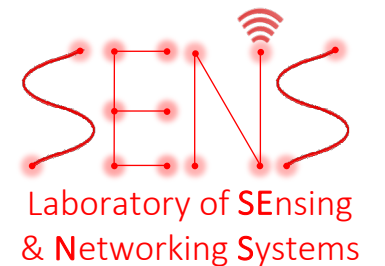
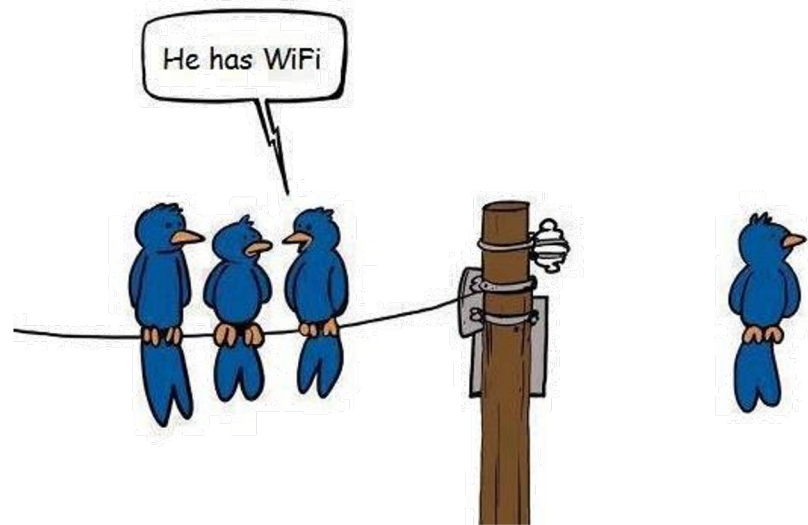


COM-405: Mobile Networks

Lecture 4.1: Data Rates, Coding, & Rate Adaptation Haitham Hassanieh



Wireless Data Rates

- **Data Rate**
 - *Bandwidth: B Samples/sec*
 - *Modulation: k Bits/sample*

$$\text{Data Rate} = B \text{ Samples/sec} \times k \text{ Bits/sample}$$

e.g. Bandwidth: $B = 10 \text{ MHz}$

BPSK:

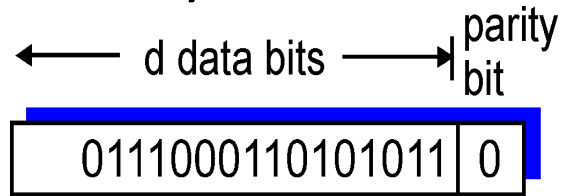
$$\text{Data Rate} = 10 \text{ MS/s} \times 1 \text{ bps} = 10 \text{ Mbps}$$

64 QAM:

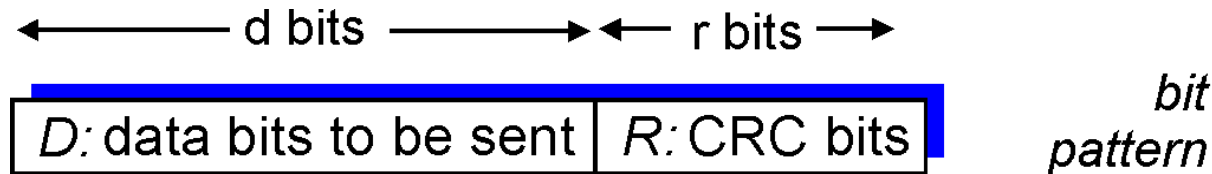
$$\text{Data Rate} = 10 \text{ MS/s} \times 6 \text{ bps} = 60 \text{ Mbps}$$

Error Detection

- Add Redundant bits in order to detect errors at receiver
- Checksums → Detect Errors
 - Parity Check



- CRC: Cyclic Redundancy Check



Error Detection

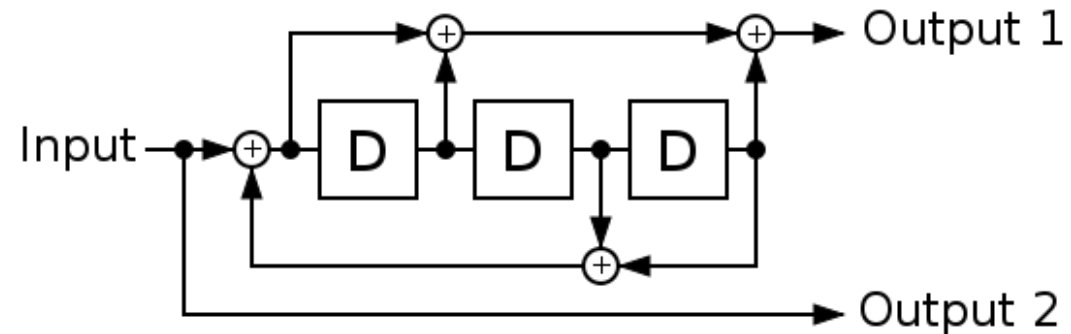
- Checksum added at the end of each packet.
- If packet (frame, sequence of bits) fails the checksum test
→ Packet is dropped → Packet loss.
- Actual Throughput < Data Rate

Coding: Forward Error Correction (FEC)

- Add Redundant bits in order to correct errors at receiver

- FEC: Forward Error Correction

- Repetition Code
- Convolutional codes
- Reed Solomon codes
- Turbo codes
- Polar codes
- LDPC codes

