ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 11 Modern Digital Communications
Midterm Exam November 6, 2024

Name:

Note:

- You have 2 hours to work on the exam.
- The exam is closed book, but you are allowed one sheet (one single-sided A4 page) of handwritten notes, with NO processing (plain and pure handwriting on paper). Resources from the internet as well as code written outside this exam are not allowed (unless the code is written on the sheet of handwritten notes).
- The code will be evaluated according to the usual criteria, namely correctness, speed, form, and readability. Short comments that allow us to follow what you are doing will improve readability.
- The problems can be solved in any order.
- You will upload (to Moodle) your solution to the problems that require writing MATLAB or Python code. Do so in a single archive.

To get started with the exam, do the following:

1. Close all the windows and programs on your laptop.
2. Launch the MATLAB/Python editor and close all the tabs (previously written code).
3. From Moodle, download the file `mdc_midterm_2024.zip`. Unzip the file to create the directory `mdc_midterm_2024`. For the rest of the exam you are required to work inside that directory. The MATLAB/Python files for Problem n, are found in subfolder pn.
4. Turn your WiFi off until you are ready to upload your solutions.
5. Wait until you receive the go-ahead signal.

Problem 1. 12 points (Paper and Pencil)

Assume that $s(t)$ is a continuous $\mathcal{L}_2$ function, and its Fourier transform $s_{\mathcal{F}}(f) = 0$, $f \notin [-B, B]$.

1. Specify the argument of the sinc() in the equation below, such that this equation represents the reconstruction formula of the sampling theorem. Specify the sampling time T, such that the conditions of the sampling theorem are satisfied.

$$s(t) = \sum_i s(iT)\,\mathrm{sinc}(\cdots)$$

2. Specify the scalar A and the argument of the sinc() in the equation below, such that $\psi(t)$ fulfills the Nyquist criterion.

$$\psi(t) = A\,\mathrm{sinc}(\cdots)$$

3. Write down $\psi_{\mathcal{F}}(f)$, the Fourier transform of $\psi(t)$.

4. Using $\psi_{\mathcal{F}}(f)$, prove that the function $\psi(t)$ you found above fulfills indeed the Nyquist criterion.

5. Write down the time-domain condition which $\psi(t)$ should satisfy such that it is a Nyquist pulse.

6. Specify s_i and the argument of the function $\psi()$, such that the equation below represents the sampling theorem written as an orthonormal expansion.

$$s(t) = \sum_i s_i \psi(\cdots)$$

PROBLEM 2. 14 points (MATLAB/Python)

Consider symbol-by-symbol on a pulse train simulated at the sample level. The symbols take values in $\{\pm 1\}$ and the pulses are shifted replicas of `pulse`. Subsequent pulses are non-overlapping and without space in between. Somewhere inside the symbol sequence there is the `preamble`. The signal is transmitted over a channel that multiplies the samples by $e^{j\phi_0}$ and adds white Gaussian noise. The phase $\phi_0 \in (-\pi, \pi]$ is unknown to the receiver. The channel output sequence is stored in `rxSamples`, but an initial and a final part of the channel output have been missed.

1. Estimate the phase ϕ_0 and the position where the first full pulse is contained in the received sequence. Note that the first full pulse might start before the start of the preamble.

2. From `rxSamples`, remove the excess samples before the start of the first full pulse, and remove the samples after the last full pulse.

3. Using your estimate of ϕ_0, remove the rotation introduced by the channel.

4. Treat the result `r` as the output of the AWGN channel (without rotation) and, by means of a matched filter, construct a vector of sufficient statistics for the symbol sequence contained in `r`. The preamble is considered as part of the useful data, so you do not have to remove it. Plot the resulting constellation.

5. From your sufficient statistics, estimate the symbol sequence and compute the symbol error rate (SER). If your receiver was properly implemented, you should obtain the sequence stored in `txSymbols` and SER = 0.

Problem 3. 14 points (Paper and Pencil + MATLAB/Python)

Let $p(t)$ be a known signal and let $r(t) = p(t-\tau)e^{j2\pi dt}$. We view τ and d as a delay and a Doppler frequency, respectively, both of which are unknown. We are interested in estimating d. Let $p_{\mathcal{F}}(f)$ and $r_{\mathcal{F}}(f)$ be the Fourier transforms of $p(t)$ and $r(t)$ respectively.

In case you need it, recall that the Fourier transform of a function $x(t)$ is defined as $x_{\mathcal{F}}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$.

1. Express $r_{\mathcal{F}}(f)$ as a function of $p_{\mathcal{F}}(f)$.
2. Describe a method for estimating d from $|r_{\mathcal{F}}(f)|$.
3. The files `p.mat` and `r.mat` contain the samples of $p(t)$ and $r(t)$, respectively. Write a MATLAB/Python routine that estimates d according to the method of question 2. Choose the DFT size N so that the estimate is quantized in steps of 1 Hz or better. The sampling time is $T_s = 10^{-5}$ seconds.

   Hint: Do not truncate the result of the correlations you compute, and plot them to check for a clear peak. You can then correct the indices to obtain the ones which you need.
4. Correct $r(t)$ for the Doppler and do a scatterplot of the result. If your routine works correctly, you should see a noisy BPSK constellation (with no rotation).