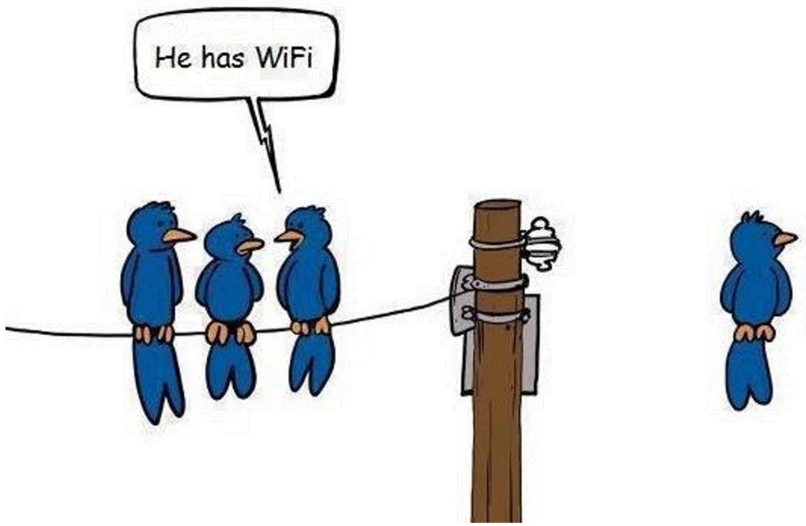
COM-405: Mobile Networks

Lecture 4.0: Modulation II
Haitham Hassanieh



EPFL

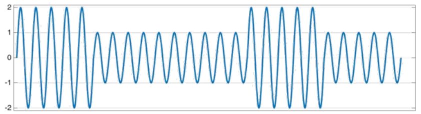
Laboratory of SEnsing & Networking Systems

Modulation

Based on how the bits are encoded

ASK

Amplitude Shift Keying

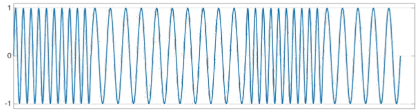


1 0 0 1 0

OOK, ASK

FSK

Frequency Shift Keying

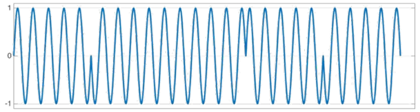


1 0 0 1 0

CFSK, MSK, GMSK

PSK

Phase Shift Keying



1 0 0 1 0

BPSK, QPSK, CPM

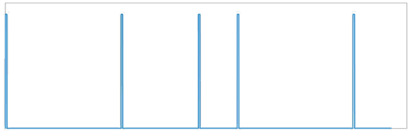
QAM

Phase & Amplitude Modulation (APSK)