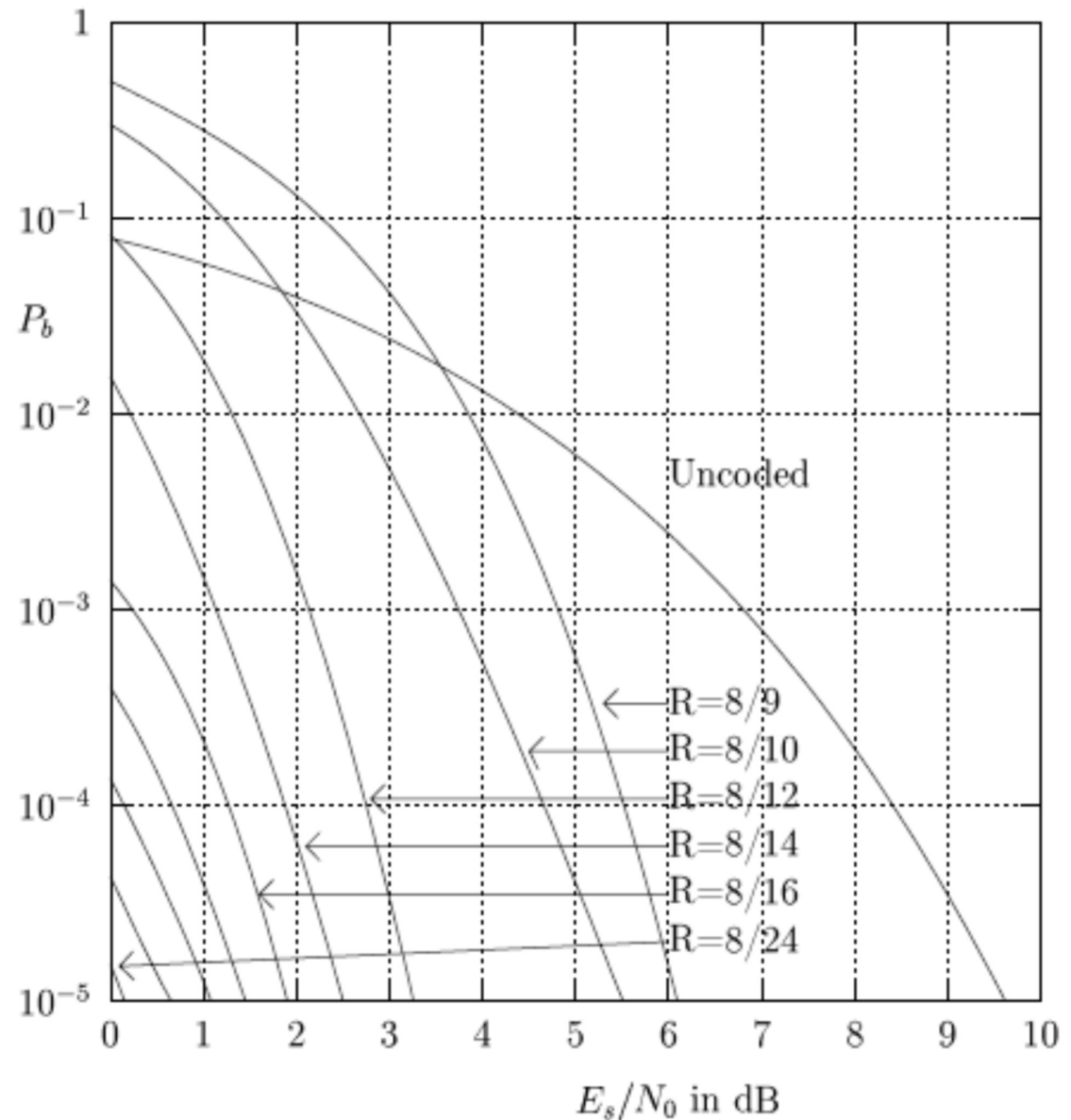


Coding: Forward Error Correction (FEC)

- Code Rate: $R = \frac{\text{\# of data bits (uncoded)}}{\text{\# of transmitted bits (coded)}}$
 - E.g. coding rate 2/3, for every 2 data bits, we transmit 3 bits.
 - ↗ code rate → ↘ redundancy → ↗ bit errors
 - ↘ code rate → ↗ redundancy → ↘ bit errors

Coding: Forward Error Correction (FEC)

- Convolutional code over BPSK with rate R
- BER vs SNR curve for different R .



Wireless Data Rates

- **Data Rate**
 - Bandwidth: B Samples/sec
 - Modulation: k Bits/sample
 - Coding Rate: R Data Bits/ Coded Bits

$$\begin{aligned}\text{Data Rate} &= B \text{ Samples/sec} \times k \text{ Coded Bits/sample} \times R \text{ Data Bits /Coded Bits} \\ &= B \times k \times R \text{ bits/sec}\end{aligned}$$

BPSK with code rate 1/2:

$$\text{Data Rate} = 10 \text{ MS/s} \times 1 \text{ bps} \times 1/2 = 5 \text{ Mbps}$$

64 QAM with code rate 1/3:

$$\text{Data Rate} = 10 \text{ MS/s} \times 6 \text{ bps} \times 1/3 = 20 \text{ Mbps}$$

Wireless Data Rates

- *Data Rate*
 - *Bandwidth: B Samples/sec*
 - *Modulation: k Bits/sample*
 - *Coding Rate: R Data Bits/ Coded Bits*

$$\begin{aligned}\text{Data Rate} &= B \text{ Samples/sec} \times k \text{ Coded Bits/sample} \times R \text{ Data Bits /Coded Bits} \\ &= B \times k \times R \text{ bits/sec}\end{aligned}$$

- *Capacity*
 - *Maximum Achievable Data Rate*
 - *Shannon Capacity Theorem: $\text{Capacity} = \text{Bandwidth} \times \log_2(1 + \text{SNR})$*

Wireless Data Rates

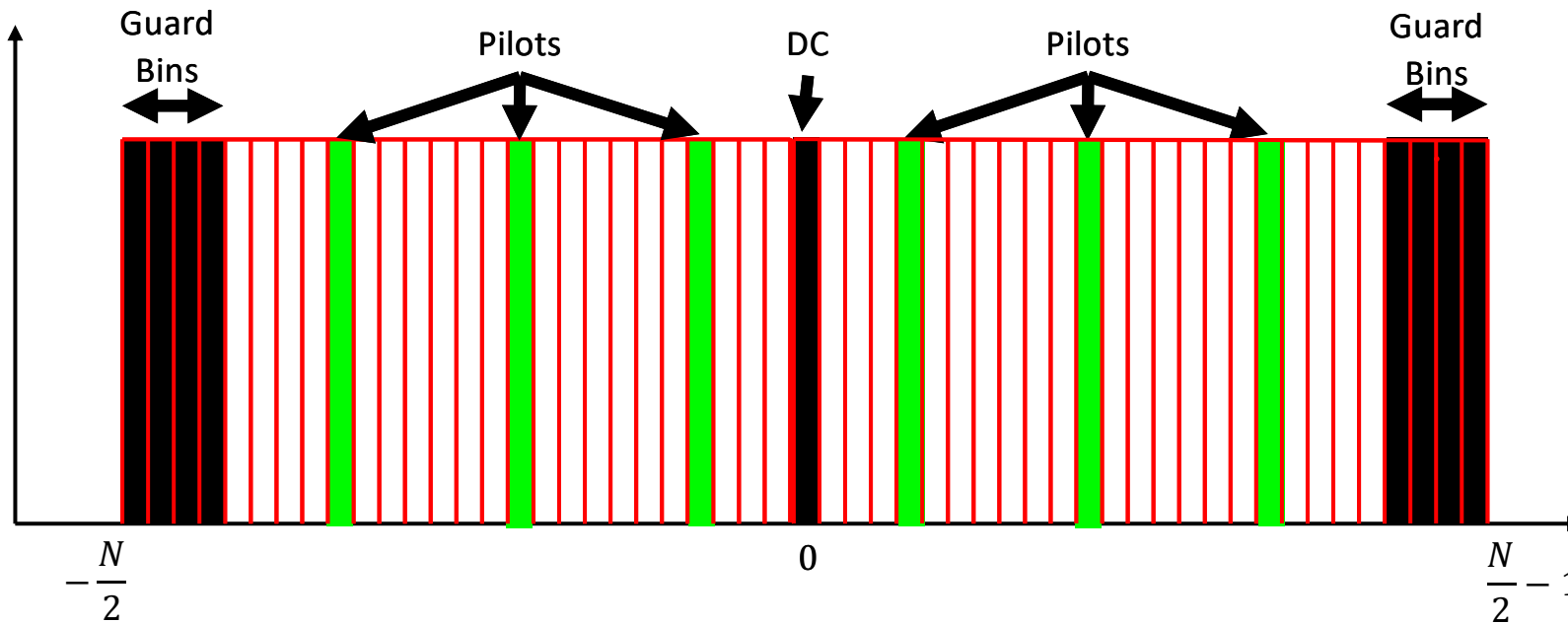
- *Practical Systems cannot support any coding and modulation!*
- *Predefined set of modulation and coding schemes (MCS) baked into the hardware.*

