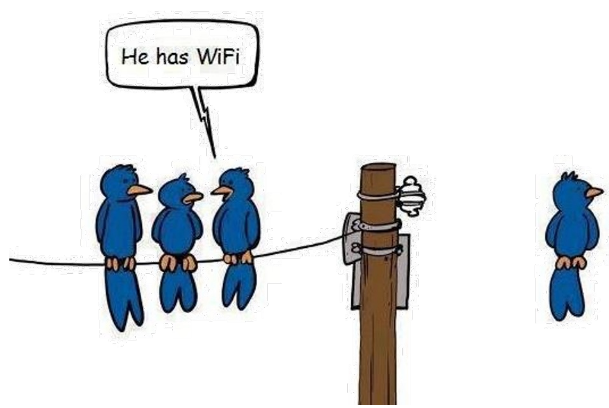
COM-405: Mobile Networks

Lecture 5.0: MIMO

Haitham Hassanieh



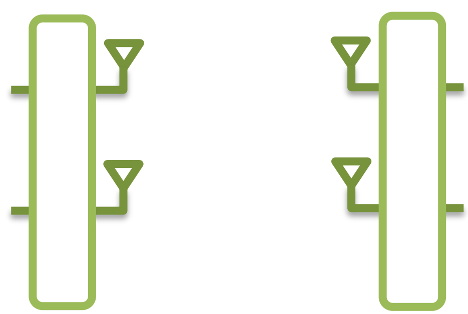
EPFL

Laboratory of SEnsing & Networking Systems

MIMO: Multiple Input Multiple Output

So far: single input single output

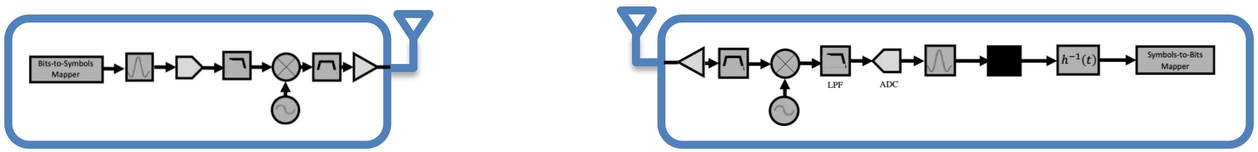
MIMO: multiple input multiple output



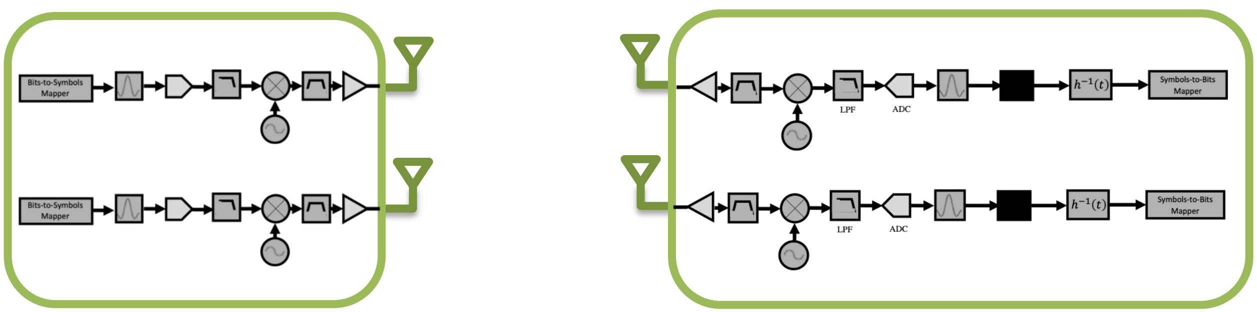
Increase capacity of channel using multiple TX and RX antennas.

MIMO: Multiple Input Multiple Output

So far: single input single output



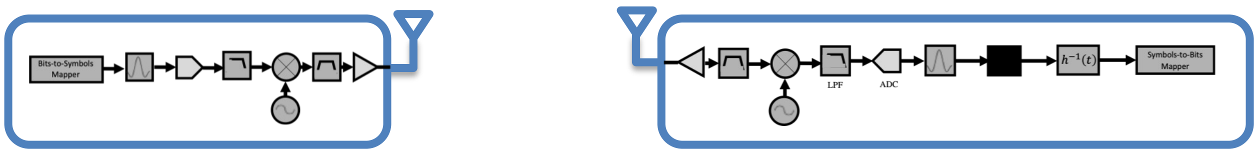
MIMO: multiple input multiple output



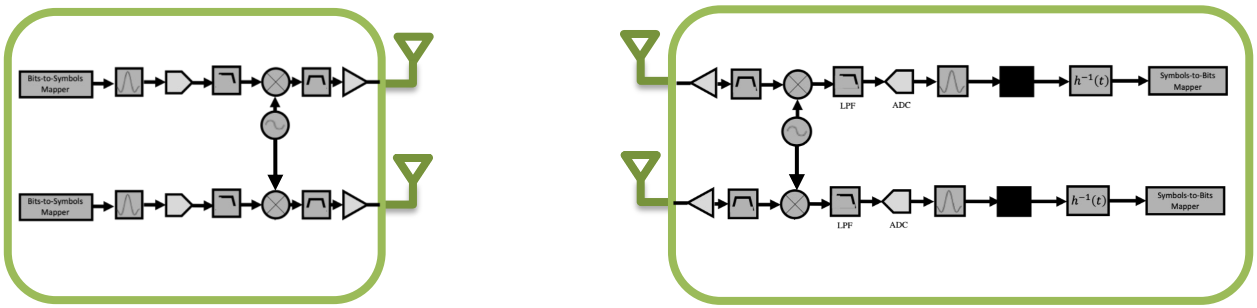
Increase capacity of channel using multiple TX and RX antennas.

MIMO: Multiple Input Multiple Output

So far: single input single output

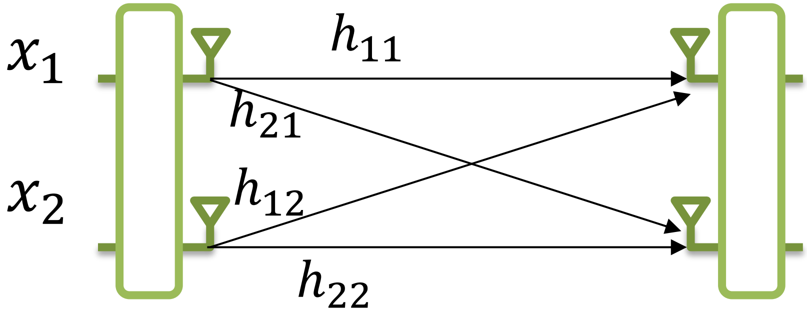


MIMO: multiple input multiple output



Increase capacity of channel using multiple TX and RX antennas.

MIMO: Multiple TX-RX streams



$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Rightarrow \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

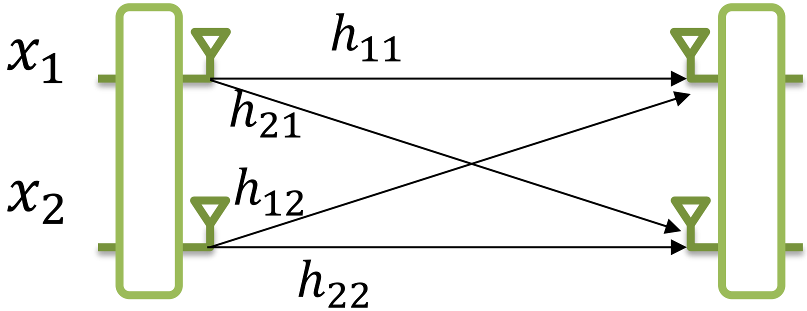
How to recover x_1 and x_2?

Estimate $\mathbf{H}$, compute $\mathbf{H}^{-1}$ and invert the channel!

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Transmit 2 packets at the same time!

MIMO: Multiple TX-RX streams



$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Rightarrow \mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

For N antennas, $\mathbf{H}$ is $N \times N$ matrix → $O(N^3)$

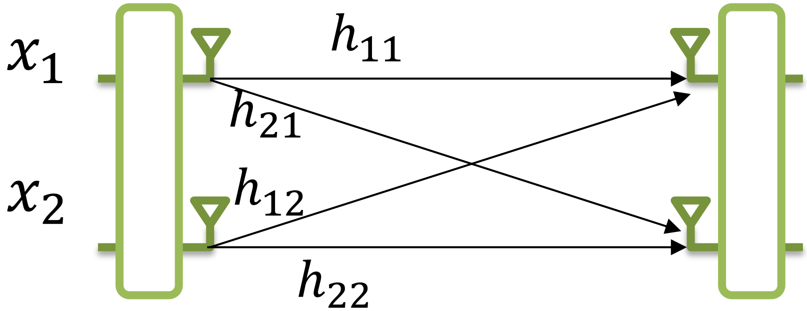
How to recover x_1 and x_2?

Estimate $\mathbf{H}$, compute $\mathbf{H}^{-1}$ and invert the channel!

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Noise amplification

MIMO: Vector Representation



$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

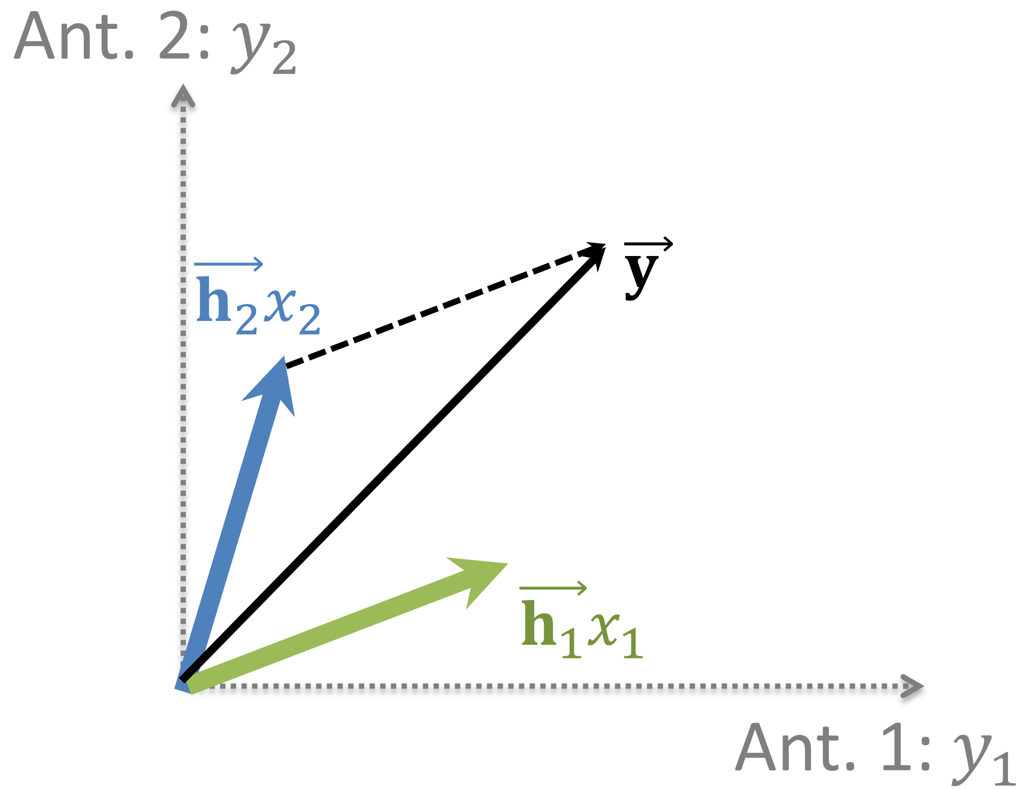
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Rightarrow \mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

How to recover x_1 and x_2?

$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2$$

MIMO: Antenna Space

$$\overrightarrow{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2$$



MIMO: Antenna Space

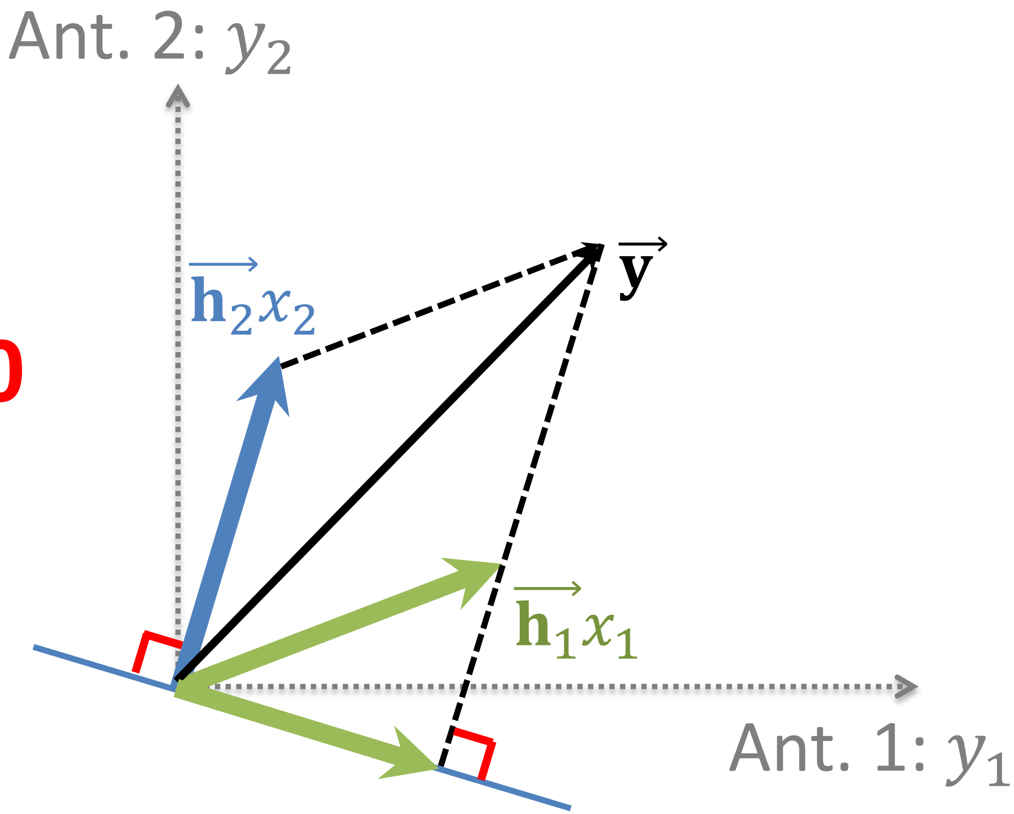
$$\vec{\mathbf{y}} = \vec{\mathbf{h}_1} x_1 + \vec{\mathbf{h}_2} x_2$$

To decode x_1, project on a vector $\vec{\mathbf{h}_2}^{\perp}$ orthogonal to $\vec{\mathbf{h}_2}$

$$\vec{\mathbf{h}_2}^{\perp} \vec{\mathbf{y}} = \vec{\mathbf{h}_2}^{\perp} \vec{\mathbf{h}_1} x_1 + \vec{\mathbf{h}_2}^{\perp} \vec{\mathbf{h}_2} x_2$$