PAM, M-QAM

PPM

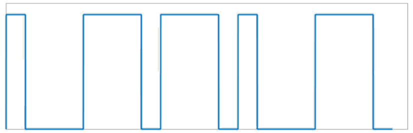
Pulse Position Modulation



1 0 0 1 0

PWM

Pulse Width Modulation



1 0 0 1 0

DQPSK

Differential QPSK

DBPSK, DQPSK,

DQAM

Differential QAM

DQAM

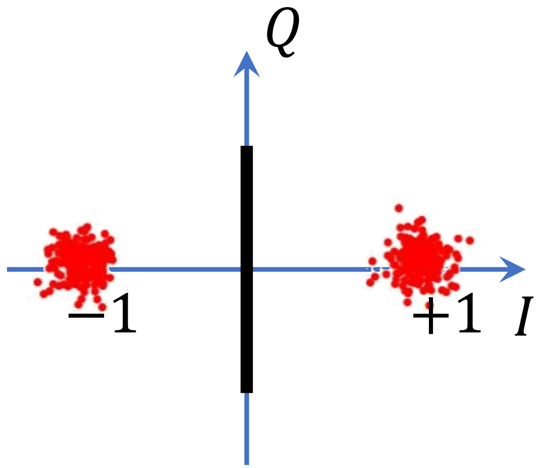
Maximum Likelihood Decoder

$$x = \pm 1 \longrightarrow y = x + v$$

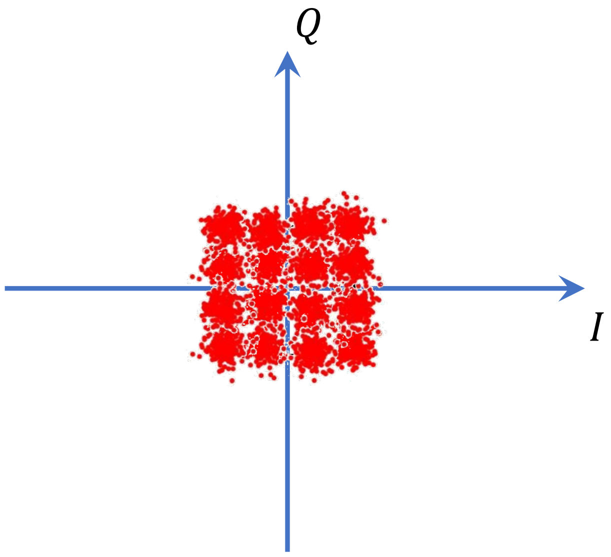
$$P(y|x=+1) \underset{0}{\overset{1}{\gtrless}} P(y|x=-1)$$

$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

$0 \rightarrow -1$
$1 \rightarrow +1$

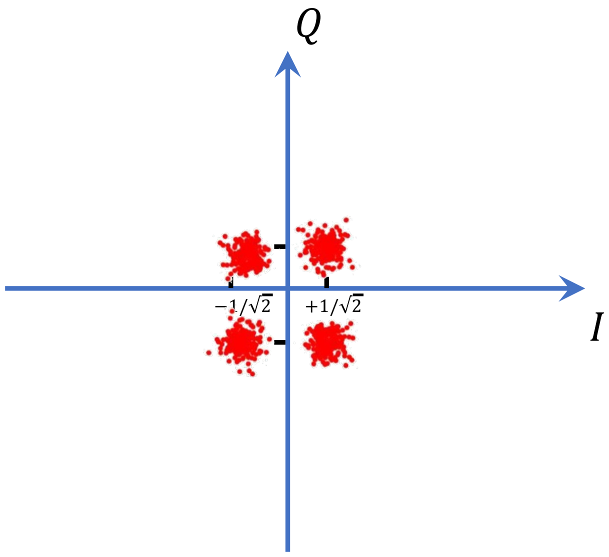


16-QAM



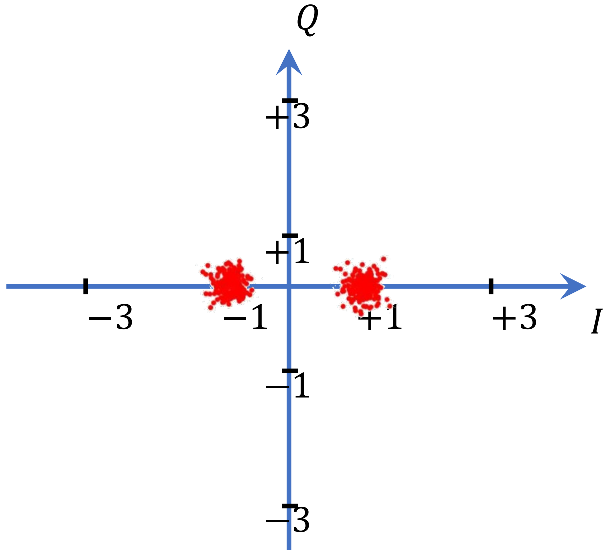
4 bits/symbol

4-QAM



2 bits/symbol

BPSK



1 bit/symbol

Higher Order Modulation:

- Needs higher SNR to decode correctly.
- Achieves higher Bit Rate

Given an SNR, choose highest order modulation that guarantees minimal Bit Error Rate (BER)