802.11g: WiFi 3

MCS index(read as little endian)	RATE bits R1-R4	Modulation type	Coding rate	Data rate (Mbit/s)
11	1101	BPSK	1/2	6
15	1111	BPSK	3/4	9
10	0101	QPSK	1/2	12
14	0111	QPSK	3/4	18
9	1001	16-QAM	1/2	24
13	1011	16-QAM	3/4	36
8	0001	64-QAM	2/3	48
12	0011	64-QAM	3/4	54

Wireless Data Rates

- *OFDM Data Rate* = *Bandwidth* $\times$ *Modulation Rate* $\times$ *Code Rate* $\times$ (1 – *Overhead*)



$$\text{Overhead} = \frac{\# \text{Guard Bins} + \# \text{Pilot Bins} + \text{DC Bin} + CP}{N + CP}$$

WiFi 802.11ac MCS (Modulations and Coding Schemes)

MCS Index			Spatial Stream	Modulation	Coding	20MHz		40MHz		80MHz		160MHz	
HT	VHT	HE				0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI
0	0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65
1	1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130
2	2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195
3	3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260
4	4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390
5	5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520
6	6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585
7	7	7	1	64-QAM	5/6	65	72.2	135	150	292.5	325	585	650
	8	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780

- *OFDM Data Rate* = *Bandwidth* × *Modulation Rate* × *Code Rate* × (1 – *Overhead*)

MCS 0 with 20 MHz channel and 0.8μs Guard Interval (CP)

$$CP = 0.8\mu s \times 20 \text{ MS/s} = 16 \text{ samples} \quad N = 64, \quad 4 \text{ Pilots}, \quad 7 \text{ Guard bins}, \quad 1 \text{ DC bin}$$

$$\text{Overhead} = \frac{\#Guard \text{ Bins} + \#Pilot \text{ Bins} + DC \text{ Bin} + CP}{N + CP} = \frac{7 + 4 + 1 + 16}{64 + 16} = \frac{28}{80}$$

$$\text{OFDM Data Rate} = 20 \text{ MS/s} \times 1 \text{ bit/sample} \times 1/2 \times (1 - 28/80) = 6.5 \text{ Mb/s}$$

WiFi 802.11ac MCS (Modulations and Coding Schemes)

MCS Index			Spatial Stream	Modulation	Coding	20MHz		40MHz		80MHz		160MHz	
HT	VHT	HE				0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI
0	0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65
1	1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130
2	2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195
3	3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260
4	4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390
5	5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520
6	6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585
7	7	7	1	64-QAM	5/6	65	72.2	135	150	292.5	325	585	650
	8	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780

- *OFDM Data Rate* = *Bandwidth* × *Modulation Rate* × *Code Rate* × (1 − *Overhead*)

MCS 6 with 40 MHz channel and 0.8μs Guard Interval (CP)

$$CP = 0.8\mu s \times 40 \text{ MS/s} = 32 \text{ samples} \quad N = 128, \quad 6 \text{ Pilots}, \quad 11 \text{ Guard bins}, \quad 3 \text{ DC bin}$$

$$\text{Overhead} = \frac{\#Guard \text{ Bins} + \#Pilot \text{ Bins} + DC \text{ Bin} + CP}{N + CP} = \frac{11 + 6 + 3 + 32}{128 + 32} = \frac{52}{160}$$

$$\text{OFDM Data Rate} = 40 \text{ MS/s} \times 6 \text{ bit/sample} \times 3/4 \times (1 - 52/160) = 121.5 \text{ Mb/s}$$

MCS Index			Spatial Stream	Modulation	Coding	20MHz		40MHz		80MHz		160MHz	
HT	VH T	HE				0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI
0	0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65
1	1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130
2	2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195
3	3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260
4	4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390
5	5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520
6	6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585
7	7	7	1	64-QAM	5/6	65	72.2	135	150	292.5	325	585	650
	8	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780

MCS Index			Spatial Stream	Modulation	Coding	20MHz		40MHz		80MHz		160MHz	
HT	VH T	HE				0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI
0	0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65
1	1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130
2	2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195
3	3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260
4	4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390
5	5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520
6	6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585
7	7	7	1	64-QAM	5/6	65	72.2	135	150	292.5	325	585	650
	8	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780

