

Modern Digital Communications: A Hands-On Approach

Implementing Diversity

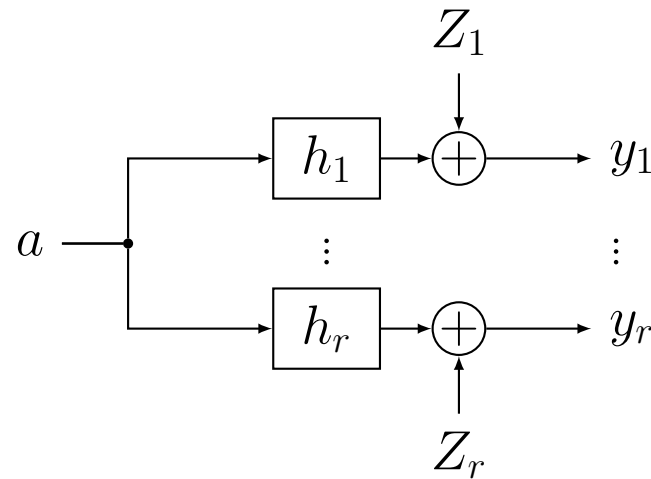
Dr. Nicolae Chiurtu

- Course material of Prof. Bixio Rimoldi -

Last revision: Sept. 13, 2017

Ways to Implement Diversity: Receiver Diversity

The simplest and most natural way to implement diversity is to have a transmitter with a single antenna and a receiver with two (or more) antennas.



If there is a single path between the transmit antenna and each of the receive antennas, then the symbol-level channel model is as described earlier, i.e.,

$$\mathbf{y} = \mathbf{h}a + \mathbf{Z}.$$

If there is multipath, then the output of each symbol-level channel is a convolution of the input sequence with an impulse response. We have not studied this case, but if we implement OFDM, then we are back in business.

Ways to Implement Diversity: Transmit Diversity

How to achieve diversity with two (or more) transmit antennas and one receive antenna is less obvious at first. Hereafter we describe Alamouti's technique.

- In the first time interval, the transmitter sends symbol a_1 across antenna 1 and a_2 across antenna 2.
- The receiver observes $Y_1 = a_1h_1 + a_2h_2 + Z_1$.
- In the second time interval, it sends $-a_2^*$ on antenna 1 and a_1^* on antenna 2.
- The receiver observes $Y_2 = -a_2^*h_1 + a_1^*h_2 + Z_2$.

Notice: we are using two time slots to get two symbols across the channel.

It is convenient to define $\mathbf{Y} = (Y_1, Y_2^*)^T$ (notice the complex conjugation) .

$$\mathbf{Y} = \underbrace{\begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}}_H \underbrace{(a_1, a_2)^T}_a + \underbrace{(Z_1, Z_2^*)^T}_Z .$$

where $\mathbf{Z} \sim \mathcal{CN}(0, N_0 I_2)$.

Key is the fact that $H^\dagger H = \|\mathbf{h}\|^2 I_2$, where $\mathbf{h} = (h_1, h_2)^T$.

Hence,

$$\tilde{\mathbf{Y}} = \frac{1}{\|\mathbf{h}\|} H^\dagger \mathbf{Y} = \|\mathbf{h}\| I_2 \mathbf{a} + \frac{1}{\|\mathbf{h}\|} H^\dagger \mathbf{Z}$$

showing that the multiplication by $H^\dagger$ at the receiver has diagonalized the channel.

Explicitly

$$\begin{aligned}\tilde{Y}_1 &= \|\mathbf{h}\|a_1 + \tilde{Z}_1 \\ \tilde{Y}_2 &= \|\mathbf{h}\|a_2 + \tilde{Z}_2\end{aligned}$$

where $\tilde{Z}_1$ and $\tilde{Z}_2$ are iid $\sim \mathcal{CN}(0, N_0)$.

We see that $\tilde{Y}_i$, $i = 1, 2$, is statistically equivalent to the output of a maximal-ratio combiner of a diversity-2 system.

Notice that we communicate 2 symbols using 2 time slots, but we actually transmit 4 symbols.

In comparison, with receiver diversity, we communicate 1 symbol per time slot, by transmitting 1 symbol.

The data rate and the error probability are the same for both systems, but the Alamouti scheme requires twice as much energy.

Alamouti's scheme might be of interest when it is much more convenient to have two antennas at the transmitter than at the receiver.

This is the case, for instance, when the sender is a base station and the receiver is a portable device.

We emphasize that in Alamouti's scheme, the transmitter has no knowledge of the channel coefficients h_i , $i = 1, 2$.