Bit Error Rate (BER) of BPSK

Encoding:

$$b = 0 \rightarrow x = -\sqrt{E_s}$$
$$b = 1 \rightarrow x = +\sqrt{E_s}$$

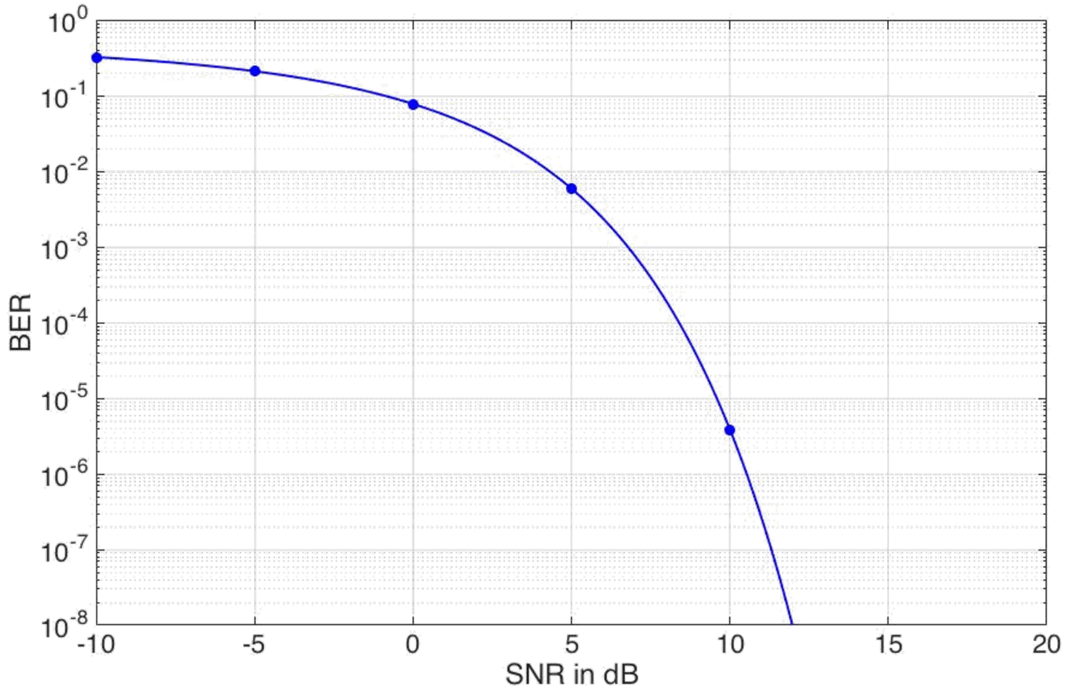
$$\longrightarrow \quad y = x + v$$

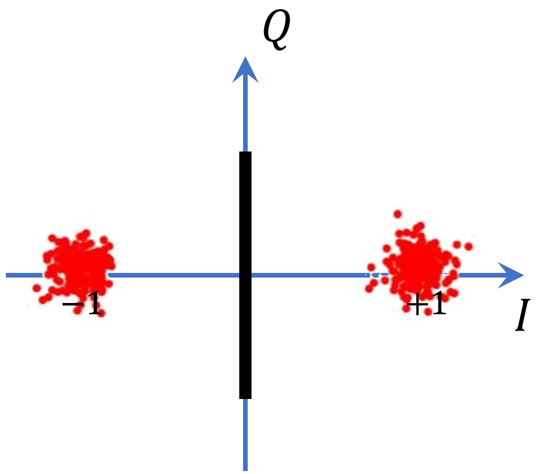
Decoding:

$$P\left(y|x = +\sqrt{E_s}\right) \underset{0}{\overset{1}{\gtrless}} P\left(y|x = -\sqrt{E_s}\right)$$