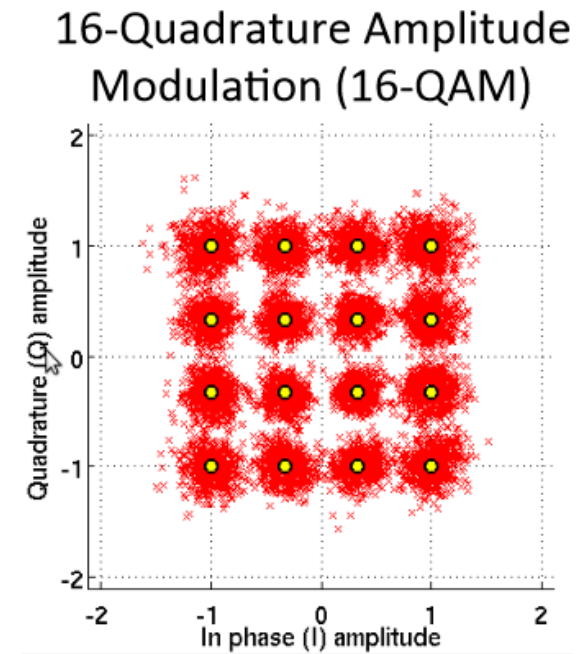
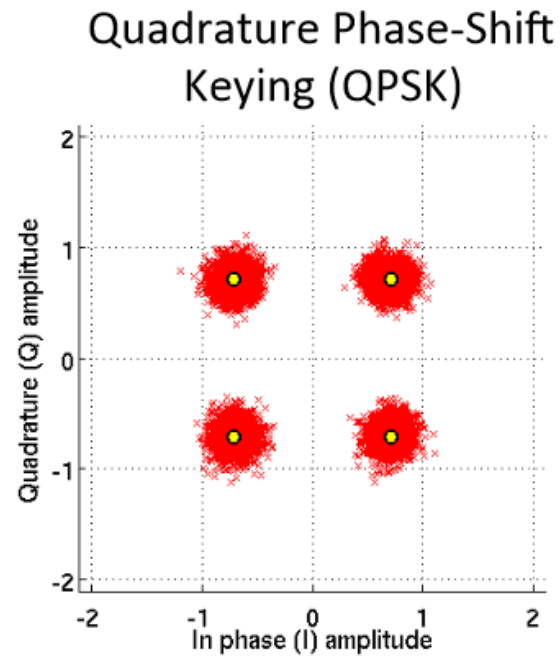
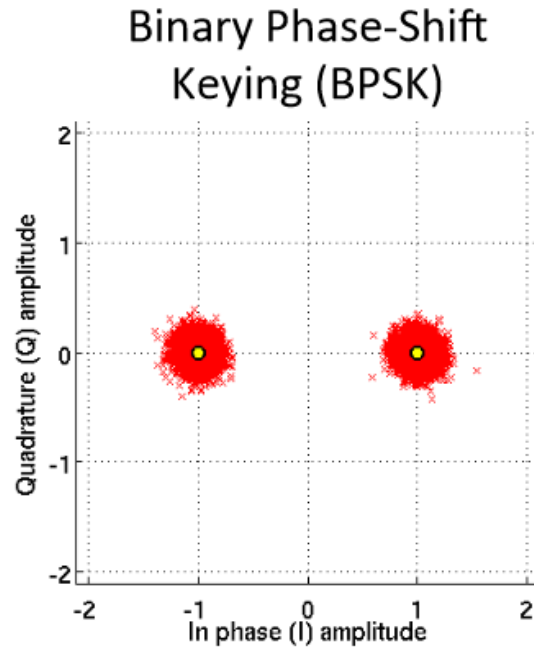
MCS Index			Spatial Stream	Modulation	Coding	OFDM (Prior 11ax)								OFDM (802.11ax)											
						20MHz		40MHz		80MHz		160MHz		20MHz			40MHz			80MHz			160MHz		
HT	VH T	HE				0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	0.4μs GI	0.8μs GI	1.6μs GI	3.2μs GI	0.8μs GI	1.6μs GI	3.2μs GI	0.8μs GI	1.6μs GI	3.2μs GI	0.8μs GI	1.6μs GI	3.2μs GI
0	0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65	8.6	8.1	7.3	17.2	16.3	14.6	36	34	30.6	72.1	68.1	61.3
1	1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130	17.2	16.3	14.6	34.4	32.5	29.3	72.1	68.1	61.3	144.1	136.1	122.5
2	2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195	25.8	24.4	21.9	51.6	48.8	43.9	108.1	102.1	91.9	216.2	204.2	183.8
3	3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260	34.4	32.5	29.3	68.8	65	58.5	144.1	136.1	122.5	288.2	272.2	245
4	4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390	51.6	48.8	43.9	103.2	97.5	87.8	216.2	204.2	183.8	432.4	408.3	367.5
5	5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520	68.8	65	58.5	137.6	130	117	288.2	272.2	245	576.5	544.4	490
6	6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585	77.4	73.1	65.8	154.9	146.3	131.6	324.3	306.3	275.6	648.5	612.5	551.3
7	7	7	1	64-QAM	5/6	65	72.2	135	150	292.5	325	585	650	86	81.3	73.1	172.1	162.5	146.3	360.3	340.3	306.3	720.6	680.6	612.5
	8	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
	9	9	1	256-QAM	5/6	N/A	N/A	180	200	390	433.3	780	866.7	114.7	108.3	97.5	229.4	216.7	195	480.4	453.7	408.3	960.8	907.4	816.7
		10	1	1024-QAM	3/4									129	121.9	109.7	258.1	243.8	219.4	540.4	510.4	459.4	1080.9	1020.8	918.8
		11	1	1024-QAM	5/6									143.4	135.4	121.9	286.8	270.8	243.8	600.5	567.1	510.4	1201	1134.3	1020.8
8	0	0	2	BPSK	1/2	13	14.4	27	30	58.5	65	117	130	17.2	16.3	14.6	34.4	32.5	29.3	72.1	68.1	61.3	144.1	136.1	122.5
9	1	1	2	QPSK	1/2	26	28.9	54	60	117	130	234	260	34.4	32.5	29.3	68.8	65	58.5	144.1	136.1	122.5	288.2	272.2	245
10	2	2	2	QPSK	3/4	39	43.3	81	90	175.5	195	351	390	51.6	48.8	43.9	103.2	97.5	87.8	216.2	204.2	183.8	432.4	408.3	367.5
11	3	3	2	16-QAM	1/2	52	57.8	108	120	234	260	468	520	68.8	65	58.5	137.6	130	117	288.2	272.2	245	576.5	544.4	490
12	4	4	2	16-QAM	3/4	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
13	5	5	2	64-QAM	2/3	104	115.6	216	240	468	520	936	1040	137.6	130	117	275.3	260	234	576.5	544.4	490	1152.9	1088.9	980
14	6	6	2	64-QAM	3/4	117	130	243	270	526.5	585	1053	1170	154.9	146.3	131.6	309.7	292.5	263.3	648.5	612.5	551.3	1297.1	1225	1102.5
15	7	7	2	64-QAM	5/6	130	144.4	270	300	585	650	1170	1300	172.1	162.5	146.3	344.1	325	292.5	720.6	680.6	612.5	1441.2	1361.1	1225
	8	8	2	256-QAM	3/4	156	173.3	324	360	702	780	1404	1560	206.5	195	175.5	412.9	390	351	864.7	816.7	735	1729.4	1633.3	1470
	9	9	2	256-QAM	5/6	N/A	N/A	360	400	780	866.7	1560	1733.3	229.4	216.7	195	458.8	433.3	390	960.8	907.4	816.7	1921.6	1814.8	1633.3
		10	2	1024-QAM	3/4									258.1	243.8	219.4	516.2	487.5	438.8	1080.9	1020.8	918.8	2161.8	2041.7	1837.5
		11	2	1024-QAM	5/6									286.8	270.8	243.8	573.5	541.7	487.5	1201	1134.3	1020.8	2402	2268.5	2041.7
16	0	0	3	BPSK	1/2	19.5	21.7	40.5	45	87.8	97.5	175.5	195	25.8	24.4	21.9	51.6	48.8	43.9	108.1	102.1	91.9	216.2	204.2	183.8
17	1	1	3	QPSK	1/2	39	43.3	81	90	175.5	195	351	390	51.6	48.8	43.9	103.2	97.5	87.8	216.2	204.2	183.8	432.4	408.3	367.5
18	2	2	3	QPSK	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585	77.4	73.1	65.8	154.9	146.3	131.6	324.3	306.3	275.6	648.5	612.5	551.3
19	3	3	3	16-QAM	1/2	78	86.7	162	180	351	390	702	780	103.2	97.5	87.8	206.5	195	175.5	432.4	408.3	367.5	864.7	816.7	735
20	4	4	3	16-QAM	3/4	117	130	243	270	526.5	585	1053	1170	154.9	146.3	131.6	309.7	292.5	263.3	648.5	612.5	551.3	1297.1	1225	1102.5
21	5	5	3	64-QAM	2/3	156	173.3	324	360	702	780	1404	1560	206.5	195	175.5	412.9	390	351	864.7	816.7	735	1729.4	1633.3	1470
22	6	6	3	64-QAM	3/4	175.5	195	364.5	405	N/A	N/A	1579.5	1755	232.3	219.4	197.4	464.6	438.8	394.9	972.8	918.8	826.9	1945.6	1837.5	1653.8
23	7	7	3	64-QAM	5/6	195	216.7	405	450	877.5	975	1755	1950	258.1	243.8	219.4	516.2	487.5	438.8	1080.9	1020.8	918.8	2161.8	2041.7	1837.5
	8	8	3	256-QAM	3/4	234	260	486	540	1053	1170	2106	2340	309.7	292.5	263.3	619.4	585	526.5	1297.1	1225	1102.5	2594.1	2450	2205
	9	9	3	256-QAM	5/6	260	288.9	540	600	1170	1300	N/A	N/A	344.1	325	292.5	688.2	650	585	1441.2	1361.1	1225	2882.4	2722.2	2450
		10	3	1024-QAM	3/4									387.1	365.6	329.1	774.3	731.3	658.1	1621.3	1531.3	1378.1	3242.6	3062.5	2756.3
		11	3	1024-QAM	5/6									430.1	406.3	365.6	860.3	812.5	731.3	1801.5	1701.4	1531.3	3602.9	3402.8	3062.5

