ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 12 Modern Digital Communications
Midterm Solutions November 6, 2024

Problem 1. 12 points (Paper and Pencil)

1. $$s(t) = \sum_i s(iT) \operatorname{sinc}\left(\frac{t - iT}{T}\right)$$
   for any $T < \frac{1}{2B}$.
2. $$\psi(t) = \frac{1}{\sqrt{T}} \operatorname{sinc}\left(\frac{t}{T}\right)$$
3. $$\psi_{\mathcal{F}}(f) = \begin{cases} \sqrt{T}, & |f| \le \frac{1}{2T} \\ 0, & \text{otherwise.} \end{cases}$$
4. Using the above, we can immediately see that $\psi_{\mathcal{F}}(f)$ fulfills the Nyquist criterion:
   $$\text{l.i.m.} \sum_{k=-\infty}^{\infty} \left| \psi_{\mathcal{F}}(f - \frac{k}{T}) \right|^2 = T \quad \text{for } f \in \mathbb{R}$$
5. $$\langle \psi(t), \psi(t - kT) \rangle = \delta_k.$$
6. $$\begin{aligned} s(t) &= \sum_i s(iT) \operatorname{sinc}\left(\frac{t - iT}{T}\right) && \textit{(sampling thm)} \\ &= \sum_i s_i \psi(t - iT) && \textit{(orthonormal expansion)} \end{aligned}$$
   where $s_i = s(iT)\sqrt{T}$.

Problem 3. 14 points (Paper and Pencil + MATLAB/Python)

1. $\mathcal{F}\{p(t-\tau)\} = p_{\mathcal{F}}(f) e^{-j2\pi f \tau}$. So $r_{\mathcal{F}}(f) = \mathcal{F}\{p(t-\tau) e^{j2\pi dt}\} = p_{\mathcal{F}}(f-d) e^{-j2\pi(f-d)\tau}$.
2. $|r_{\mathcal{F}}(f)| = |p_{\mathcal{F}}(f-d)|$. The correlation between $|r_{\mathcal{F}}(f)|$ and $|p_{\mathcal{F}}(f)|$ will have its maximum at d.
3. With this method, we can estimate d with a resolution of $f_s/N = 1/(NT_s)$. For a resolution of 1 Hz or better, we need to take $N = \lceil 1/T_s \rceil$.
4. See the attached MATLAB/Python routine.