$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

$$BER = Q\left(\sqrt{2SNR}\right) = Q\left(\sqrt{2E_s/N_0}\right)$$





Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- Signal-to-Noise Ratio: SNR
- SNR per symbol: $\frac{E_s}{N_0}$
- SNR per bit: $\frac{E_b}{N_0} = ??$

A. $\log_2 M \frac{E_s}{N_0}$ B. $\frac{1}{\log_2 M} \frac{E_s}{N_0}$ C. $M \frac{E_s}{N_0}$ D. $\frac{1}{M} \frac{E_s}{N_0}$

Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- Signal-to-Noise Ratio: SNR
- SNR per symbol: $\frac{E_s}{N_0}$
- SNR per bit: $\frac{E_b}{N_0} = \frac{1}{\log_2 M} \frac{E_s}{N_0}$
- Symbol Error Rate: $SER = P_s$
- Bit Error Rate: $BER = P_b$

- Relation between P_s and P_b?

A. $P_b \leq P_s$ B. $P_b = P_s$ C. $P_b > P_s$ D. Depends on Modulation Used

Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- $P_b \leq P_s$?

1 symbol error can at most generate $\log_2 M$ bit errors.

$$P_b = \frac{Number\ of\ Error\ Bits}{Total\ Number\ of\ Bits} \leq \frac{Number\ of\ Error\ Symbols \times \log_2 M}{Total\ Number\ of\ Symbols \times \log_2 M} = P_s$$

How much is P_b less than P_s?

Depends on the code used: How we assign bits to symbol

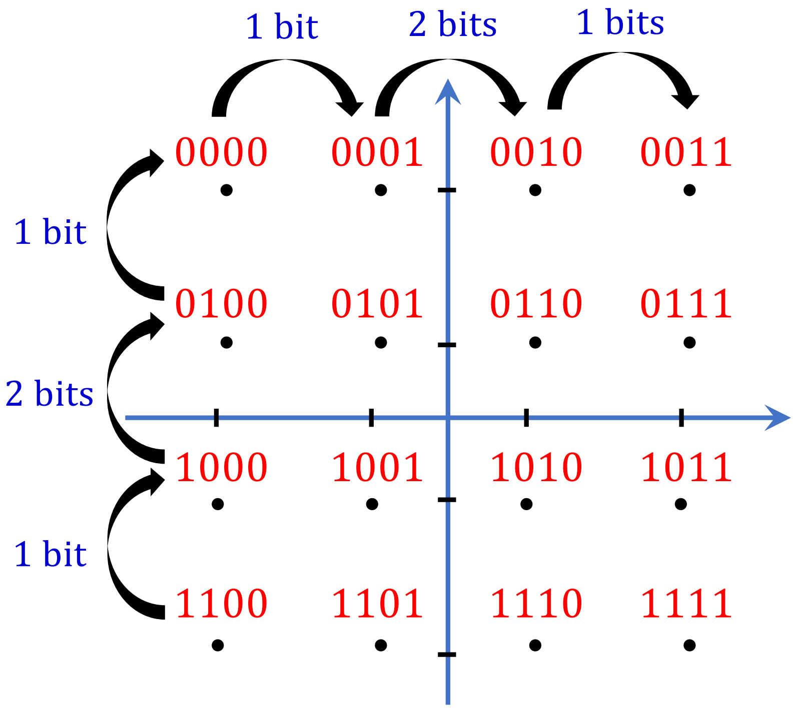
How much is P_b less than P_s?