antennas

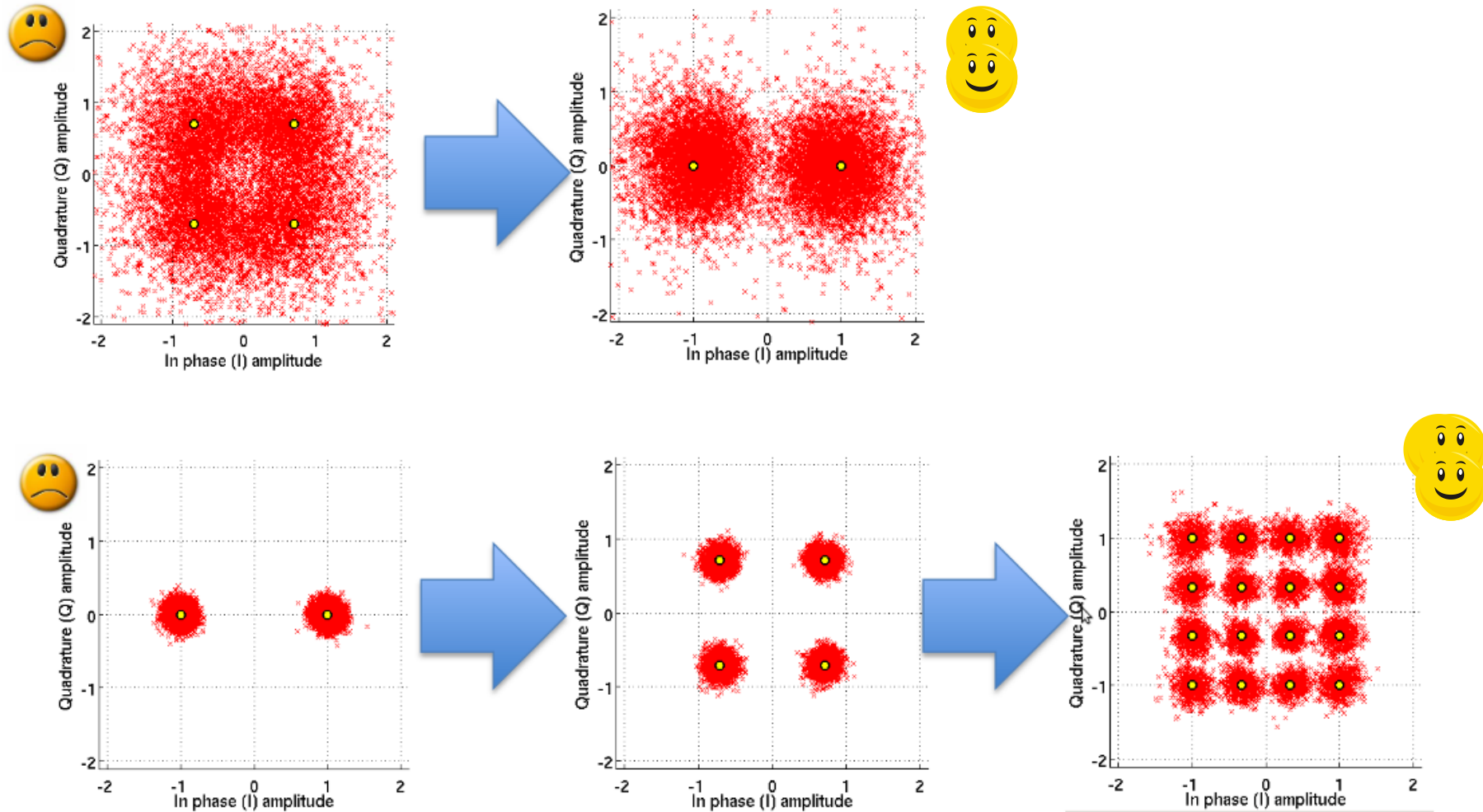
WiFi 5

WiFi 6

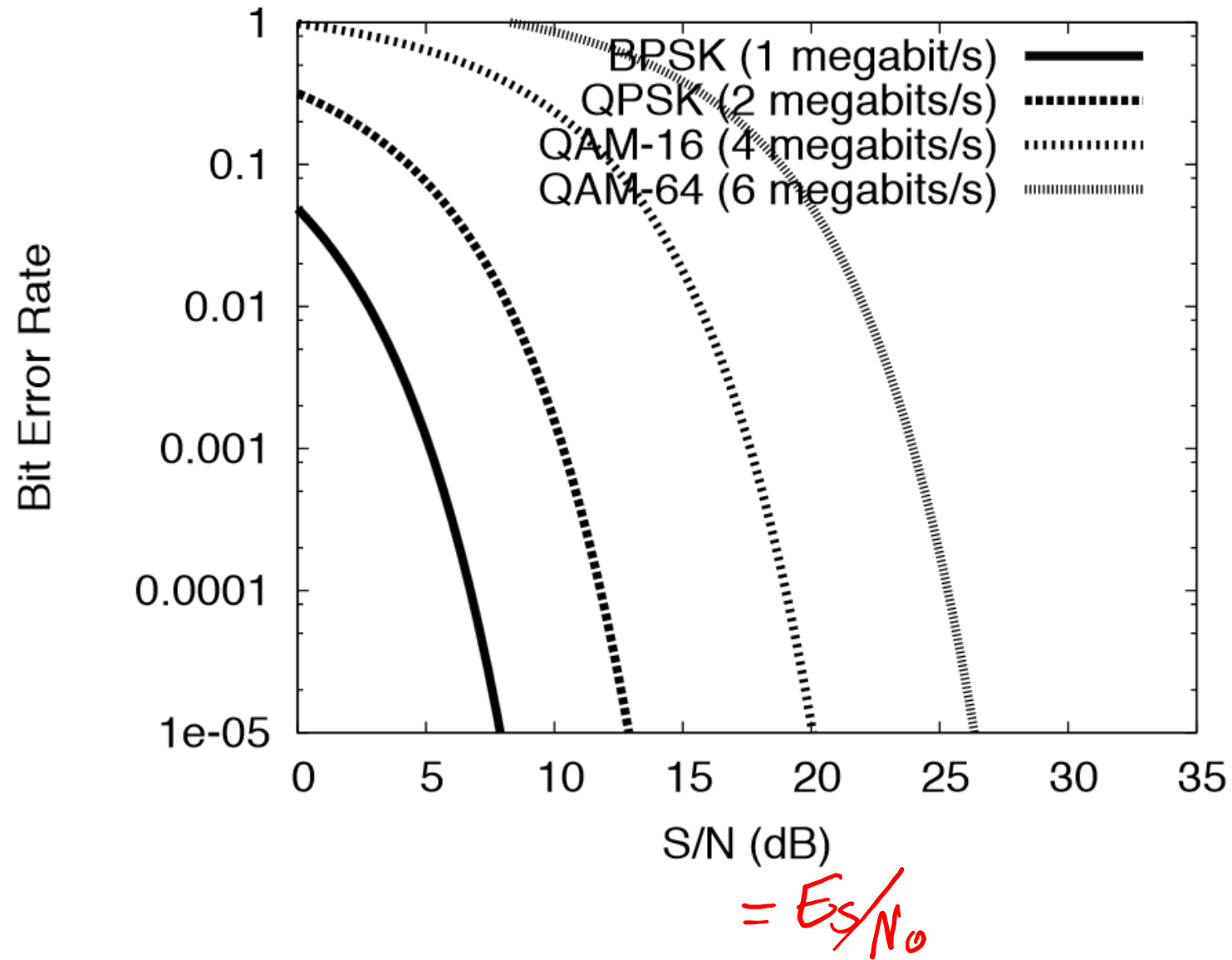
Modulation Schemes



Choice of Modulation



BER vs SNR



Throughput

Throughput: number of data bits correctly received per second.

M : Number of bits per packet