0

$$= \vec{\mathbf{h}_2}^{\perp} \vec{\mathbf{h}_1} x_1$$



$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2$$

$$\vec{\mathbf{h}_2}^{\perp} = \begin{bmatrix} h_{22} & -h_{12} \end{bmatrix}$$

$$\vec{\mathbf{h}_2}^{\perp} \vec{\mathbf{y}} = h_{22} h_{11} x_1 - h_{12} h_{21} x_1$$

$$+ h_{22} h_{12} x_2 - h_{12} h_{22} x_2$$

0

$$= (h_{22} h_{11} - h_{12} h_{21})\, x_1$$

MIMO: Antenna Space

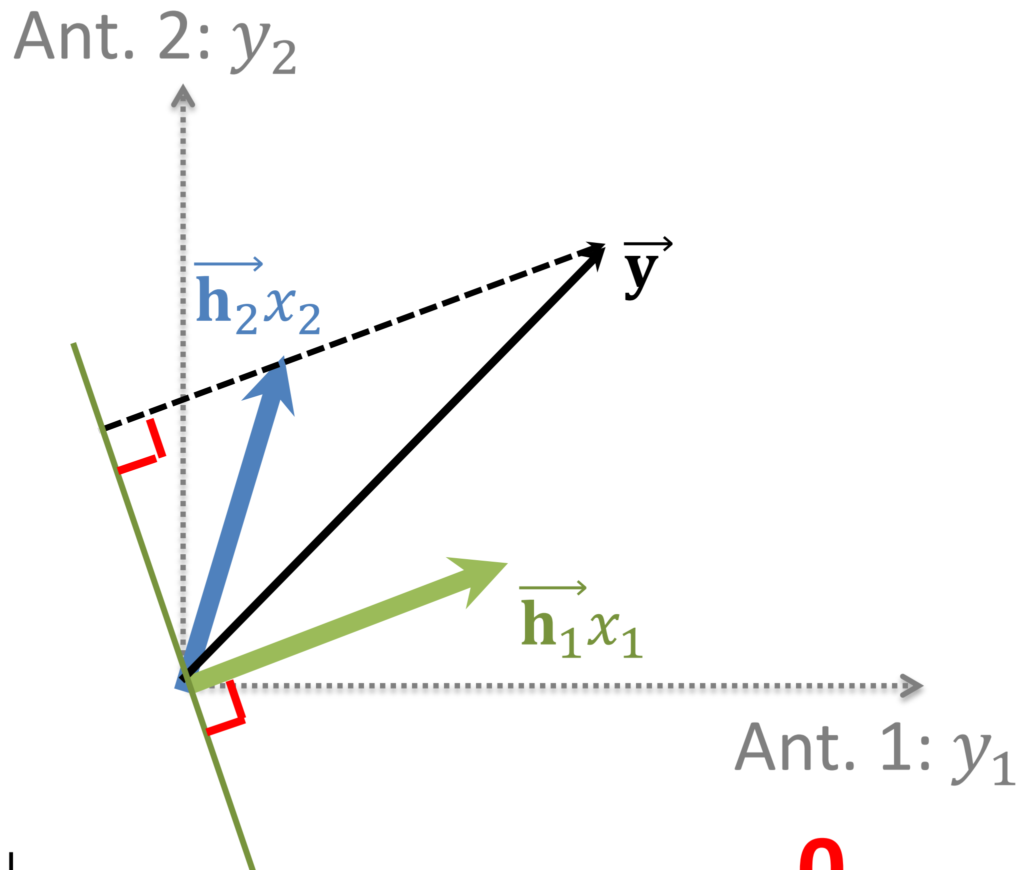
$$\vec{\mathbf{y}} = \vec{\mathbf{h}}_1 x_1 + \vec{\mathbf{h}}_2 x_2$$

To decode x_2, project on a vector $\vec{\mathbf{h}}_1^{\perp}$ orthogonal to $\vec{\mathbf{h}}_1$

$$\vec{\mathbf{h}}_1^{\perp}\vec{\mathbf{y}} = \vec{\mathbf{h}}_1^{\perp}\vec{\mathbf{h}}_1 x_1 + \vec{\mathbf{h}}_1^{\perp}\vec{\mathbf{h}}_2 x_2$$

0

$$= \vec{\mathbf{h}}_1^{\perp}\vec{\mathbf{h}}_2 x_2$$



$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2$$

$$\vec{\mathbf{h}}_1^{\perp} = \begin{bmatrix} h_{21} & -h_{11} \end{bmatrix}$$

$$\vec{\mathbf{h}}_1^{\perp}\vec{\mathbf{y}} = h_{21}h_{11}x_1 - h_{21}h_{11}x_1$$

0

$$+ h_{21}h_{12}x_2 - h_{11}h_{22}x_2$$

$$= (h_{21}h_{12} - h_{11}h_{22})\, x_2$$

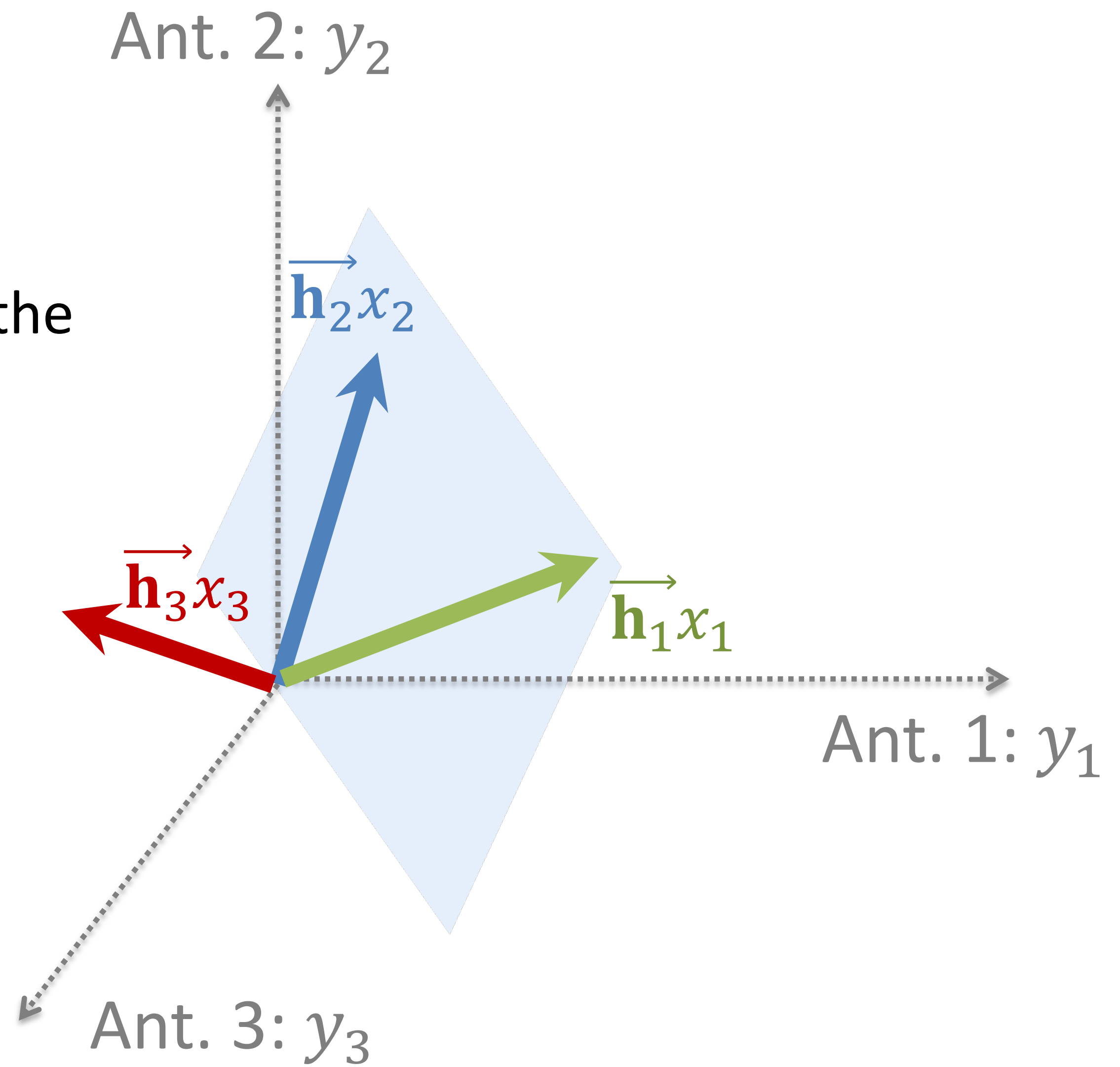
MIMO: Antenna Space

What about MIMO with more antennas?

$$\vec{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2 + \overrightarrow{\mathbf{h}_3} x_3$$

To decode x_1, project on a vector $\overrightarrow{\mathbf{h}_{23}}^{\perp}$ orthogonal to the plane formed by $\overrightarrow{\mathbf{h}_2}$ and $\overrightarrow{\mathbf{h}_3}$

$$\overrightarrow{\mathbf{h}_{23}}^{\perp} = \overrightarrow{\mathbf{h}_2} \times \overrightarrow{\mathbf{h}_3}$$



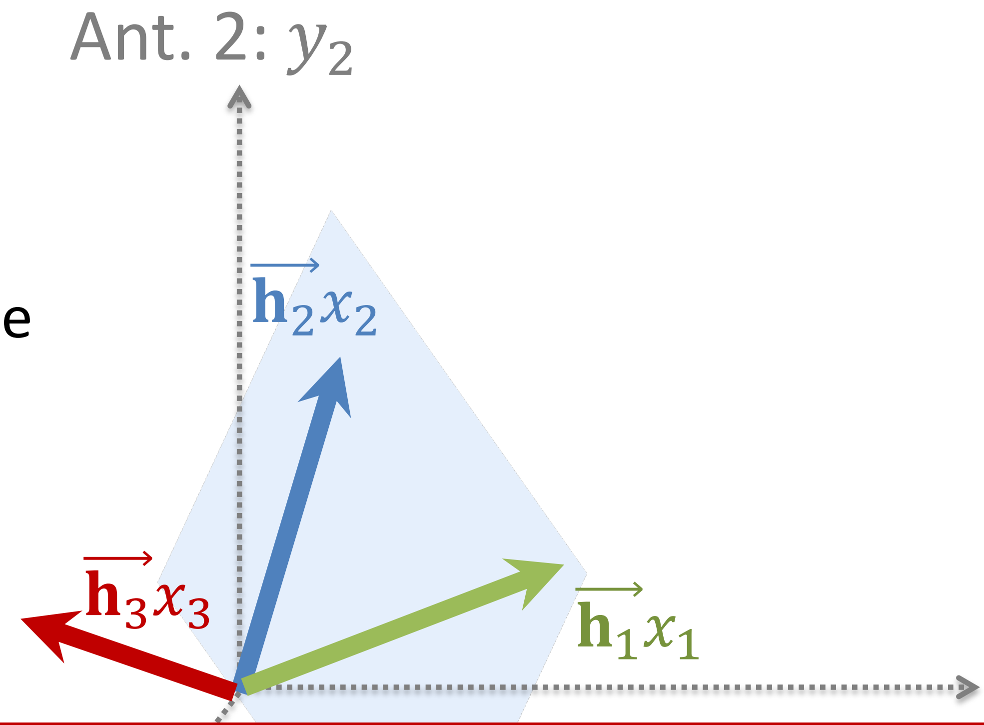
MIMO: Antenna Space

What about MIMO with more antennas?

$$\vec{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2 + \overrightarrow{\mathbf{h}_3} x_3$$

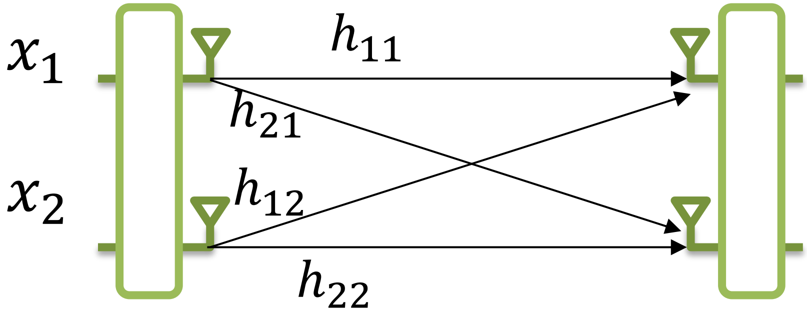
To decode x_1, project on a vector $\overrightarrow{\mathbf{h}_{23}}^{\perp}$ orthogonal to the plane formed by $\overrightarrow{\mathbf{h}_2}$ and $\overrightarrow{\mathbf{h}_3}$

$$\overrightarrow{\mathbf{h}_{23}}^{\perp} = \overrightarrow{\mathbf{h}_2} \times \overrightarrow{\mathbf{h}_3}$$



N antenna MIMO, receives signals in N dimensional space
→ Can decode N parallel signals
Decoding complexity scales $O(N^2)$ operations with the number of antennas

MIMO Channel

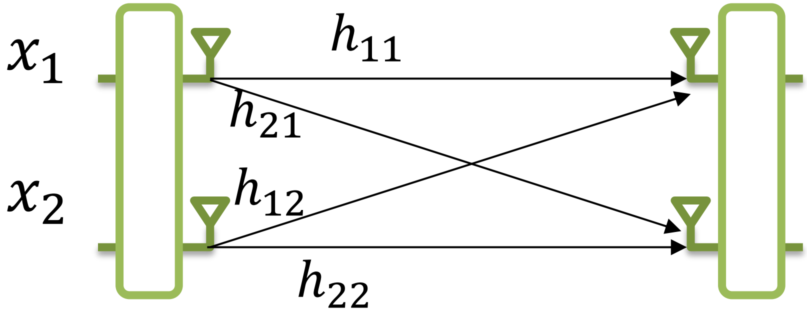


$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

How to estimate the channels: $h_{11}, h_{12}, h_{21}, h_{22}$?

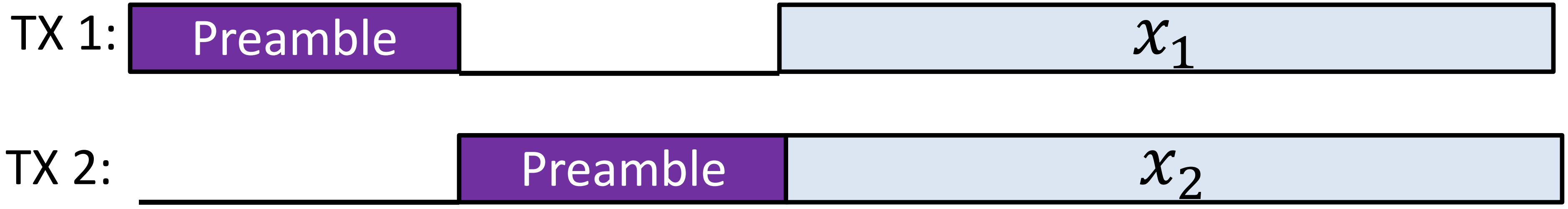
MIMO Channel



$y_1 = h_{11}x_1 + h_{12}x_2$

$y_2 = h_{21}x_1 + h_{22}x_2$

How to estimate the channels: $h_{11}, h_{12}, h_{21}, h_{22}$?



MIMO Gains

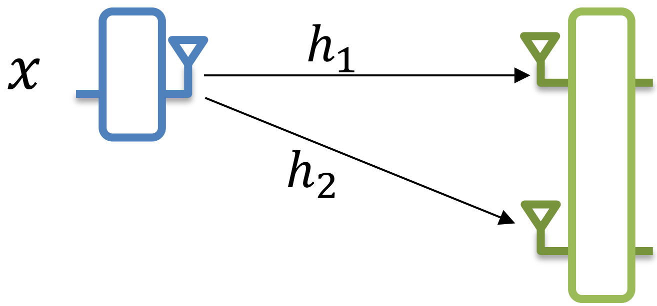
Multiplexing Gain:

- Send multiple packets at the same time
- $N \times N$ MIMO → $N \times$ more packets

Diversity Gain:

- Send/Receive the same packet on multiple antennas
- Increase SNR of the received packets → transmit at higher data rates

Receiver Diversity



$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

How to best decode x?