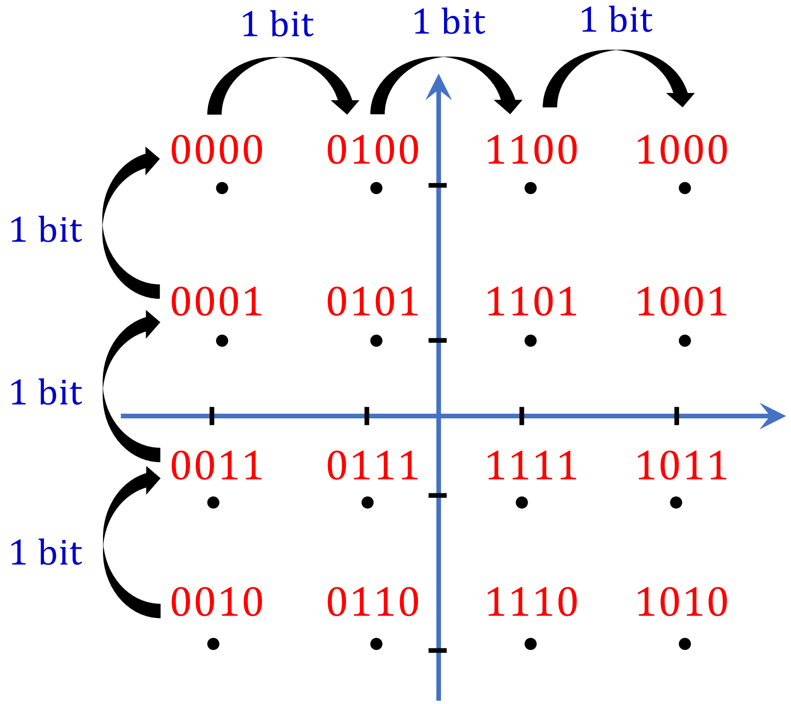
Depends on the code used: How we assign bits to symbol

$$P_b \approx \frac{1}{3} P_s$$

16-QAM

$$P_b \approx \frac{1}{4} P_s$$





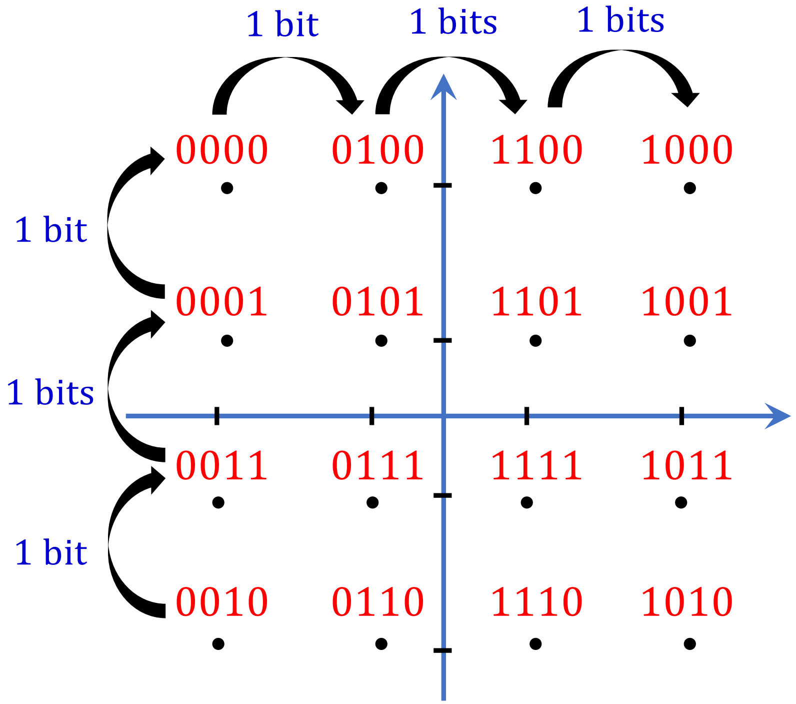
Most likely error occurs between nearest neighbor constellation points!

Minimize bit flips between nearest neighbors.

How much is P_b less than P_s?