P_b : Probability of bit error after error correcting codes = function(BER , $Code Rate$, $Code Scheme$)

$$Packet Loss Rate = 1 - (1 - P_b)^M \approx M \times P_b$$

$$Throughput = Data Rate \times (1 - Loss Rate)$$

$$Throughput \leq Data Rate < Capacity$$

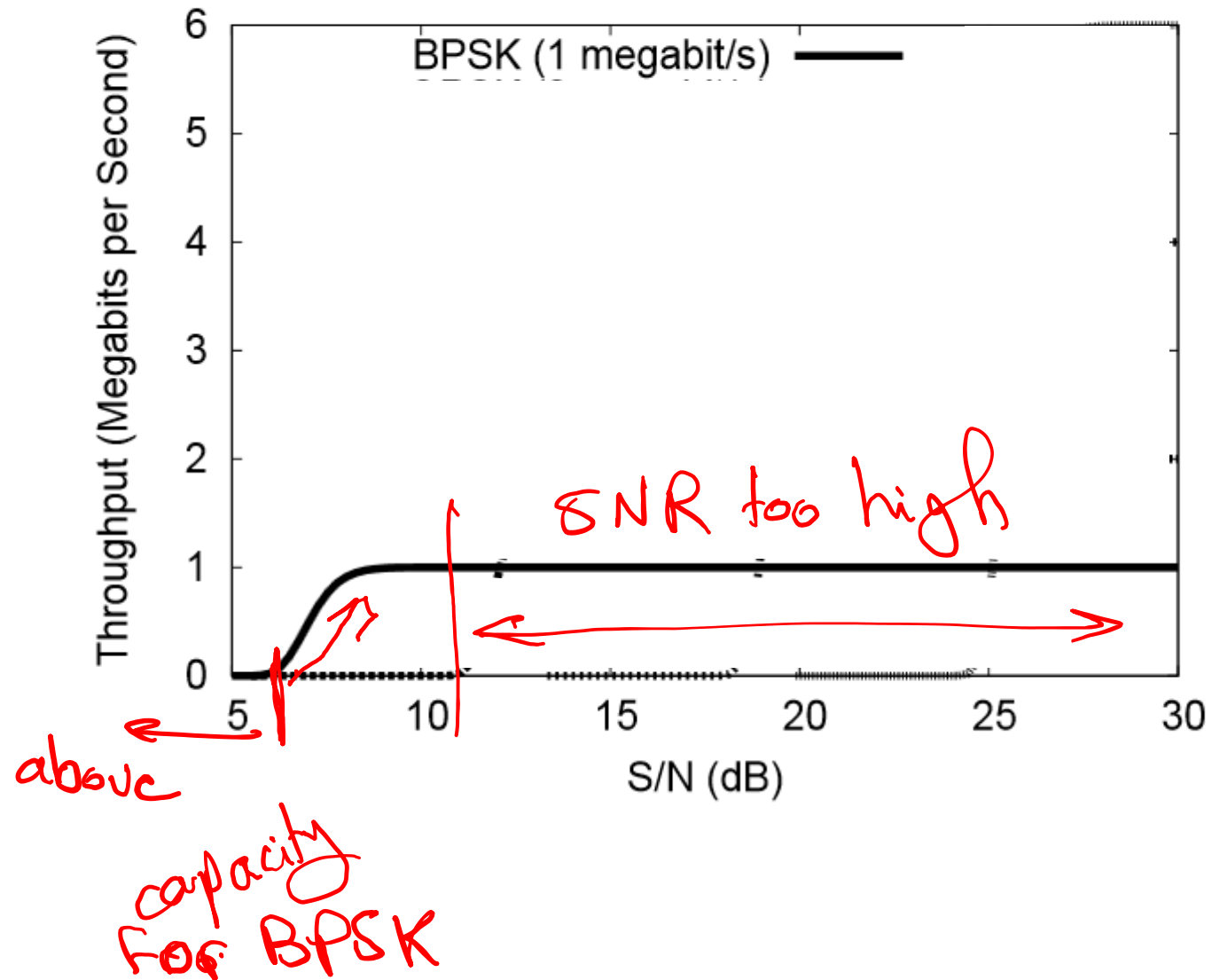
If we transmit above capacity:

- Lots of bit errors
- All packets loss
- Throughput = 0

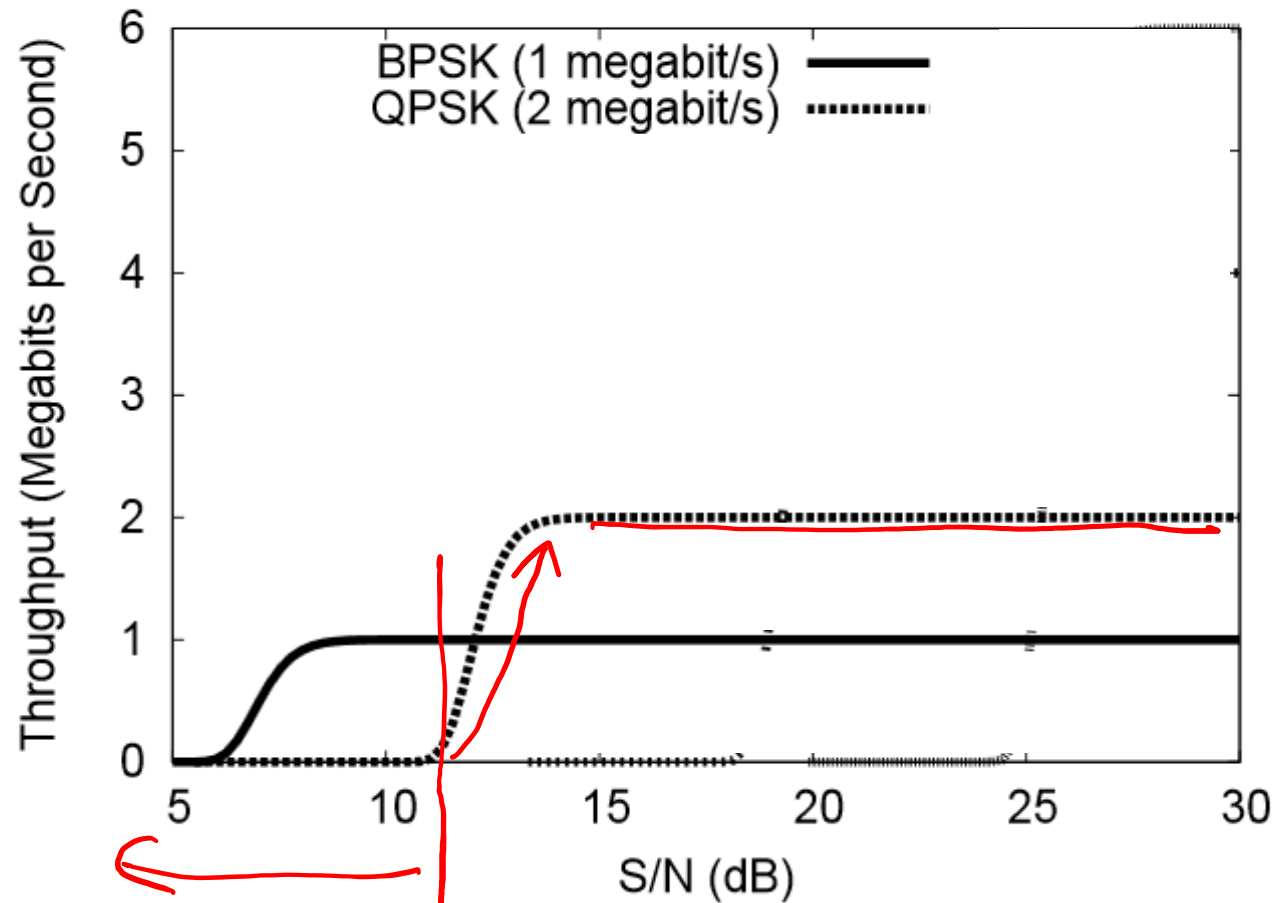
If we transmit below capacity:

- Small number of bit errors
- Bit errors corrected by coding
- Throughput $\approx$ Data Rate