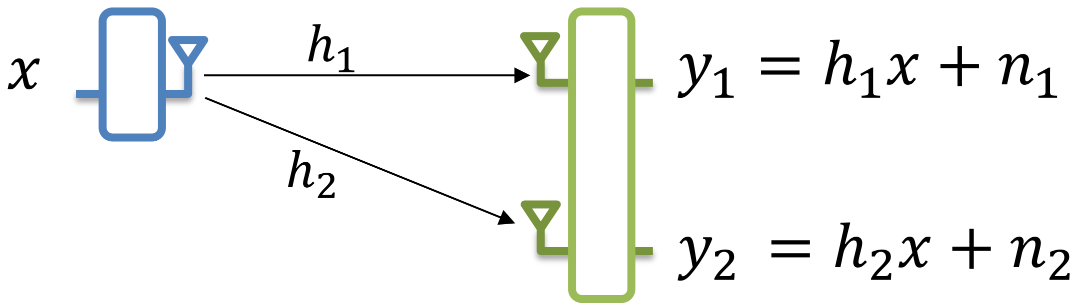
Option 1: Add the received signals

$$y_1 + y_2 = h_1 x + n_1 + h_2 x + n_2$$

$$= (h_1 + h_2)x + n_1 + n_2$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Receiver Diversity



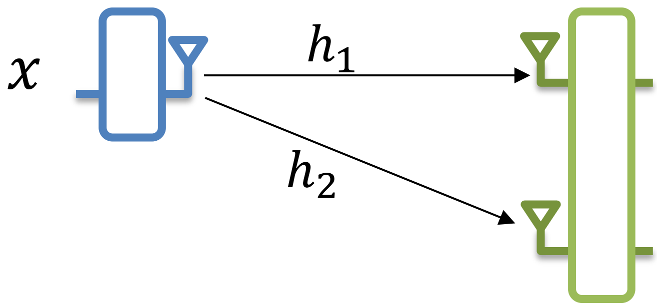
How to best decode x?

Option 1: Add the received signals

Option 2: Decode independently

Sub-optimal!

Receiver Diversity



$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

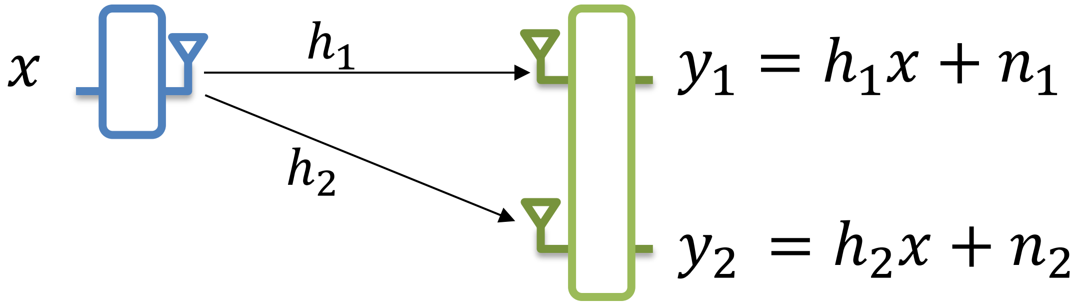
How to best decode x?

Option 1: Add the received signals

Option 2: Decode independently

Optimal Solution: Maximum Ratio Combining (MRC)

Maximum Ratio Combining



$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

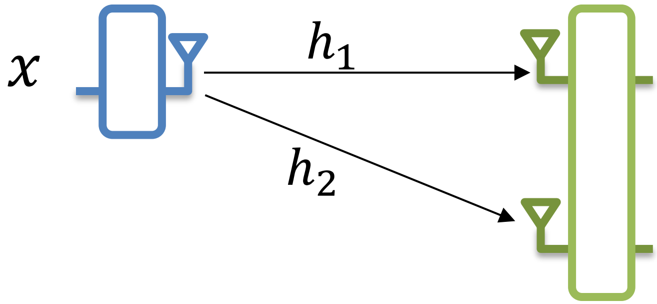
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = \mathbb{E}[|(|h_1|^2 + |h_2|^2)x|^2]$$

$$= (|h_1|^2 + |h_2|^2)^2\ \mathbb{E}[|x|^2]$$

$$= (|h_1|^2 + |h_2|^2)^2\ P$$

Maximum Ratio Combining



$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2) x + h_1^* n_1 + h_2^* n_2$$

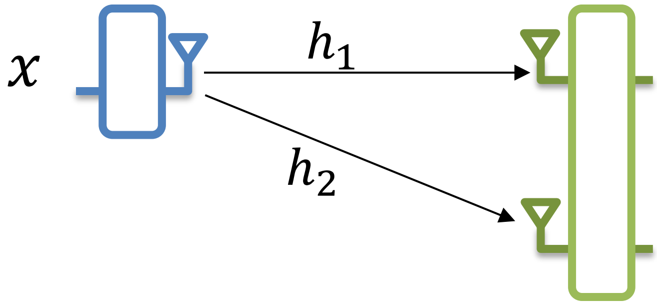
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = (|h_1|^2 + |h_2|^2)^2\, P$$

$$Noise\ Power = \mathbb{E}[|h_1^* n_1 + h_2^* n_2|^2] = \mathbb{E}[|h_1^* n_1|^2] + \mathbb{E}[|h_2^* n_2|^2]$$

$$= |h_1|^2 \mathbb{E}[|n_1|^2] + |h_2|^2 \mathbb{E}[|n_2|^2] = (|h_1|^2 + |h_2|^2)\sigma^2$$

Maximum Ratio Combining



$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

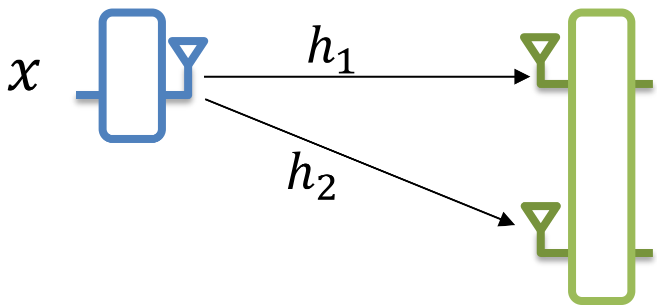
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = (|h_1|^2 + |h_2|^2)^2\, P$$

$$Noise\ Power = (|h_1|^2 + |h_2|^2)\sigma^2$$

$$SNR = \frac{(|h_1|^2 + |h_2|^2)^2\, P}{(|h_1|^2 + |h_2|^2)\sigma^2} = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$$

Maximum Ratio Combining



$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

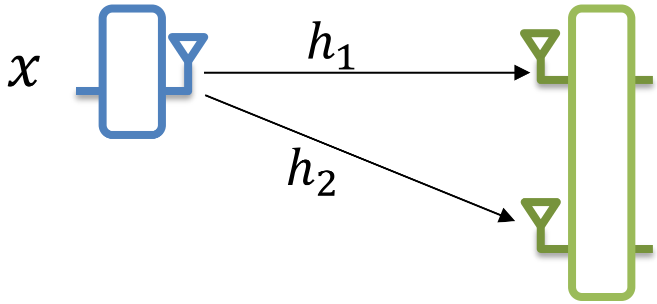
$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

With Receiver Diversity: $SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

Single Receiver: $SNR = |h_1|^2 \frac{P}{\sigma^2}$

Receiver Diversity Gain



$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

With Receiver Diversity: $SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

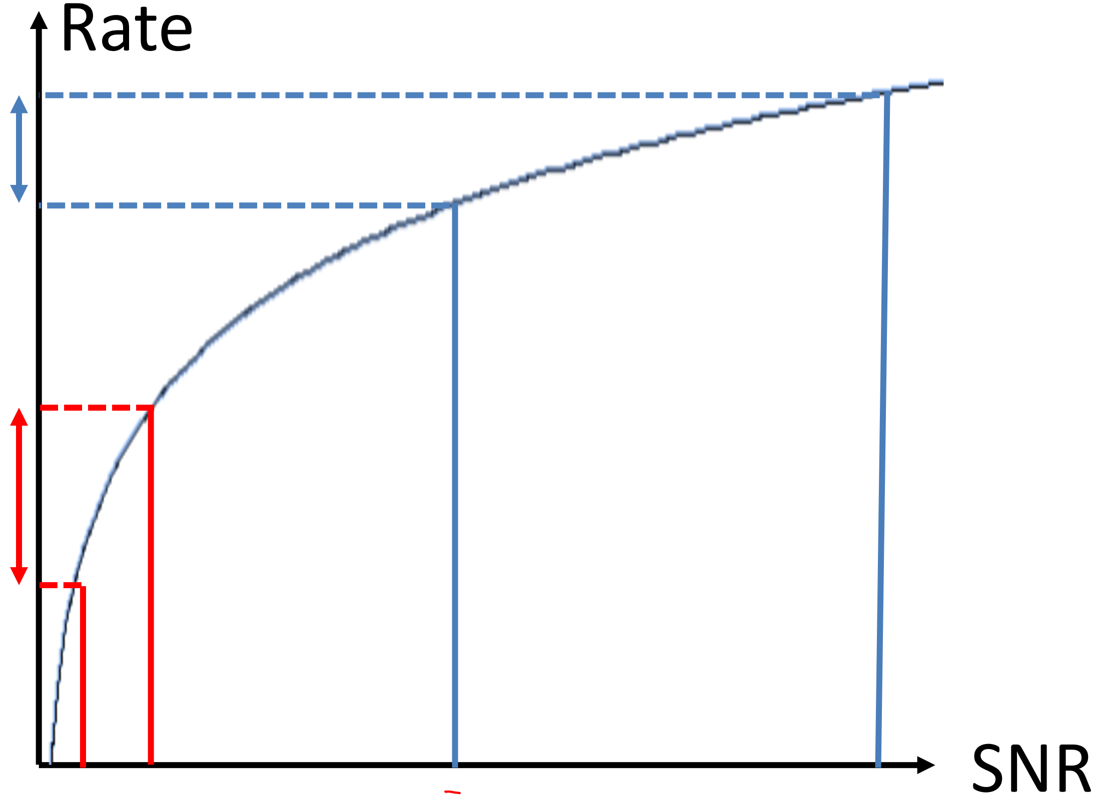
Single Receiver: $SNR = |h_1|^2 \frac{P}{\sigma^2}$

- $|h_1|^2 \approx |h_2|^2$ → Can double SNR!

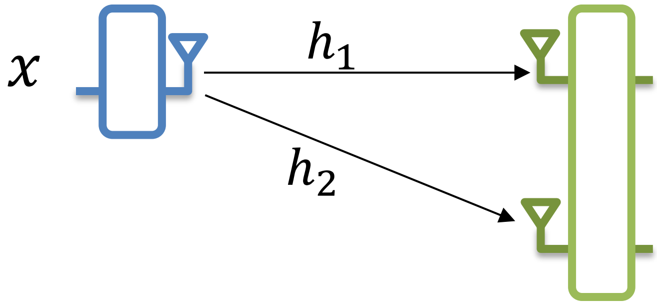
Receiver Diversity Gain

Do we care about doubling the SNR?

$$Capacity \propto \log(SNR)$$



Receiver Diversity Gain



$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

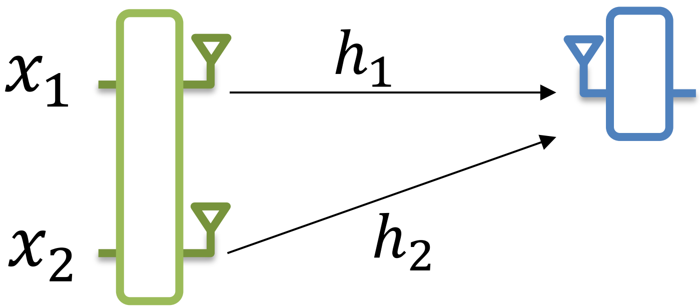
With Receiver Diversity: $SNR_{2RX} = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

Single Receiver: $SNR_{1RX} = |h_1|^2 \frac{P}{\sigma^2}$

- $|h_1|^2 \approx |h_2|^2$ → Can double SNR!
- $|h_1|^2 \ll |h_2|^2$ → Huge Gain in SNR
- $|h_1|^2 \gg |h_2|^2$ → Little Gain in SNR

It is unlikely that both antennas experience channel fading.

Transmitter Diversity



$y = h_1 x_1 + h_2 x_2 + n$

What should we transmit on each antenna?

Option 1: transmit the same thing x on both antennas

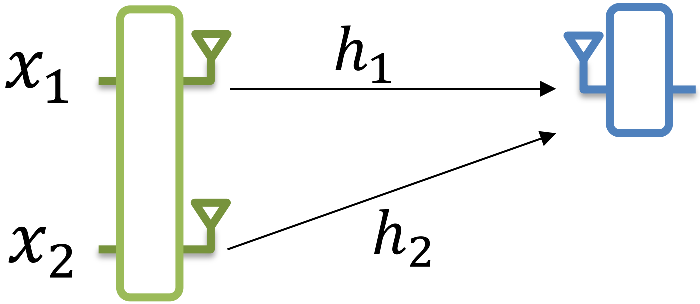
$$y = (h_1 + h_2)x + n$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Total transmit power $= 2P$ → Doubled TX power

→ Why not use 1 TX with $2P$

Transmitter Diversity



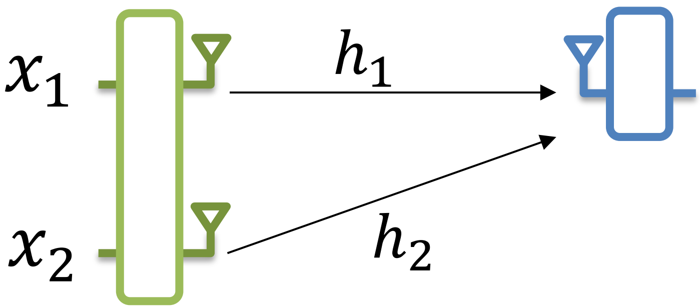
$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Option 1: transmit the same thing x on both antennas

- Must ensure signals sum up constructively.
- Must ensure total TX power $= \mathbb{E}[|x_1|^2] + \mathbb{E}[|x_2|^2]$
  $= \mathbb{E}[|x|^2] = P$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

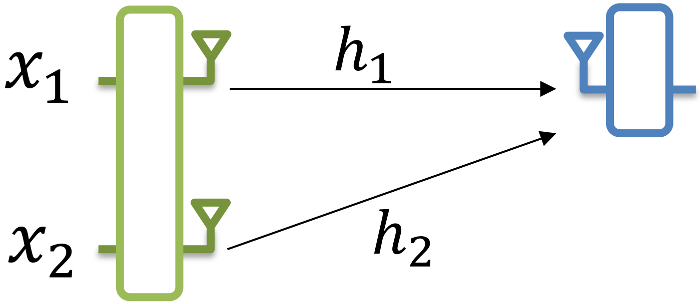
Option 2: Maximum Ratio Combining (MRC)

$$\left.\begin{aligned} x_1 &= \alpha_1 x \\ x_2 &= \alpha_2\, x \end{aligned}\right\} y = (\alpha_1 h_1 + \alpha_2 h_2)x + n$$

Set: $\alpha_1 = h_1^*, \alpha_2 = h_2^*$ → $y = (h_1^* h_1 + h_2^* h_2)x + n$

$$= (|h_1|^2 + |h_2|^2)x + n$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

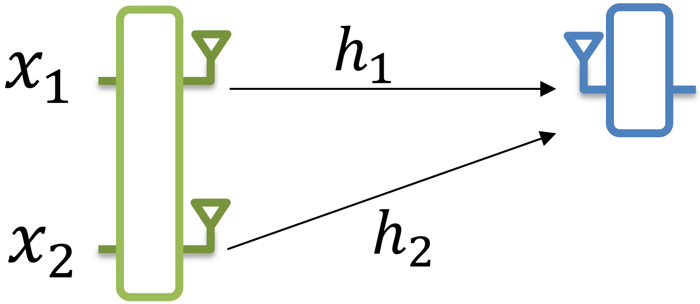
Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$Total\ TX\ Power = \mathbb{E}[|x_1|^2] + \mathbb{E}[|x_1|^2]$$

$$= \frac{|h_1|^2}{|h_1|^2 + |h_2|^2} P + \frac{|h_2|^2}{|h_1|^2 + |h_2|^2} P = P$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

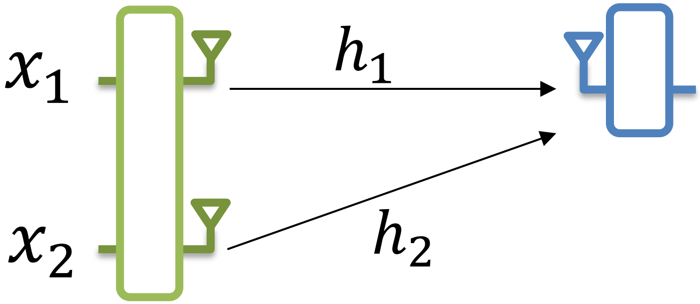
What should we transmit on each antenna?

Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$y = \frac{h_1 h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + \frac{h_2 h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + n$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

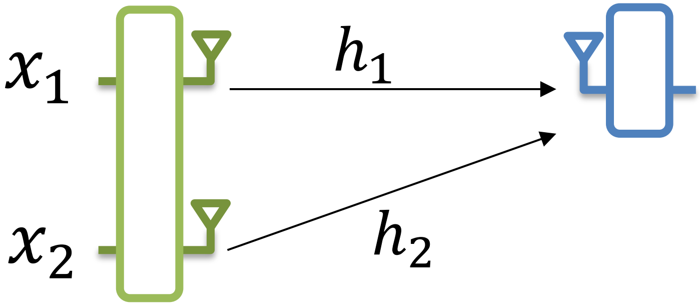
What should we transmit on each antenna?

Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}} x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}} x$$

$$y = \frac{|h_1|^2 + |h_2|^2}{\sqrt{|h_1|^2 + |h_2|^2}} x + n = \left(\sqrt{|h_1|^2 + |h_2|^2}\right) x + n$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

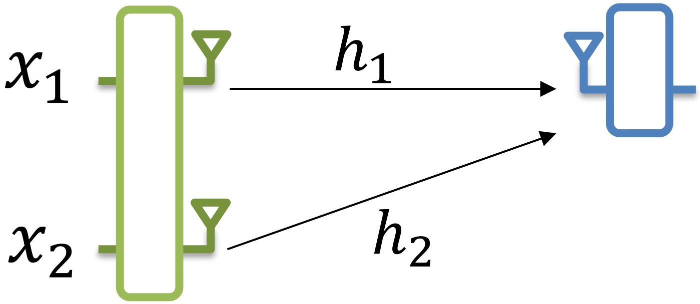
Option 2: Maximum Ratio Combining (MRC)

$$y = \left(\sqrt{|h_1|^2 + |h_2|^2}\right) x + n$$

$$SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$$ → Similar SNR Gain to RX Diversity

Caveat: MRC at TX Requires Feedback from the Receiver!

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

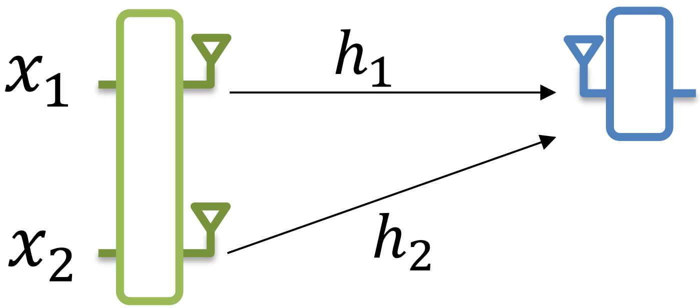
MRC codes across space only (Requires Channel Feedback):

	$t = 1$	$t = 2$
TX1: x_1	$\alpha_1 x[1]$	$\alpha_1 x[2]$
TX2: x_2	$\alpha_2 x[1]$	$\alpha_2 x[2]$

$$\alpha_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

$$\alpha_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

Transmitter Diversity

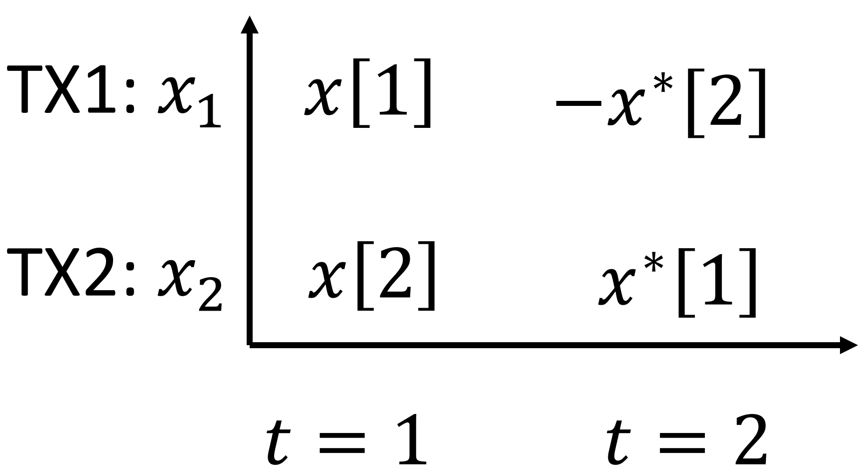


$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

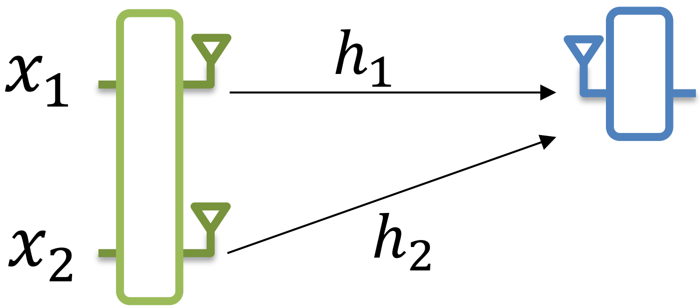
Alamouti Codes:



$$y[1] = h_1 x[1] + h_2 x[2]$$

$$y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

Alamouti Codes:

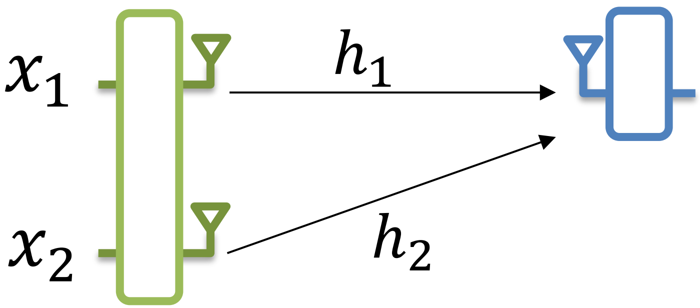
$$y[1] = h_1 x[1] + h_2 x[2] \qquad y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

$$h_1^* y[1] + h_2 y^*[2] = h_1^* h_1 x[1] + h_1^* h_2 x[2]$$

$$-h_2 h_1^* x[2] + h_2 h_2^* x[1]$$

$$= (|h_1|^2 + |h_2|^2) x[1]$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

Alamouti Codes:

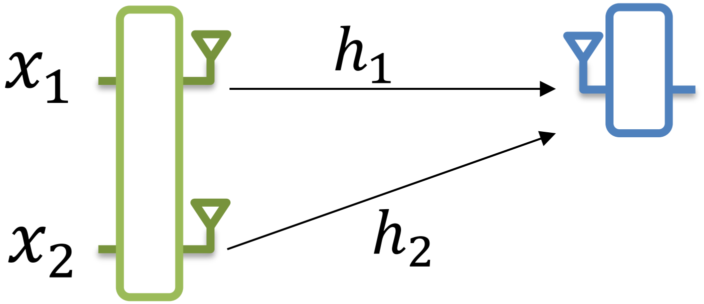
$$y[1] = h_1 x[1] + h_2 x[2] \qquad y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

$$h_2^* y[1] - h_1 y^*[2] = h_2^* h_1 x[1] + h_2^* h_2 x[2]$$

$$+ h_1 h_1^* x[2] - h_1 h_2^* x[1]$$

$$= (|h_1|^2 + |h_2|^2) x[2]$$

Transmitter Diversity

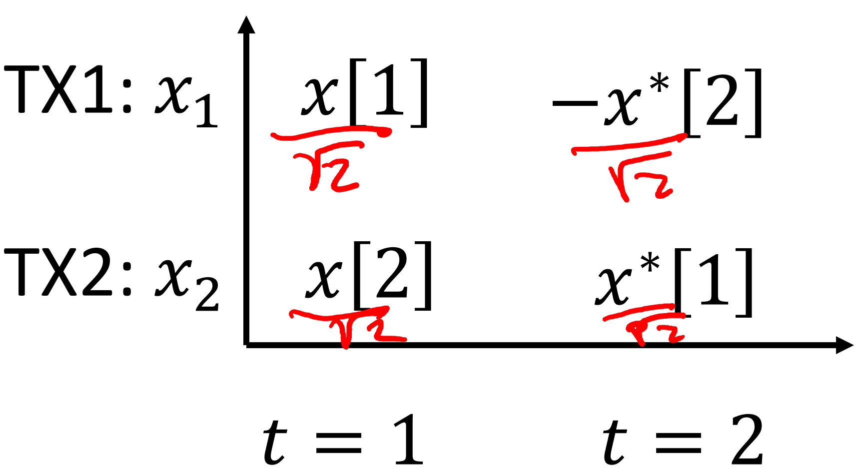


$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

Alamouti Codes:



$$h_1^* y[1] + h_2 y^*[2]$$

$$= (|h_1|^2 + |h_2|^2)\frac{x[1]}{\sqrt{2}}$$

$$h_2^* y[1] - h_1 y^*[2]$$

$$= (|h_1|^2 + |h_2|^2)\frac{x[2]}{\sqrt{2}}$$

MIMO Gains

Multiplexing Gain:

- Send multiple packets at the same time
- $N \times N$ MIMO → $\times N$ more packets
- Data Rate: $\propto N \log(SNR/N)$

Diversity Gain:

- Increase SNR of the received packets
- $N \times N$ MIMO → $\times \log N^2$ data rate
- Data Rate $\propto \log(SNR \times N^2)$

MIMO Gains

- Multipath is essential for MIMO
  - More Multipath → More Diversity
  - More Multipath → More Degress of Freedom

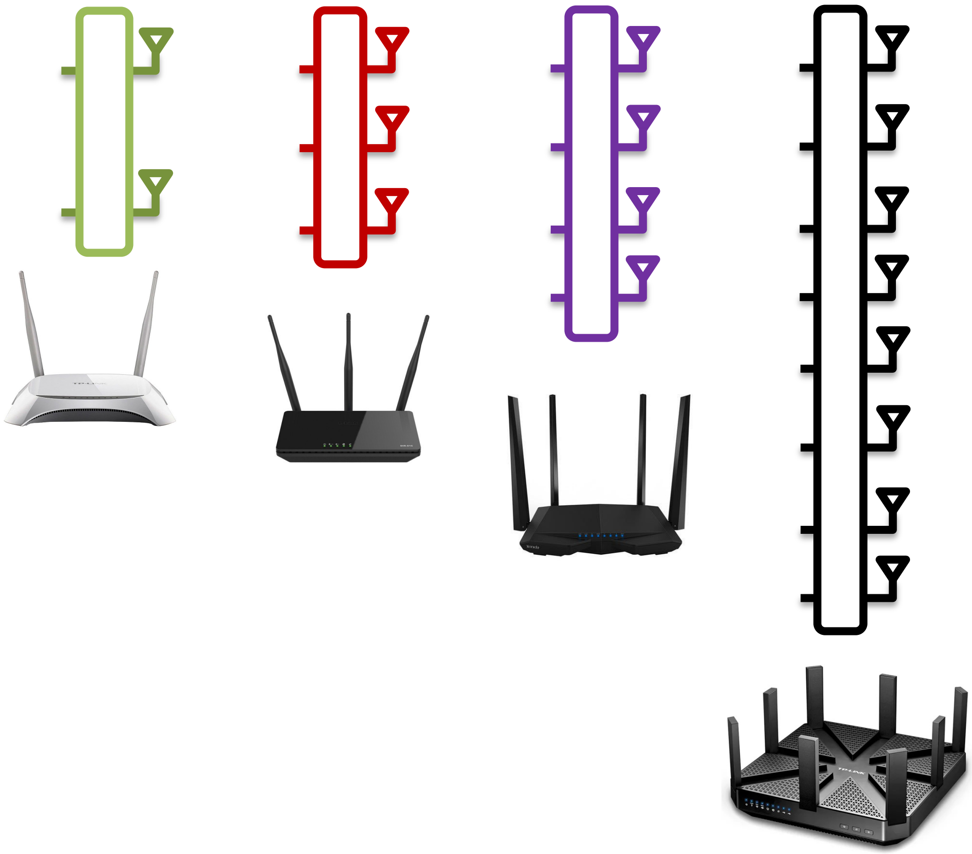
- Condition Number of Matrix $\mathbf{H}$
  - A measure of how close a matrix is to being singular
  - A matrix with large condition number is nearly singular → Unstable MIMO system, high noise amplification, not enough degrees of freedom.
  - A matrix with condition number close to 1 is far from being singular.