Depends on the code used: How we assign bits to symbol

16-QAM $\quad P_b \approx \frac{1}{4} P_s$



Gray Codes

Bit flips between nearest neighbors $= 1$

$$P_b \approx \frac{1}{\log_2 M} P_s$$

Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- Assuming Gray Code is used:

$$BER:\ P_b = \frac{1}{\log_2 M} P_s$$

- Nearest Neighbor Approximation:

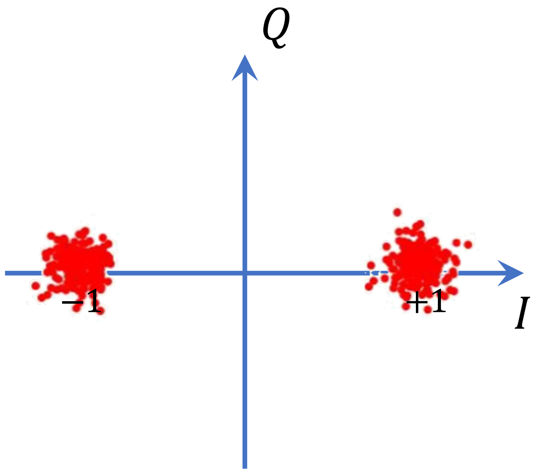
$$SER:\ P_s \approx \#\, nearest\ neighbors \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E_s}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{2E_s/N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{2E_b/N_0}\right)$$

BPSK:



$$d_{min} = 2\sqrt{E_s}$$

$$\#nn = 1$$

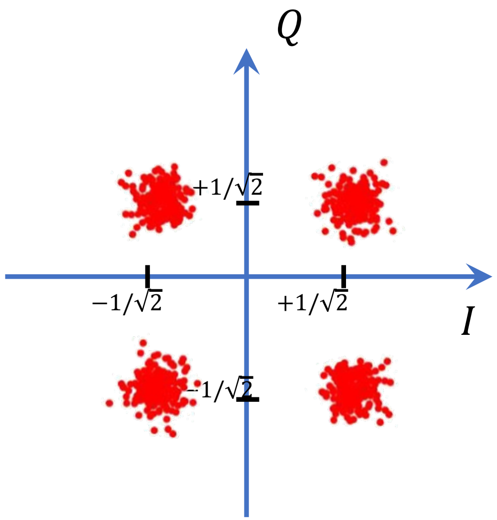
$$\log_2 M = 1$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{E_s/N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{2E_b/N_0}\right)$$

4-QAM/QPSK



$$d_{min} = \frac{2}{\sqrt{2}}\sqrt{E_s}$$

$$\#nn = 2$$

$$\log_2 M = 2$$

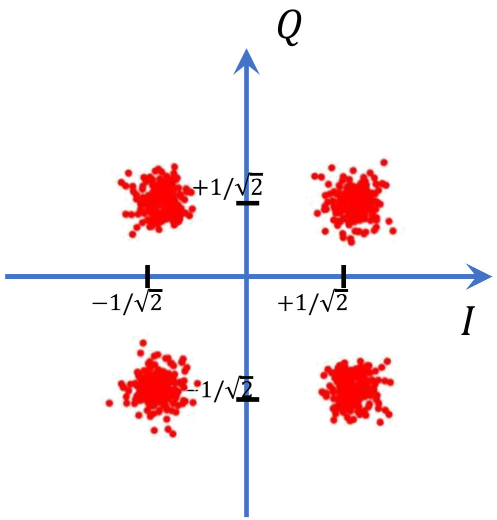
Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{E_s/N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{2E_b/N_0}\right)$$

Same as BPSK with 2× Bitrate!

