ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 18 Modern Digital Communications
Solutions to Assignment 11 December 11, 2024

See the provided MATLAB/Python routines that implement the required algorithms. For the function `correctOFDMSymbolRotation` we implement the LS estimator that uses both pilot carriers (as discussed in Exercise 4).

SOLUTION 4. The observable $\boldsymbol{Y}$ is an element of the inner-product space $\mathcal{U} = \mathbb{C}^k$ and let $\mathcal{V}$ be the subspace spanned by $\boldsymbol{X}$. We are seeking for the vector $\hat{\boldsymbol{Y}} \in \mathcal{V}$ that minimizes $\|\boldsymbol{Y} - \hat{\boldsymbol{Y}}\|^2$.

The projection theorem tells us that $\hat{\boldsymbol{Y}}$ is the projection of $\boldsymbol{Y}$ into $\mathcal{V}$. It has the property that the error vector $\boldsymbol{Y} - \hat{\boldsymbol{Y}}$ is orthogonal to every element of $\mathcal{V}$. In particular, it is orthogonal to $\boldsymbol{X}$. Hence,

$$\langle \boldsymbol{Y} - \hat{R}\boldsymbol{X}, \boldsymbol{X} \rangle = 0$$

which is the same as

$$\langle \boldsymbol{Y}, \boldsymbol{X} \rangle = \langle \hat{R}\boldsymbol{X}, \boldsymbol{X} \rangle.$$

Hence

$$\hat{R}_{\text{LS}}(\boldsymbol{Y}) = \frac{\langle \boldsymbol{Y}, \boldsymbol{X} \rangle}{\langle \boldsymbol{X}, \boldsymbol{X} \rangle} = \frac{\boldsymbol{X}^\dagger \boldsymbol{Y}}{\|\boldsymbol{X}\|^2} .$$

Obviously, since we are only interested in the phase of R, in our problem we can ignore the normalization by $\|\boldsymbol{X}\|^2$ and estimate the phase as

$$\hat{\theta}_{\text{LS}} = \arg\{\boldsymbol{X}^\dagger \boldsymbol{Y}\}.$$