Throughput vs SNR

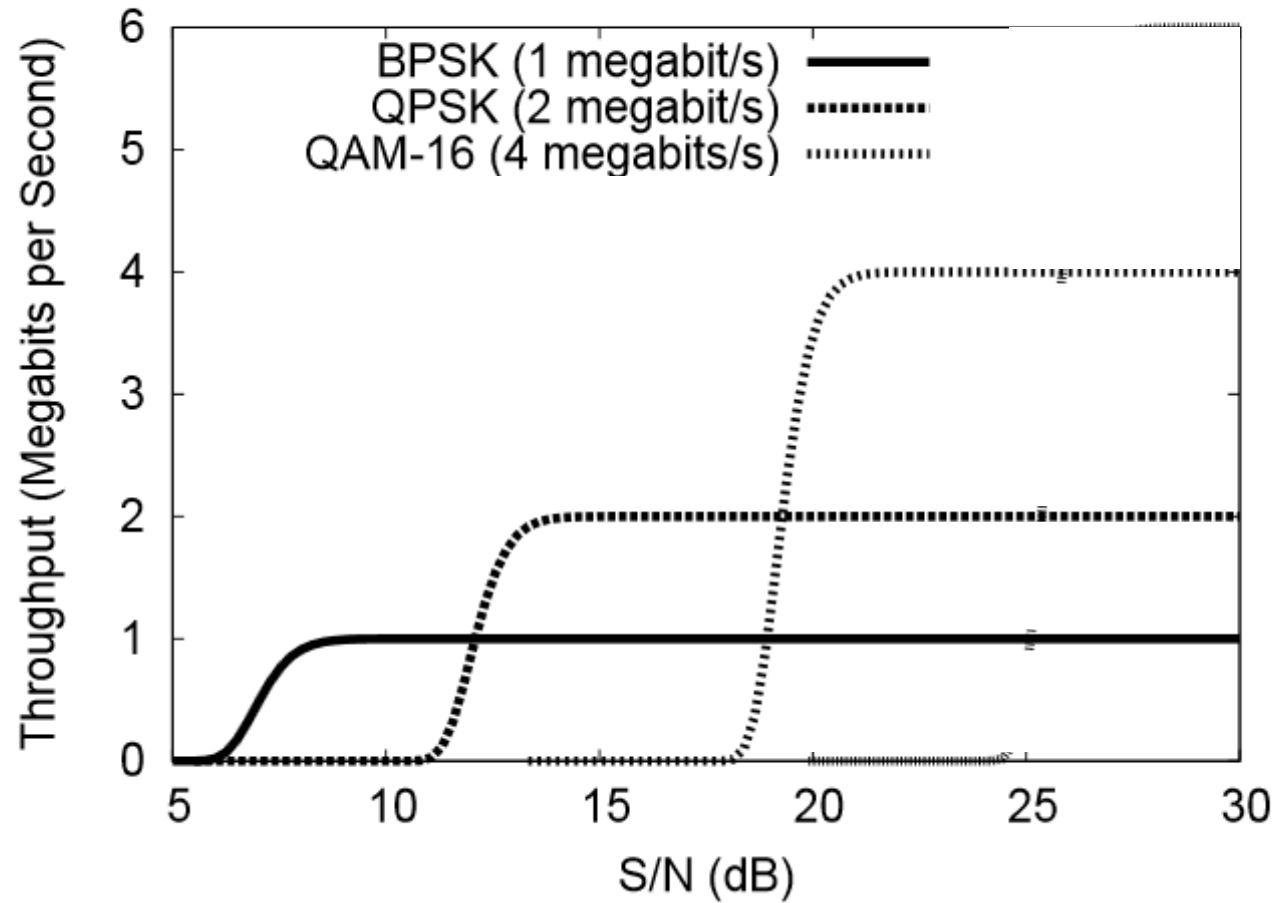


Throughput vs SNR

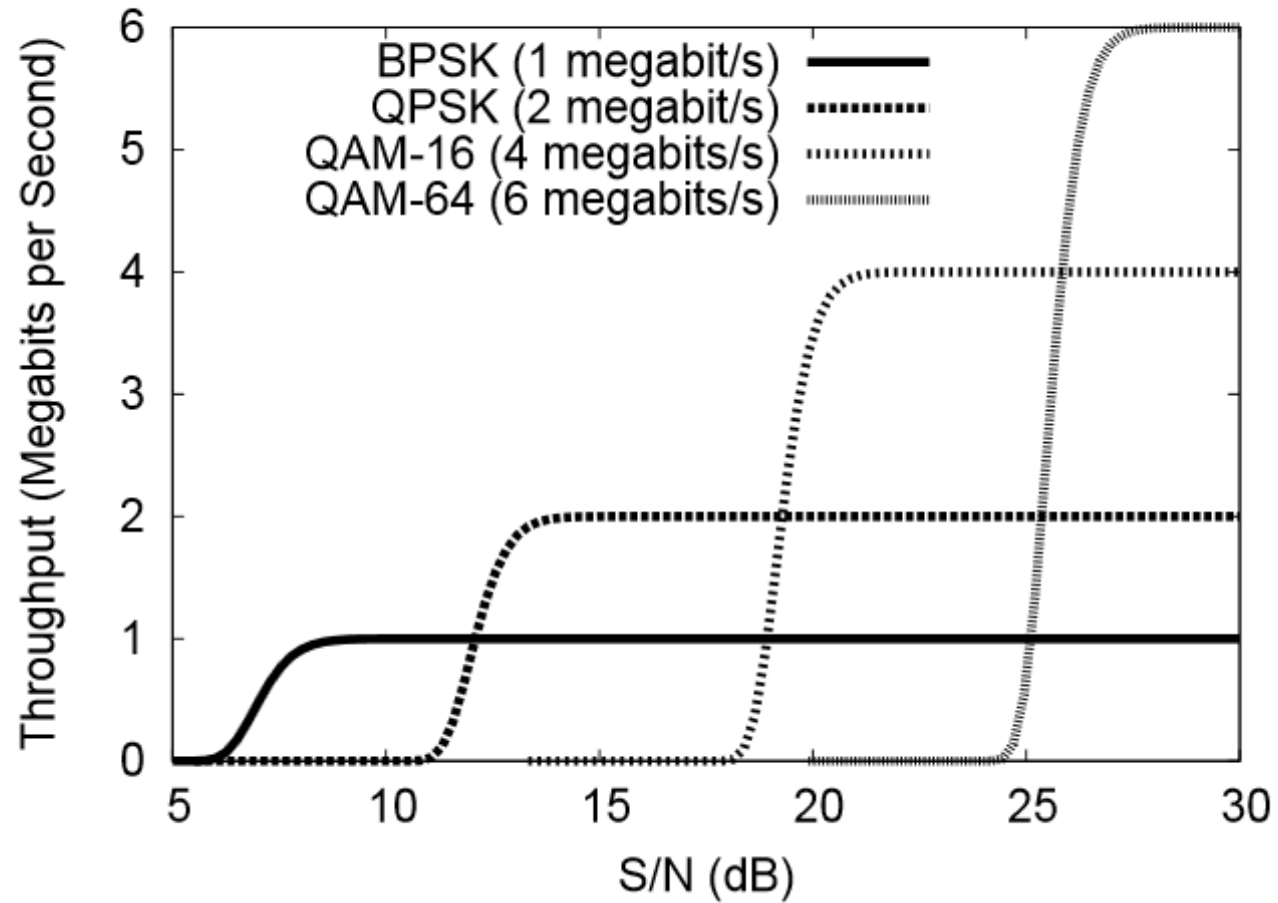


above
capacity
of PSK

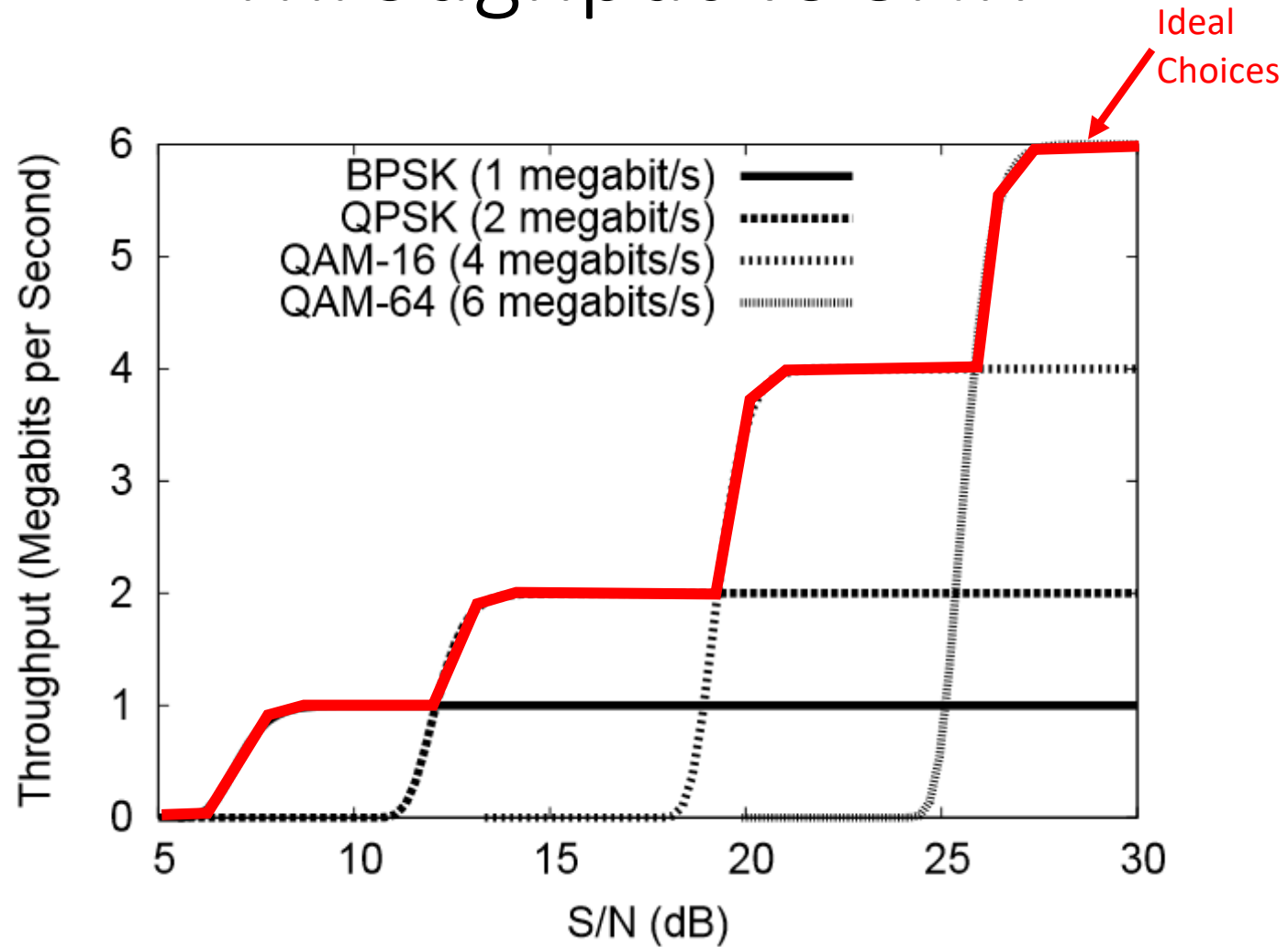
Throughput vs SNR



Throughput vs SNR



Throughput vs SNR



Rate Adaptation

Choose the best modulation and coding scheme that maximizes the throughput that can be supported by the channel.

Challenges:

- Few modulation and coding rates supported by standards/hardware → must choose for discrete set
- TX does not know the channel and noise at the RX before choosing the modulation & coding.