Here is a link to the full MCS Table: http://bit.ly/2G0DIcD

MCS Index			Spatial Stream	Modulation	Coding	OFDM (Prior 11ax)								OFDM (802.11ax)											
						20MHz		40MHz		80MHz		160MHz		20MHz			40MHz			80MHz			160MHz		
HT	VHT	HE				0.8µs GI	0.4µs GI	0.8µs GI	0.4µs GI	0.8µs GI	0.4µs GI	0.8µs GI	0.4µs GI	0.8µs GI	1.6µs GI	3.2µs GI	0.8µs GI	1.6µs GI	3.2µs GI	0.8µs GI	1.6µs GI	3.2µs GI	0.8µs GI	1.6µs GI	3.2µs GI
0	0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65	8.6	8.1	7.3	17.2	16.3	14.6	36	34	30.6	72.1	68.1	61.3
1	1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130	17.2	16.3	14.6	34.4	32.5	29.3	72.1	68.1	61.3	144.1	136.1	122.5
2	2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195	25.8	24.4	21.9	51.6	48.8	43.9	108.1	102.1	91.9	216.2	204.2	183.8
3	3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260	34.4	32.5	29.3	68.8	65	58.5	144.1	136.1	122.5	288.2	272.2	245
4	4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390	51.6	48.8	43.9	103.2	97.5	87.8	216.2	204.2	183.8	432.4	408.3	367.5
5	5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520	68.8	65	58.5	137.6	130	117	288.2	272.2	245	576.5	544.4	490
6	6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585	77.4	73.1	65.8	154.9	146.3	131.6	324.3	306.3	275.6	648.5	612.5	551.3
7	7	7	1	64-QAM	5/6	65	72.2	135	150	292.5	325	585	650	86	81.3	73.1	172.1	162.5	146.3	360.3	340.3	306.3	720.6	680.6	612.5
	8	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
	9	9	1	256-QAM	5/6	N/A	N/A	180	200	390	433.3	780	866.7	114.7	108.3	97.5	229.4	216.7	195	480.4	453.7	408.3	960.8	907.4	816.7
		10	1	1024-QAM	3/4									129	121.9	109.7	258.1	243.8	219.4	540.4	510.4	459.4	1080.9	1020.8	918.8
		11	1	1024-QAM	5/6									143.4	135.4	121.9	286.8	270.8	243.8	600.5	567.1	510.4	1201	1134.3	1020.8
8	0	0	2	BPSK	1/2	13	14.4	27	30	58.5	65	117	130	17.2	16.3	14.6	34.4	32.5	29.3	72.1	68.1	61.3	144.1	136.1	122.5
9	1	1	2	QPSK	1/2	26	28.9	54	60	117	130	234	260	34.4	32.5	29.3	68.8	65	58.5	144.1	136.1	122.5	288.2	272.2	245
10	2	2	2	QPSK	3/4	39	43.3	81	90	175.5	195	351	390	51.6	48.8	43.9	103.2	97.5	87.8	216.2	204.2	183.8	432.4	408.3	367.5
11	3	3	2	16-QAM	1/2	52	57.8	108	120	234	260	468	520	68.8	65	58.5	137.6	130	117	288.2	272.2	245	576.5	544.4	490
12	4	4	2	16-QAM	3/4	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
13	5	5	2	64-QAM	2/3	104	115.6	216	240	468	520	936	1040	137.6	130	117	275.3	260	234	576.5	544.4	490	1152.9	1088.9	980
14	6	6	2	64-QAM	3/4	117	130	243	270	526.5	585	1053	1170	154.9	146.3	131.6	309.7	292.5	263.3	648.5	612.5	551.3	1297.1	1225	1102.5
15	7	7	2	64-QAM	5/6	130	144.4	270	300	585	650	1170	1300	172.1	162.5	146.3	344.1	325	292.5	720.6	680.6	612.5	1441.2	1361.1	1225
	8	8	2	256-QAM	3/4	156	173.3	324	360	702	780	1404	1560	206.5	195	175.5	412.9	390	351	864.7	816.7	735	1729.4	1633.3	1470
	9	9	2	256-QAM	5/6	N/A	N/A	360	400	780	866.7	1560	1733.3	229.4	216.7	195	458.8	433.3	390	960.8	907.4	816.7	1921.6	1814.8	1633.3
		10	2	1024-QAM	3/4									258.1	243.8	219.4	516.2	487.5	438.8	1080.9	1020.8	918.8	2161.8	2041.7	1837.5
		11	2	1024-QAM	5/6									286.8	270.8	243.8	573.5	541.7	487.5	1201	1134.3	1020.8	2402	2268.5	2041.7
16	0	0	3	BPSK	1/2	19.5	21.7	40.5	45	87.8	97.5	175.5	195	25.8	24.4	21.9	51.6	48.8	43.9	108.1	102.1	91.9	216.2	204.2	183.8
17	1	1	3	QPSK	1/2	39	43.3	81	90	175.5	195	351	390	51.6	48.8	43.9	103.2	97.5	87.8	216.2	204.2	183.8	432.4	408.3	367.5
18	2	2	3	QPSK	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585	77.4	73.1	65.8	154.9	146.3	131.6	324.3	306.3	275.6	648.5	612.5	551.3
19	3	3	3	16-QAM	1/2	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
20	4	4	3	16-QAM	3/4	117	130	243	270	526.5	585	1053	1170	154.9	146.3	131.6	309.7	292.5	263.3	648.5	612.5	551.3	1297.1	1225	1102.5
21	5	5	3	64-QAM	2/3	156	173.3	324	360	702	780	1404	1560	206.5	195	175.5	412.9	390	351	864.7	816.7	735	1729.4	1633.3	1470
22	6	6	3	64-QAM	3/4	175.5	195	364.5	405	N/A	N/A	1579.5	1755	232.3	219.4	197.4	464.6	438.8	394.9	972.8	918.8	826.9	1945.6	1837.5	1653.8
23	7	7	3	64-QAM	5/6	195	216.7	405	450	877.5	975	1755	1950	258.1	243.8	219.4	516.2	487.5	438.8	1080.9	1020.8	918.8	2161.8	2041.7	1837.5
	8	8	3	256-QAM	3/4	234	260	486	540	1053	1170	2106	2340	309.7	292.5	263.3	619.4	585	526.5	1297.1	1225	1102.5	2594.1	2450	2205
	9	9	3	256-QAM	5/6	260	288.9	540	600	1170	1300	N/A	N/A	344.1	325	292.5	688.2	650	585	1441.2	1361.1	1225	2882.4	2722.2	2450
		10	3	1024-QAM	3/4									387.1	365.6	329.1	774.3	731.3	658.1	1621.3	1531.3	1378.1	3242.6	3062.5	2756.3
		11	3	1024-QAM	5/6									430.1	406.3	365.6	860.3	812.5	731.3	1801.5	1701.4	1531.3	3602.9	3402.8	3062.5
24	0	0	4	BPSK	1/2	26	28.9	54	60	117	130	234	260	34.4	32.5	29.3	68.8	65	58.5	144.1	136.1	122.5	288.2	272.2	245
25	1	1	4	QPSK	1/2	52	57.8	108	120	234	260	468	520	68.8	65	58.5	137.6	130	117	288.2	272.2	245	576.5	544.4	490
26	2	2	4	QPSK	3/4	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
27	3	3	4	16-QAM	1/2	104	115.6	216	240	468	520	936	1040	137.6	130	117	275.3	260	234	576.5	544.4	490	1152.9	1088.9	980
28	4	4	4	16-QAM	3/4	156	173.3	324	360	702	780	1404	1560	206.5	195	175.5	412.9	390	351	864.7	816.7	735	1729.4	1633.3	1470
29	5	5	4	64-QAM	2/3	208	231.1	432	480	936	1040	1872	2080	275.3	260	234	550.6	520	468	1152.9	1088.9	980	2305.9	2177.8	1960
30	6	6	4	64-QAM	3/4	234	260	486	540	1053	1170	2106	2340	309.7	292.5	263.3	619.4	585	526.5	1297.1	1225	1102.5	2594.1	2450	2205
31	7	7	4	64-QAM	5/6	260	288.9	540	600	1170	1300	2340	2600	344.1	325	292.5	688.2	650	585	1441.2	1361.1	1225	2882.4	2722.2	2450
	8	8	4	256-QAM	3/4	312	346.7	648	720	1404	1560	2808	3120	412.9	390	351	825.9	780	702	1729.4	1633.3	1470	3458.8	3266.7	2940
	9	9	4	256-QAM	5/6	N/A	N/A	720	800	1560	1733.3	3120	3466.7	458.8	433.3	390	917.6	866.7	780	1921.6	1814.8	1633.3	3843.1	3629.6	3266.7

MIMO: Multiple TX-RX streams

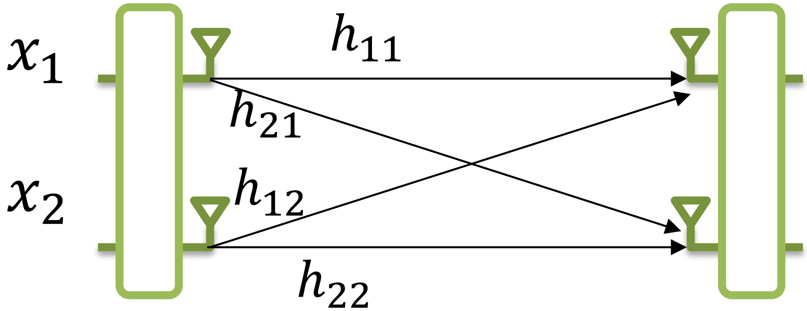
WiFi Access Points 2 to 8 MIMO



WiFi Devices 1 to 2 MIMO

- Power
- Form Factor & Antenna Separation
- Most mobile phones support 2 MIMO

MIMO



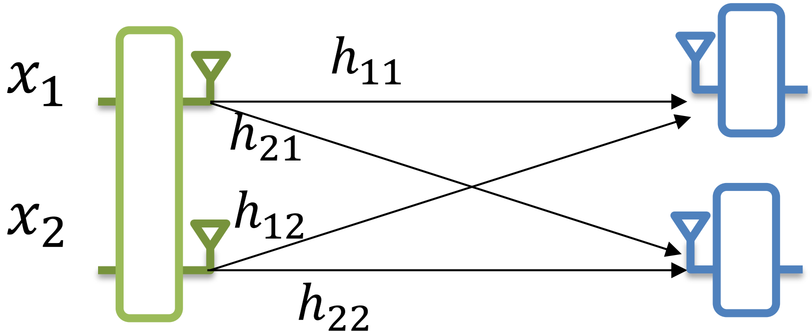
$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Multi-User MIMO



$y_1 = h_{11}x_1 + h_{12}x_2$

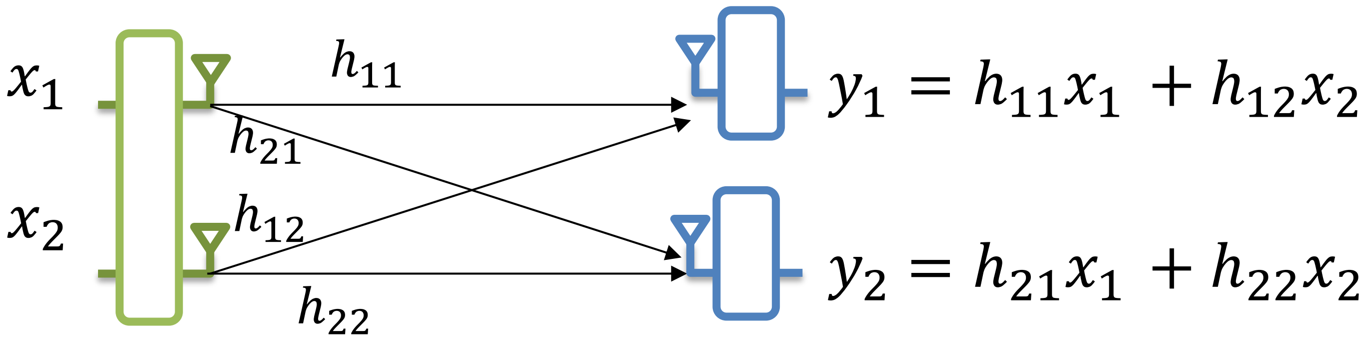
$y_2 = h_{21}x_1 + h_{22}x_2$

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Does not work. Receivers do not have access to each other's signals

Precoding



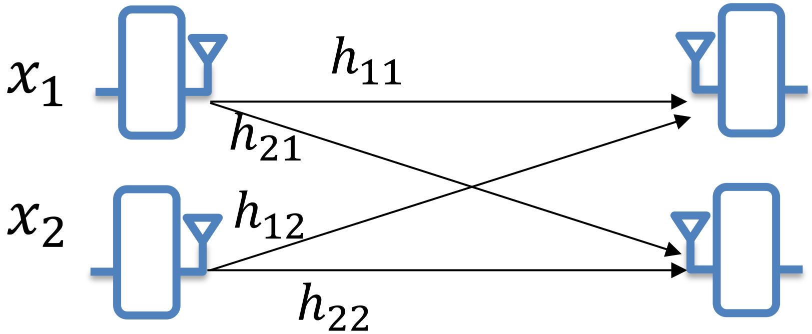
Send: $\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{x}$

Receive: $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{H}\mathbf{H}^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}$

Also known as Beamforming or Zero-Forcing

Requires Channel Feedback!

What about distributed transmitters?



$y_1 = h_{11}x_1 + h_{12}x_2$

$y_2 = h_{21}x_1 + h_{22}x_2$

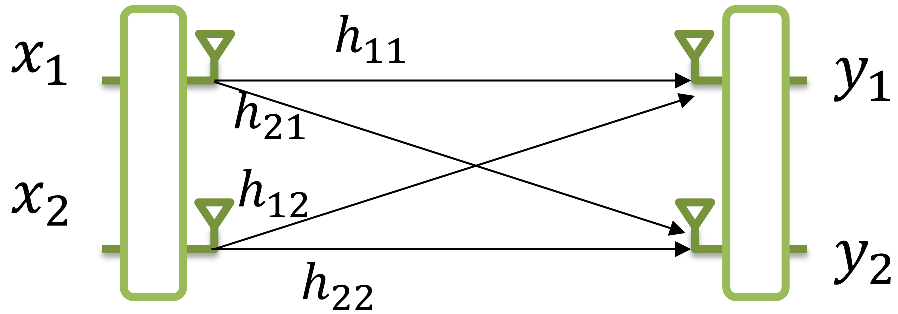
Send: $\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{x}$

Receive: $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{H}\mathbf{H}^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}$

Does not work. Transmitters do not have access to each other's signals

False: what if transmitters are APs connected over Ethernet?

Clock Synchronization in MIMO

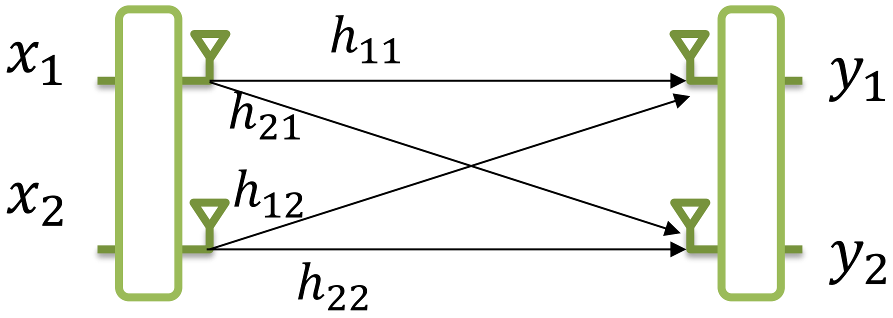


$$y_1 = h_{11}x_1e^{-j2\pi\Delta f_{11}t} + h_{12}x_2e^{-j2\pi\Delta f_{12}t}$$

$$y_2 = h_{21}x_1e^{-j2\pi\Delta f_{21}t} + h_{22}x_2e^{-j2\pi\Delta f_{22}t}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO

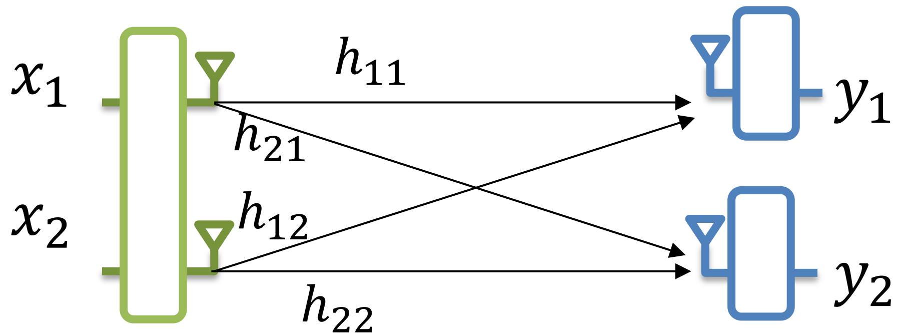


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

On chip MIMO: $\Delta f_{11} = \Delta f_{21} = \Delta f_{12} = \Delta f_{22} = \Delta f$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{-j2\pi\Delta f t} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO

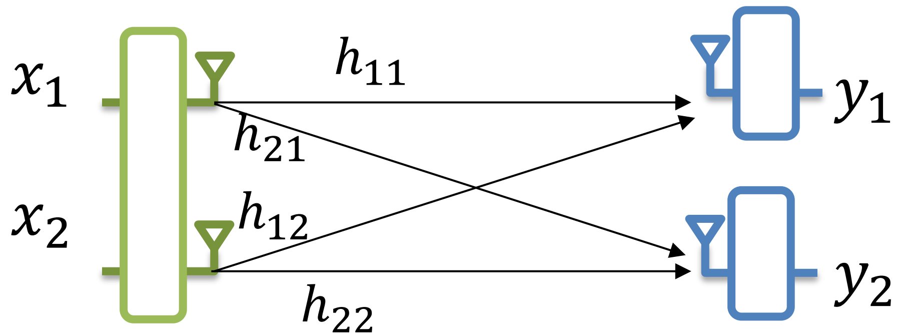


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

MU-MIMO: $\Delta f_{11} = \Delta f_{12}$ and $\Delta f_{21} = \Delta f_{22}$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_1 t} & h_{12} e^{-j2\pi\Delta f_1 t} \\ h_{21} e^{-j2\pi\Delta f_2 t} & h_{22} e^{-j2\pi\Delta f_2 t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO

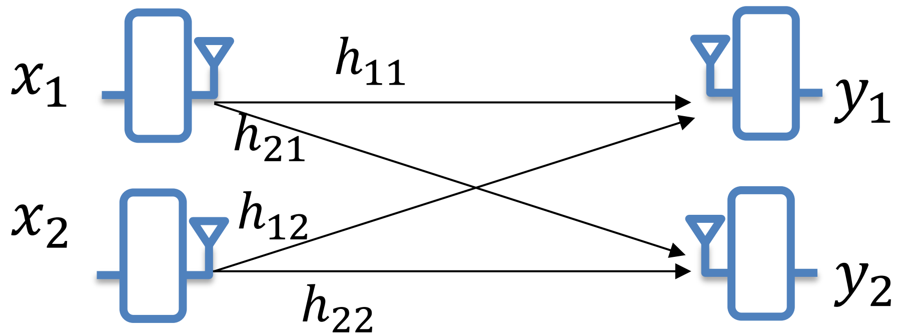


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

MU-MIMO: $\Delta f_{11} = \Delta f_{12}$ and $\Delta f_{21} = \Delta f_{22}$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} e^{-j2\pi\Delta f_1 t} & 0 \\ 0 & e^{-j2\pi\Delta f_2 t} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11}t} & h_{12}e^{-j2\pi\Delta f_{12}t} \\ h_{21}e^{-j2\pi\Delta f_{21}t} & h_{22}e^{-j2\pi\Delta f_{22}t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Distributed MIMO: $\Delta f_{11} \neq \Delta f_{12} \neq \Delta f_{21} \neq \Delta f_{22}$

Advanced Topics!

