M such that $v = Mc + b$ is the N-length vector of log likelihood ratios that consists of the messages send by the variable nodes to the parity-check factor nodes. Note: The matrix M is not the M_v used in the homework assignment.

(e) (2 p.) Specify the i-th component of $b = (b_1, \ldots, b_N)^T$.

Problem 5. (15 p.) Each part of this problem is an independent question.

(a) (3 p.) The following matrix describes a rotation by the angle α around one of the main axes of a Cartesian coordinate system, using the positive angle convention described in class.

$$\begin{pmatrix} \pm\cos(\alpha) & 0 & \pm\sin(\alpha) \\ 0 & 1 & 0 \\ \pm\sin(\alpha) & 0 & \pm\cos(\alpha) \end{pmatrix}$$

Determine the signs and the axis around which the rotation occurs. Hint: we are not expecting that you have memorized the signs.

(b) (2 p.) Assuming idealized conditions, how many constant parameters are necessary and sufficient to completely specify the position of a satellite at all times?

(c) (2 p.) How many GPS satellites are needed to determine the position of a receiver and why do we need that many?

(d) (3 p.) Let $p = (p_x, p_y, p_z)^T$ be the position of the sun at 12h00 with respect the earth-centered earth-fixed coordinate system. Describe the coordinates of the sun's position at 13h00, assuming that we can neglect the rotation of the earth around the sun. The angular velocity of the earth's rotation is $\dot{\Omega}$ [radiants per second] and you may give your answer in terms of the rotation matrices $R_1(\alpha)$, $R_2(\alpha)$ and $R_3(\alpha)$.

(e) (5 p.) Consider a GPS satellite that transmits one bit every T seconds and has clock perfectly synchronized to the GPS time. The satellite starts sending a new frame at time t_{tr}. A receiver, also with clock synchronized to the GPS time, detects the start of the same subframe at time t_1. Between time t_1 and t_2, $t_{tr} < t_1 < t_2$, the receiver detects exactly b bits. (The first bit starts at t_1 and the last bit ends at t_2.)

   (i) Describe $(t_1 - t_{tr})c$ (be precise).

   (ii) Find the distance between the receiver position at time t_2 and the satellite position when it emitted the signal received at t_2.

   (iii) Determine the (radial) speed at which the satellite approaches the receiver (it will be negative if the satellite is moving away from the receiver).