ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 20 Modern Digital Communications
Solutions to Assignment 12 December 18, 2024

SOLUTION 1.

1. See the provided MATLAB/Python routines.
2. We have
$$H^{-1} \approx \begin{bmatrix} 500e^{j0.49968\pi} & 500e^{-j0.5\pi} \\ 500e^{-j0.5\pi} & 500e^{j0.49968\pi} \end{bmatrix}$$
Hence,
$$\Sigma_{\boldsymbol{v}} = H^{-1}\Sigma_{\boldsymbol{z}}(H^{-1})^\dagger \approx \sigma^2 \begin{bmatrix} 5\times 10^5 & -5\times 10^5 \\ 5\times 10^5 & 5\times 10^5 \end{bmatrix}.$$
This means, the variance of v_1 (or v_2) is 500 000 times larger than σ^2!

SOLUTION 2.

1. We have shown in class that the LMMSE estimate of $\boldsymbol{x}$ is $\hat{\boldsymbol{x}} = B\boldsymbol{y}$, where
$$B = K_{\boldsymbol{xy}}K_{\boldsymbol{y}}^{-1} .$$
We can easily compute the above covariance matrices:
$$K_{\boldsymbol{xy}} = E\left[\boldsymbol{x}\boldsymbol{y}^\dagger\right] = E\left[\boldsymbol{x}(H\boldsymbol{x}+\boldsymbol{z})^\dagger\right] = E\left[\boldsymbol{x}\boldsymbol{x}^\dagger\right]H^\dagger = H^\dagger ,$$
and
$$K_{\boldsymbol{y}} = E\left[\boldsymbol{y}\boldsymbol{y}^\dagger\right] = E\left[(H\boldsymbol{x}+\boldsymbol{z})(H\boldsymbol{x}+\boldsymbol{z})^\dagger\right] = HE\left[\boldsymbol{x}\boldsymbol{x}^\dagger\right]H^\dagger + E\left[\boldsymbol{z}\boldsymbol{z}^\dagger\right] = HH^\dagger + \sigma^2 I_r .$$
Hence,
$$B = H^\dagger(HH^\dagger + \sigma^2 I_r)^{-1}.$$
Note that, in particular if $\sigma^2 = 0$, the LMMSE equalizer reduces to the zero-forcing equalizer of Exercise 1.
2. See the provided MATLAB/Python routines.