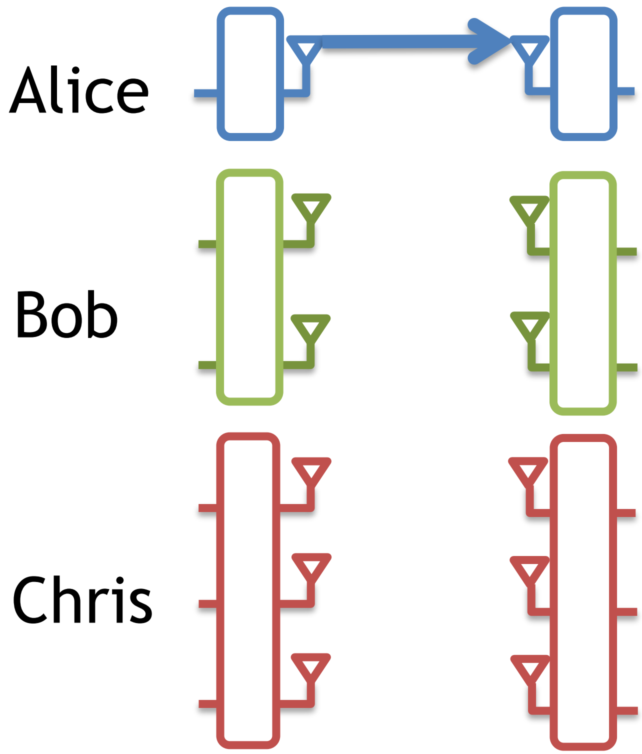
1-antenna devices

2-antenna devices

3-antenna devices

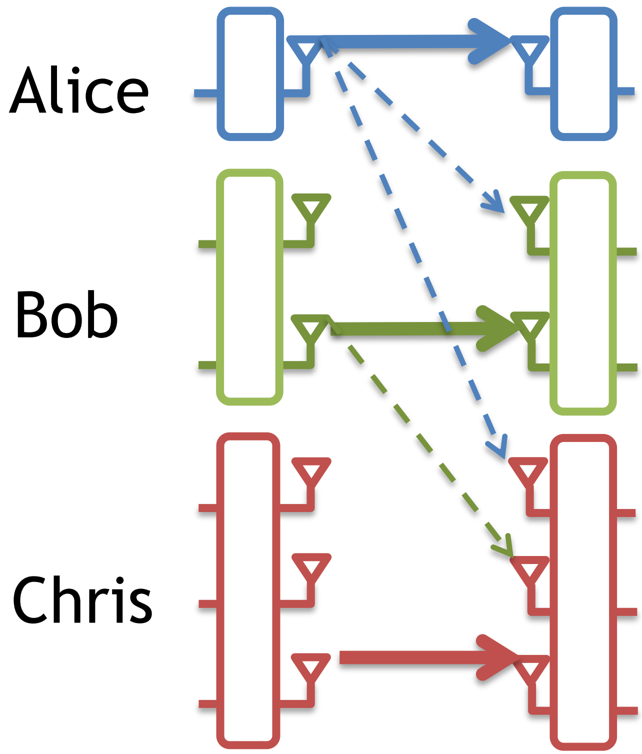
Wireless nodes increasingly have
heterogeneous numbers of antennas

Consider a scenario with Tx-Rx pairs that have a different number of antennas

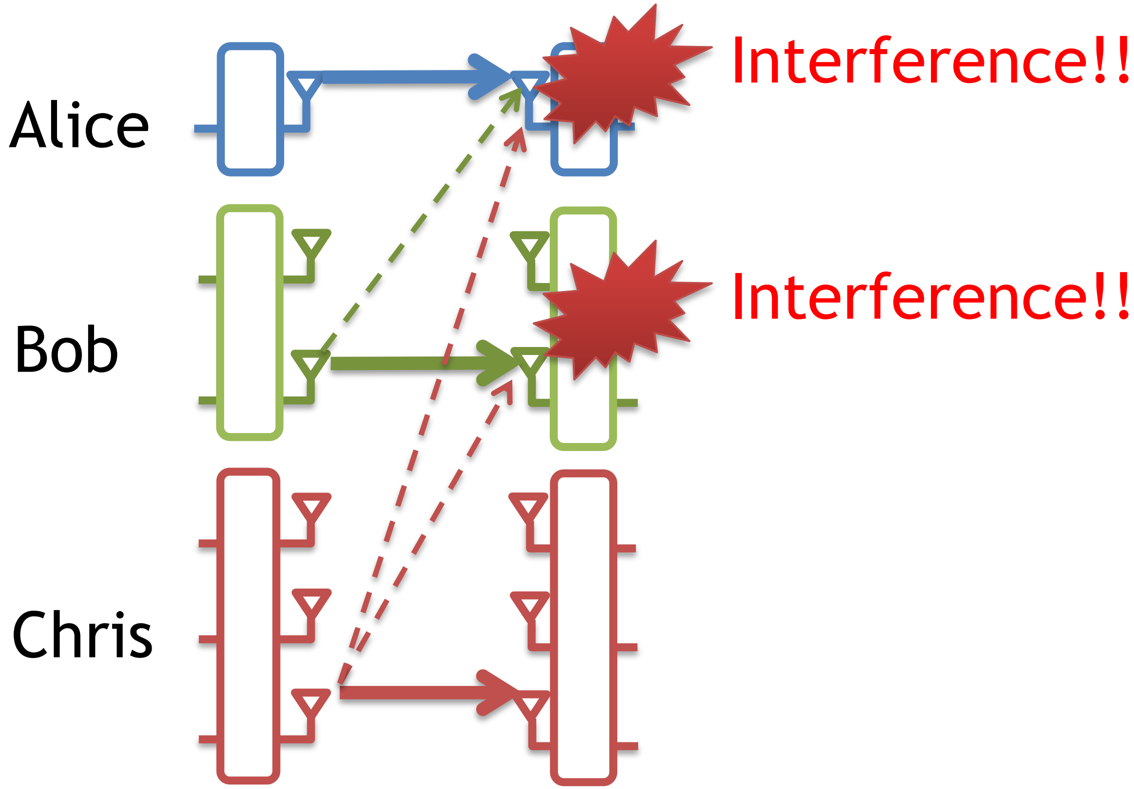


When a single-antenna node transmits, multi-antenna nodes refrain from transmitting

But, MIMO Nodes Can Receive Multiple Concurrent Streams



It's Not That Simple

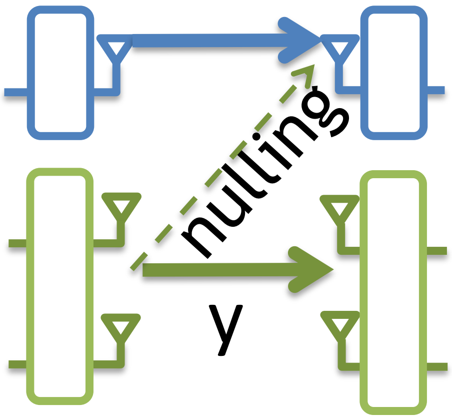


But, how do we transmit without interfering at receivers with fewer antennas?

Interference Nulling

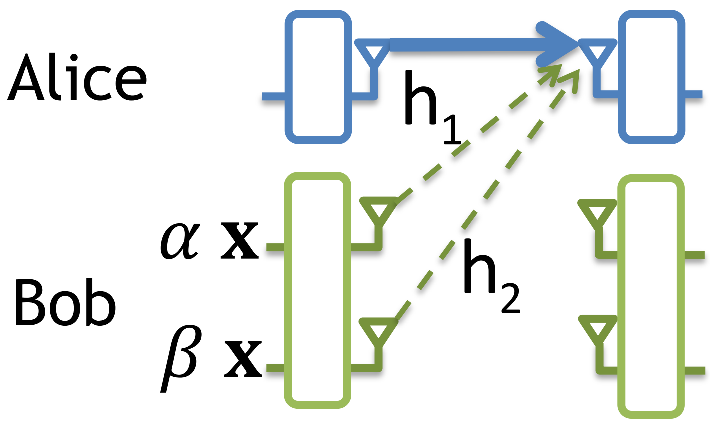
Alice

Bob



- Signals cancel each other at Alice's receiver

Interference Nulling



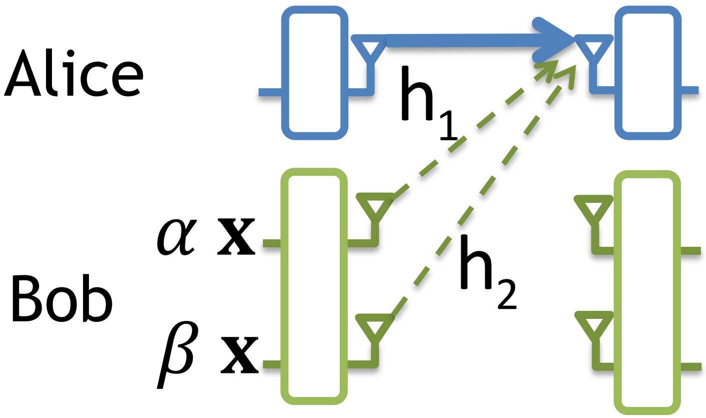
$h_1 \alpha \, \mathbf{x} + \mathrm{h}_2 \beta \, \mathbf{x} = 0$

➔ Nulling : $h_1 \alpha = -\mathrm{h}_2 \beta$

$\neq 0$

- Signals cancel each other at Alice's receiver
- Signals don't cancel each other at Bob's receiver
  - Because channels are different
- Bob's sender learns channels either by feedback from Alice's receiver or via reciprocity

Interference Nulling



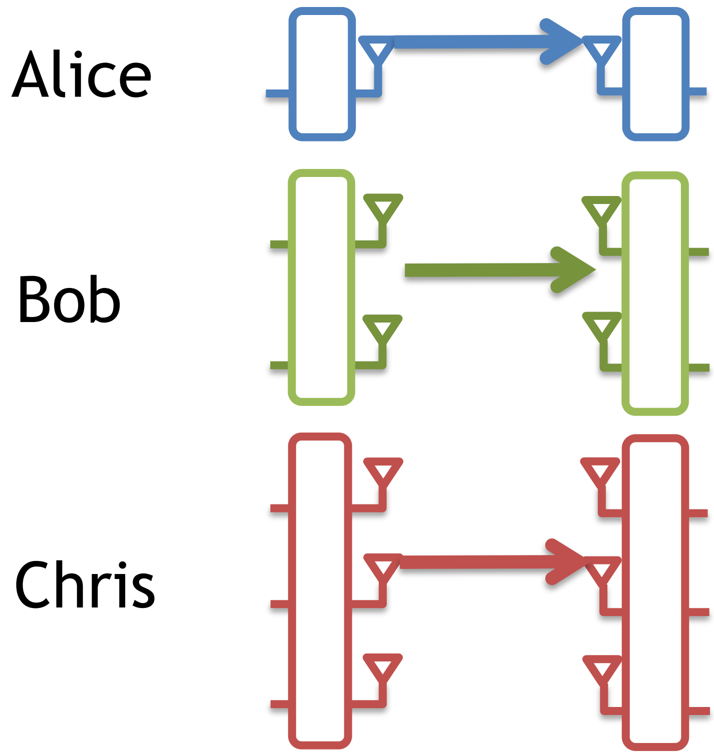
$h_1\alpha\,\mathbf{x} + \mathrm{h}_2\beta\,\mathbf{x} = 0$

➔ Nulling : $h_1\alpha = -\mathrm{h}_2\beta$

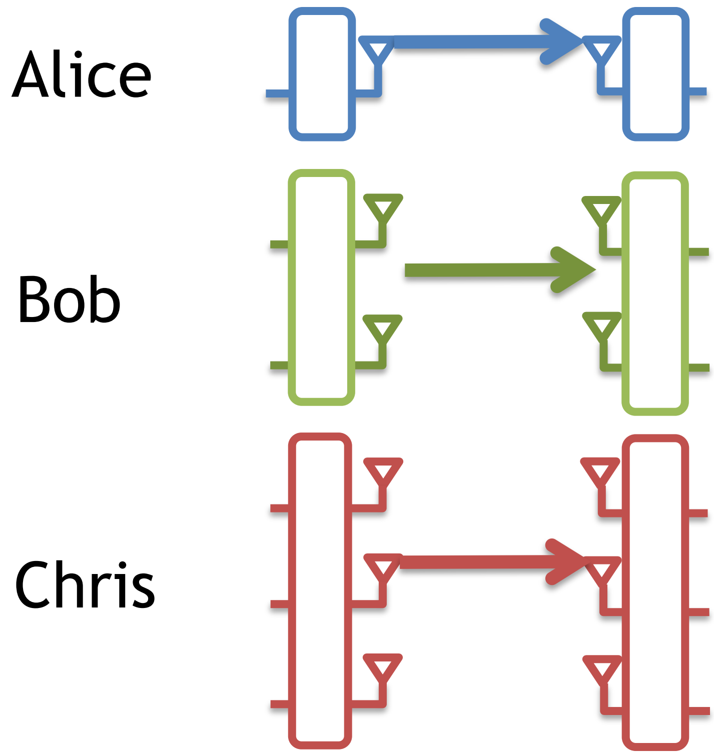
$\neq 0$

Q: How to transmit without interfering with receivers with fewer antennas?

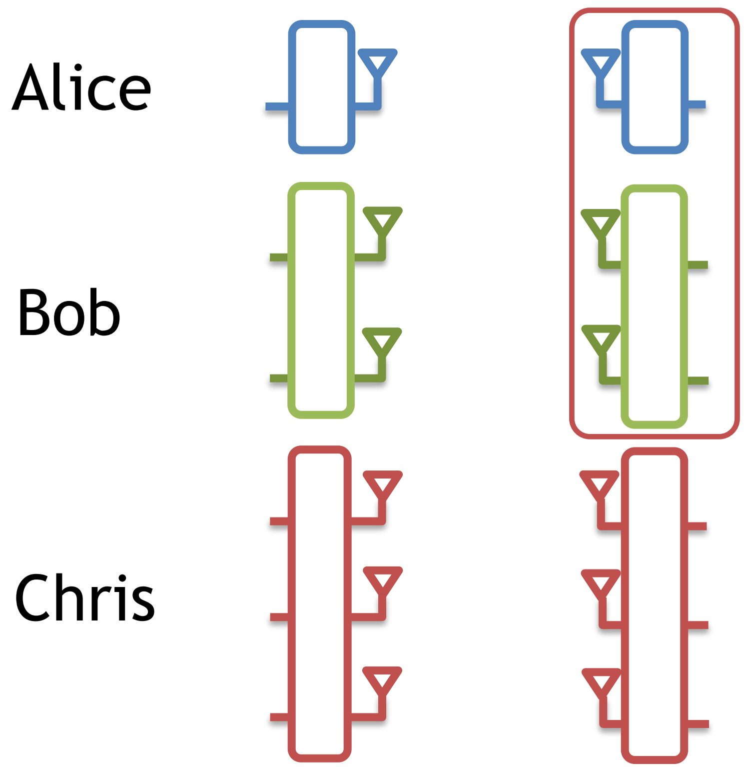
A: Nulling


Alice
Bob
Chris

Is Nulling Alone Enough?

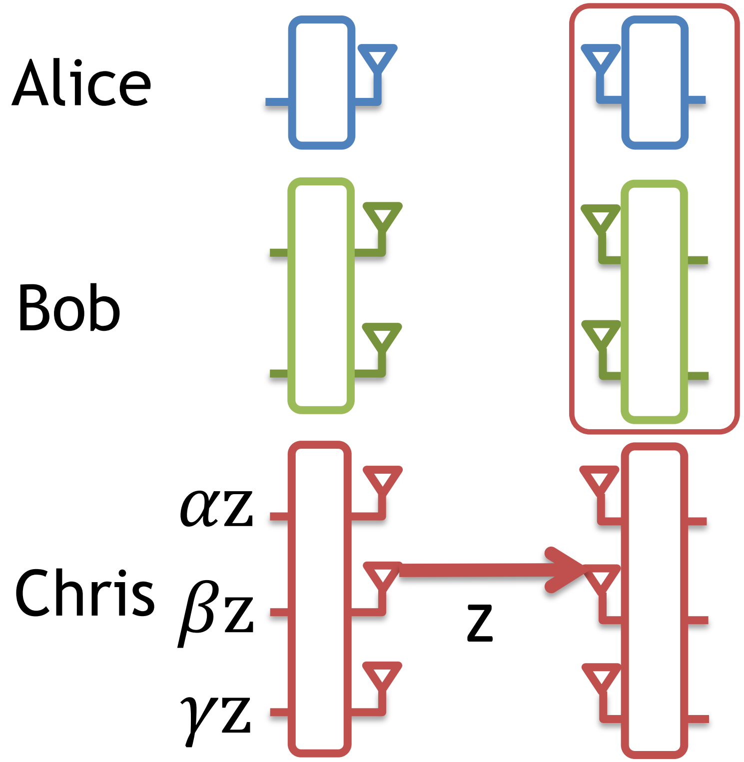


Is Nulling Alone Enough?



Chris needs to null at three antennas

Is Nulling Alone Enough? NO!



null

$$(h_{11}\alpha + h_{21}\beta + h_{31}\gamma)z = 0$$

$$(h_{12}\alpha + h_{22}\beta + h_{32}\gamma)z = 0$$

$$(h_{13}\alpha + h_{23}\beta + h_{33}\gamma)z = 0$$

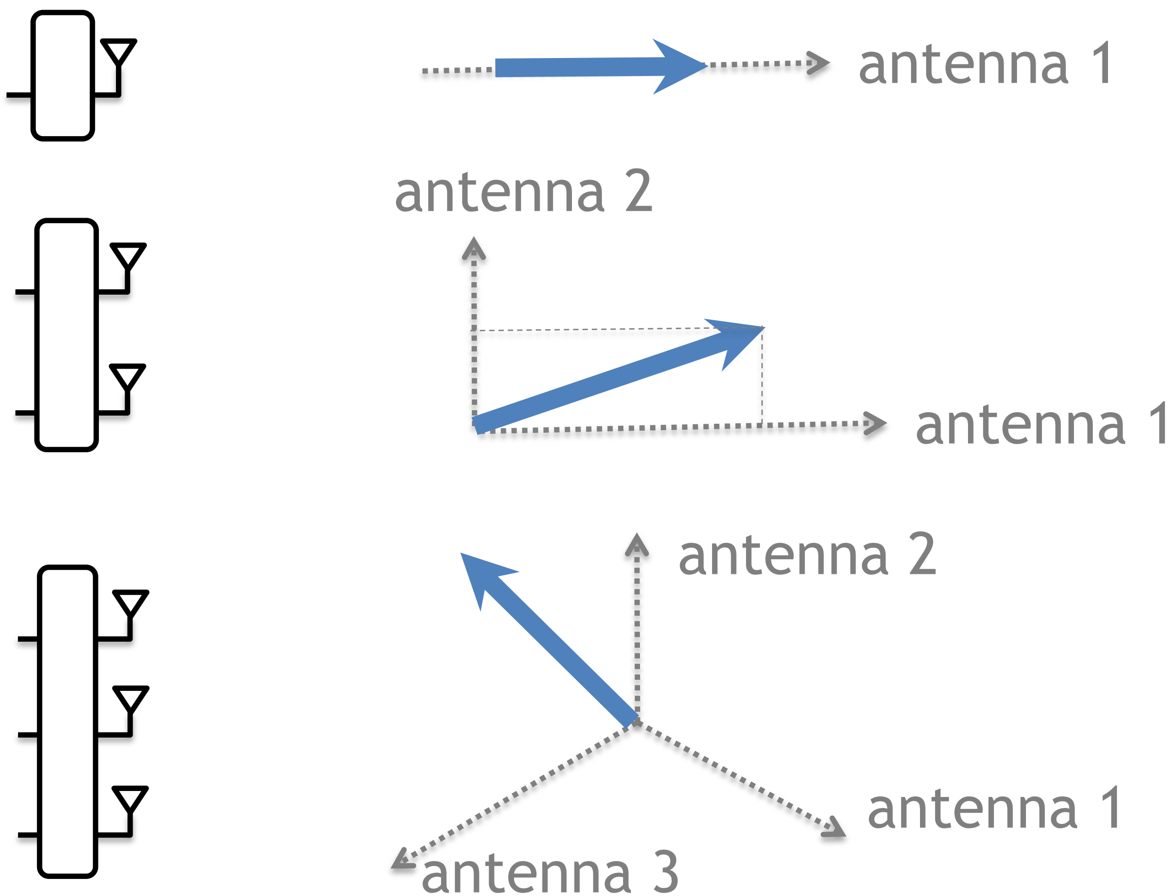
Only Solution: $\alpha = \beta = \gamma = 0$

Transmit Nothing!!!

Are we doomed?

MIMO Recap

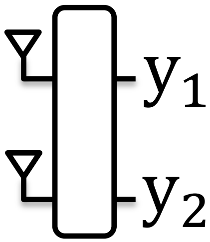
1. N-antenna node receives in N-dimensional space

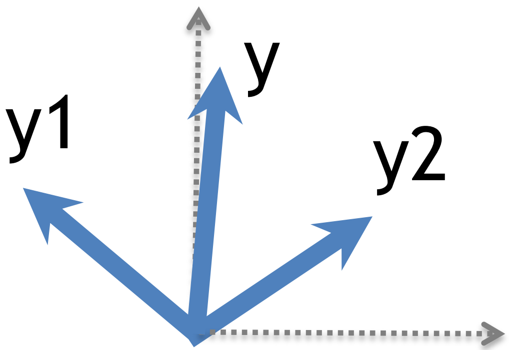


MIMO Recap

1. N-antenna node receives in N-dimensional space
2. N-antenna receiver can decode N signals

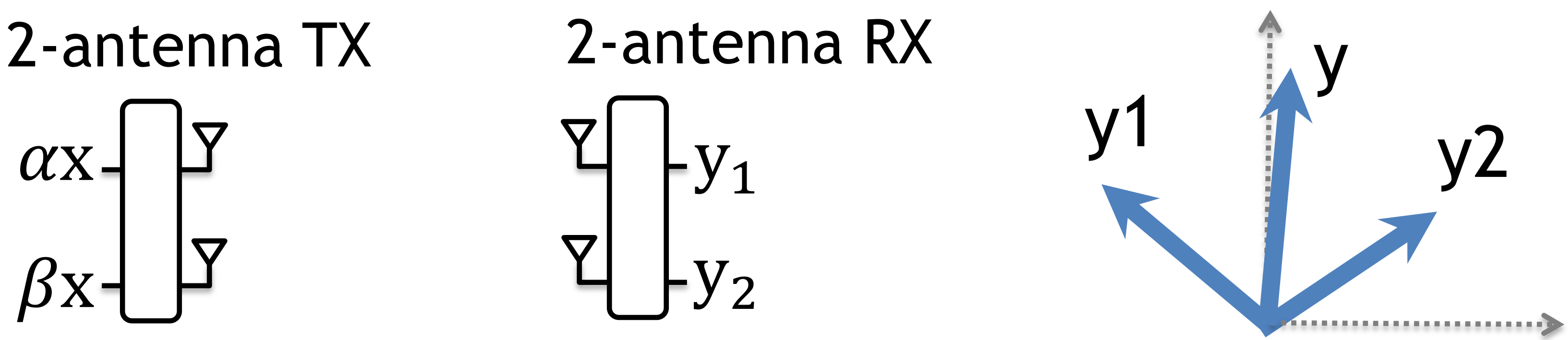
2-antenna RX





MIMO Recap

1. N-antenna node receives in N-dimensional space
2. N-antenna receiver can decode N signals
3. Transmitter can rotate the received signal

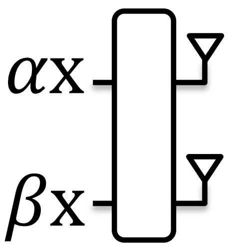


x is the sender's symbol, (y1,y2) is the received symbol, (α, β) is the pre-coding vector, and H is the channel matrix

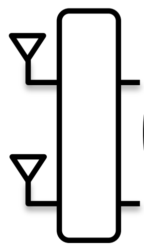
MIMO Recap

1. N-antenna node receives in N-dimensional space
2. N-antenna receiver can decode N signals
3. Transmitter can rotate the received signal

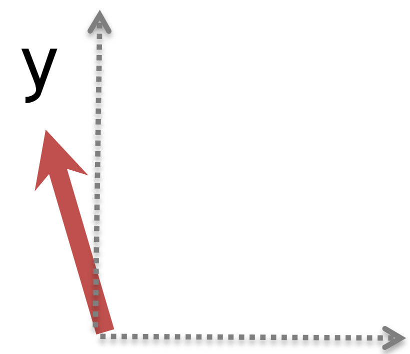
2-antenna TX



2-antenna RX



$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = H \begin{pmatrix} \alpha \\ \beta \end{pmatrix} x$$

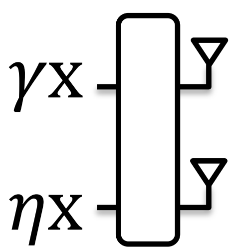


x is the sender's symbol, (y1,y2) is the received symbol, (α, β) is the pre-coding vector, and H is the channel matrix

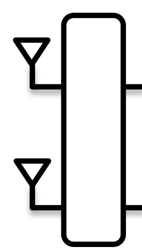
MIMO Recap

1. N-antenna node receives in N-dimensional space
2. N-antenna receiver can decode N signals
3. Transmitter can rotate the received signal

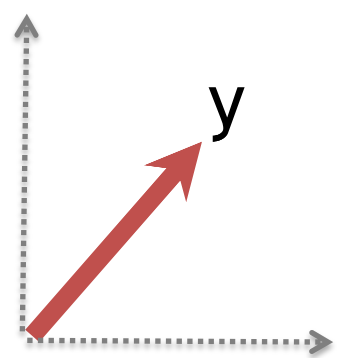
2-antenna TX



2-antenna RX



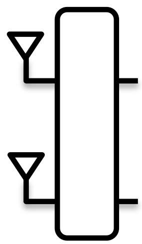
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = H \begin{pmatrix} \gamma \\ \eta \end{pmatrix} x$$

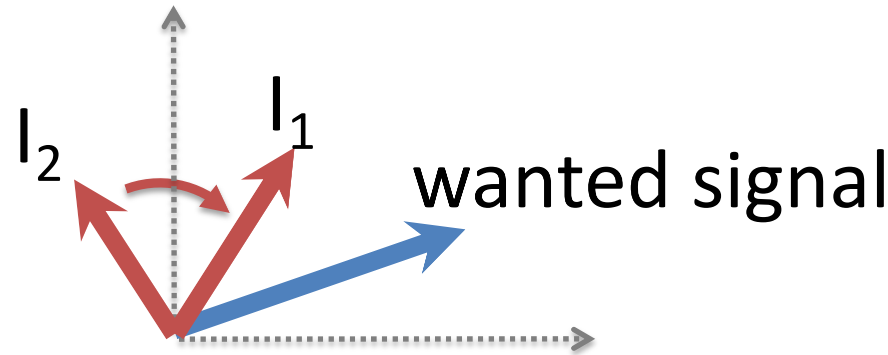


Sender can rotate the received symbol by multiplying with a different pre-coding vector; However to align along a particular direction, sender needs to know H

Interference Alignment

2-antenna receiver

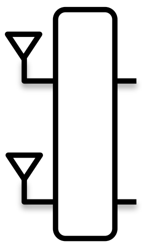


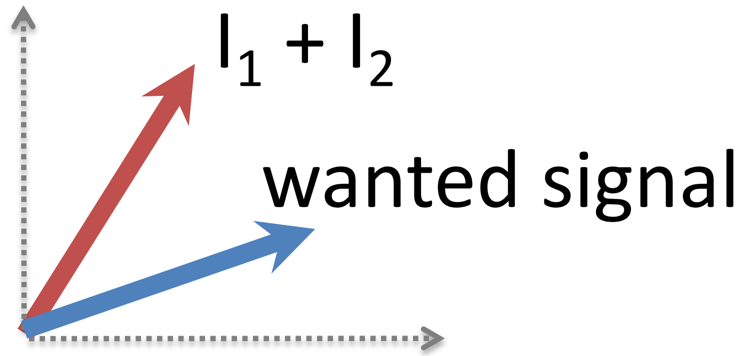


If I_1 and I_2 are aligned,

Interference Alignment

2-antenna receiver



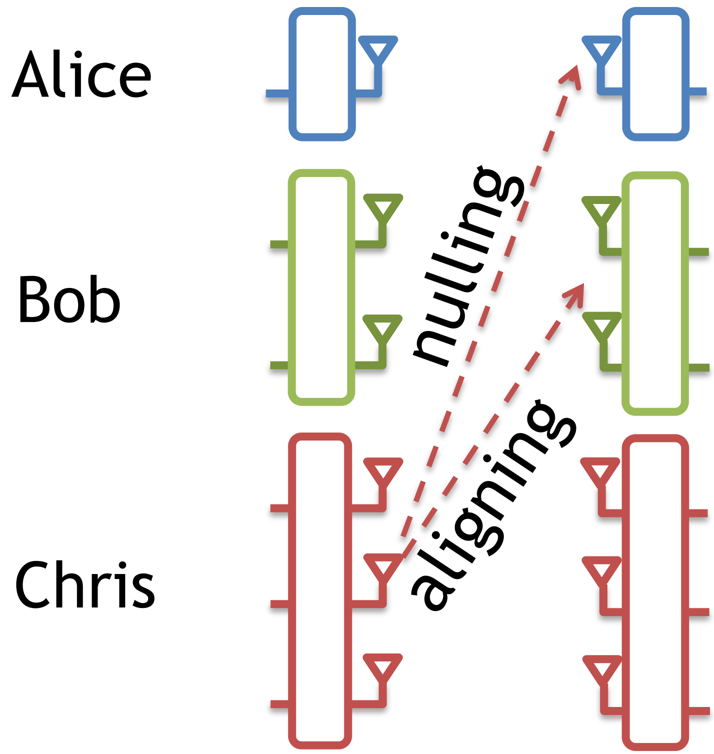


If I_1 and I_2 are aligned,

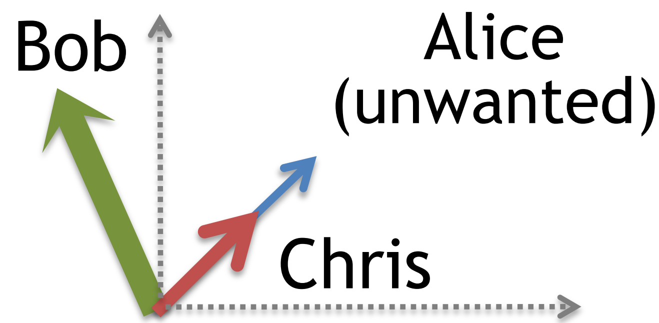
→ appear as one interferer

→ 2-antenna receiver can decode the wanted signal

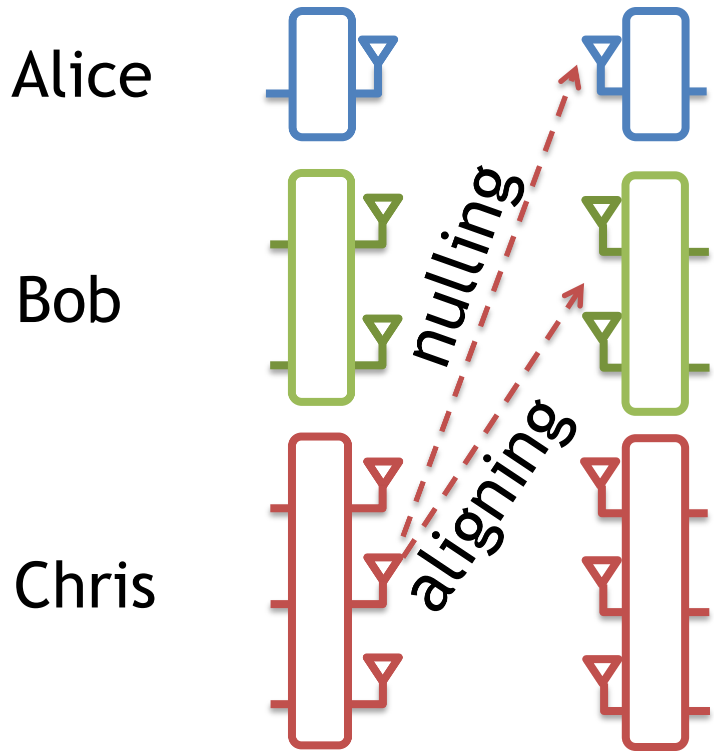
Use Nulling and Alignment

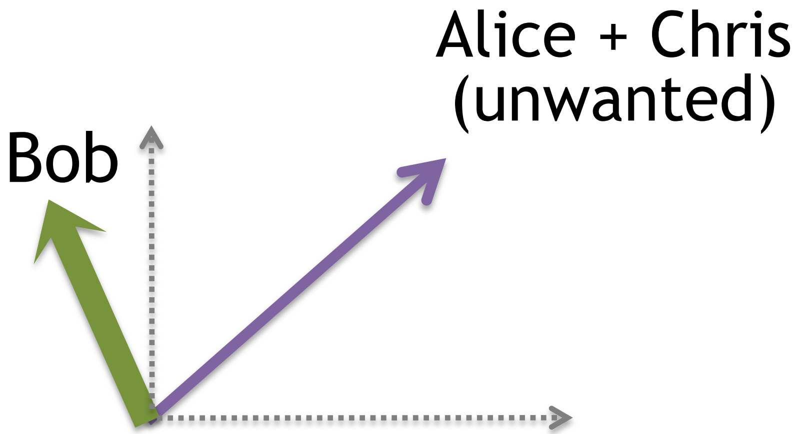


Null as before



Use Nulling and Alignment

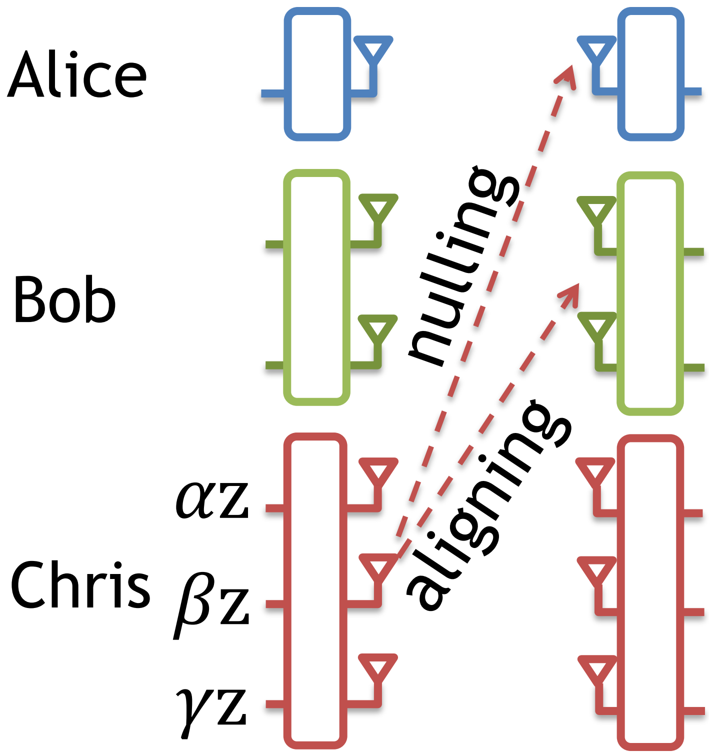




2-signals in 2D-space

➔Can decode Bob's signal

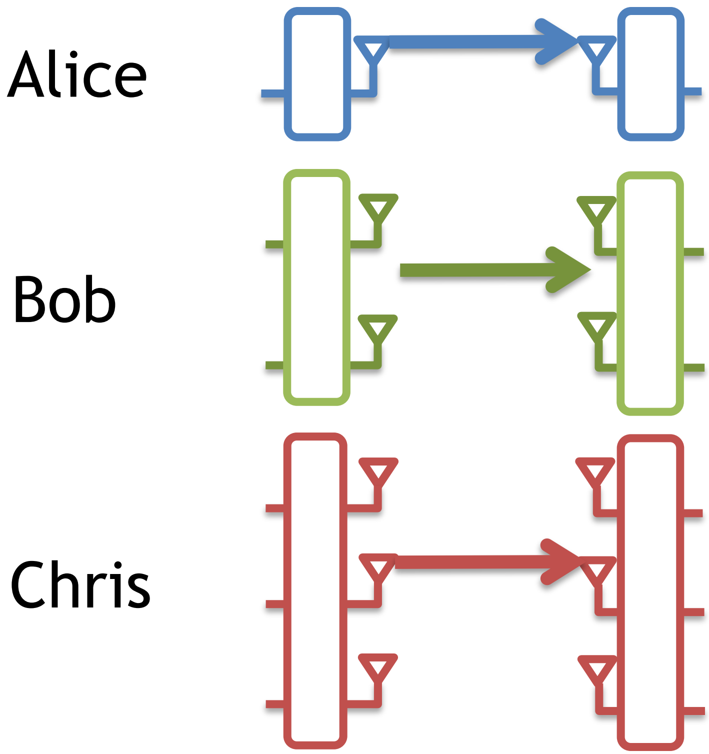
Use Nulling and Alignment



$$(h_{11}\alpha + h_{21}\beta + h_{31}\gamma)z = 0$$

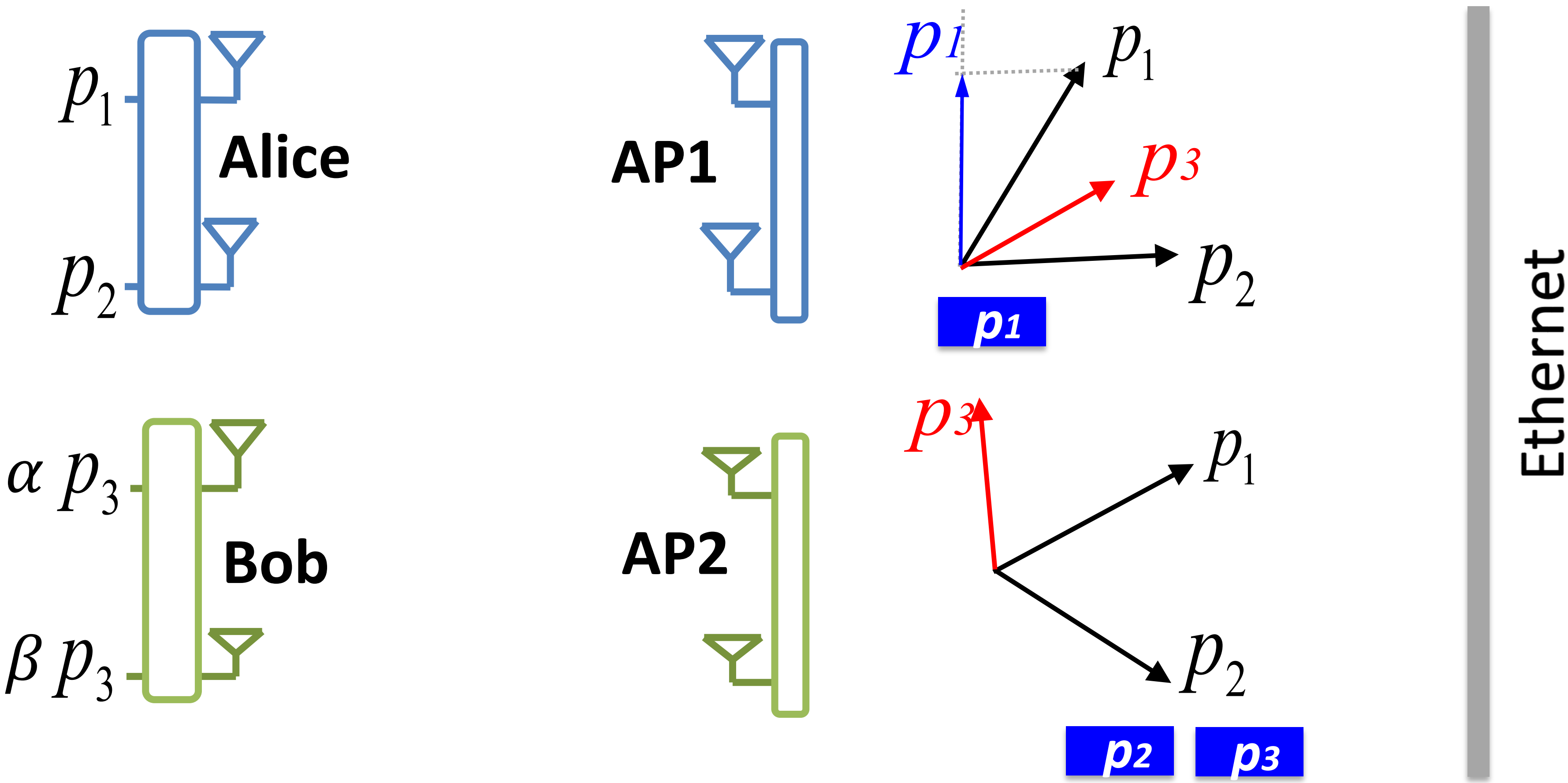
$$\begin{pmatrix} h_{12}\alpha + h_{22}\beta + h_{32}\gamma \\ h_{13}\alpha + h_{23}\beta + h_{33}\gamma \end{pmatrix} \propto \begin{pmatrix} h_{a1} \\ h_{a2} \end{pmatrix}$$

Use Nulling and Alignment



3 packets though receivers have fewer than 3 antennas

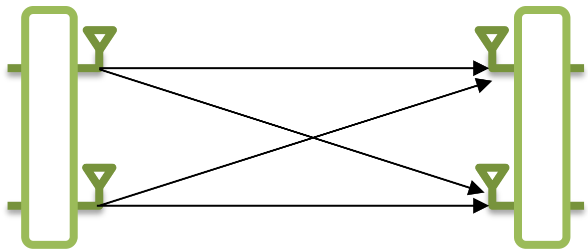
Interference Alignment and Cancellation (IAC)



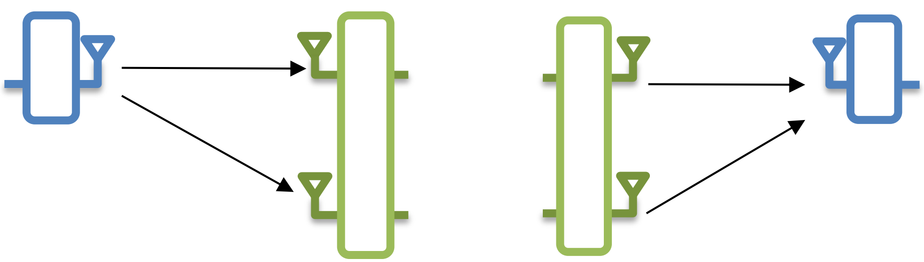
- Align P3 with P2 at AP1 → AP1 decodes P1 to its bits
- AP1 broadcasts P1 on Ethernet
- AP2 subtracts/cancels P1→ decodes P2, P3

This Lecture

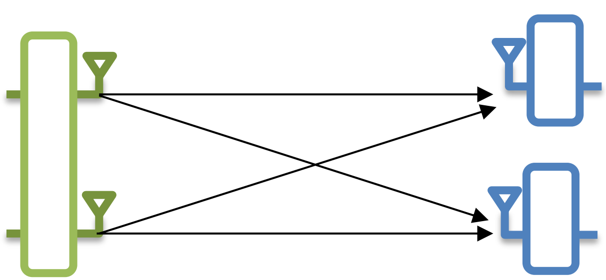
MIMO Multiplexing Gain



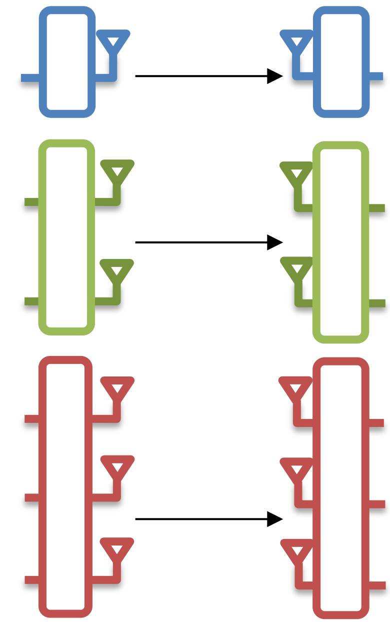
MIMO TX/RX Diversity Gain



MU-MIMO Beamforming



Heterogeneous # of antennas (Interference Nulling/Alignment)



Distributed MIMO