4-QAM/QPSK



$$d_{min} = \frac{2}{\sqrt{2}}\sqrt{E_s}$$

$$\#nn = 2$$

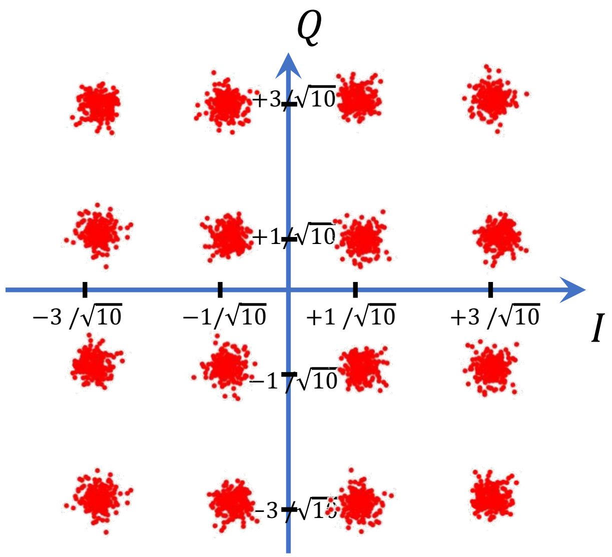
$$\log_2 M = 2$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 4Q\left(\sqrt{E_s/5N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{4E_b/5N_0}\right)$$

16-QAM



$$d_{min} = \frac{2}{\sqrt{10}}\sqrt{E_s}$$

$$\#nn = 4$$

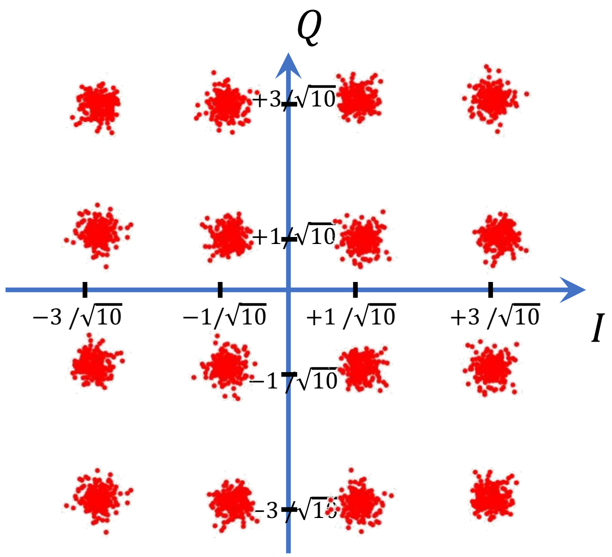
$$\log_2 M = 4$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 4Q\left(\sqrt{\frac{3E_s/N_0}{M-1}}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\log_2 M\ E_b/N_0}{M-1}}\right)$$

M-QAM



$$d_{min} = \sqrt{\frac{6E_s}{M-1}}$$

$$\#nn = 4$$

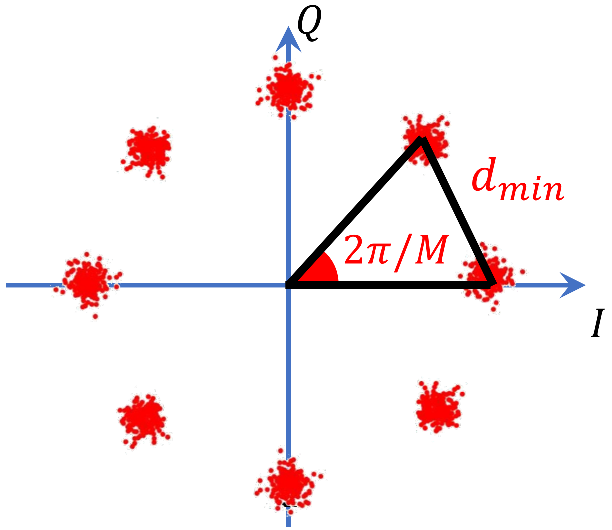
$$\log_2 M$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \quad = 2Q\left(\sqrt{2E_s/N_0}\sin\left(\frac{\pi}{M}\right)\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s \quad = \frac{2}{\log_2 M} Q\left(\sqrt{2\log_2 M\, E_b/N_0}\sin\left(\frac{\pi}{M}\right)\right)$$

M-PSK



$$d_{min} = 2\sin\left(\frac{\pi}{M}\right)\sqrt{E_s}$$

$$\#nn = 2$$

$$\log_2 M$$

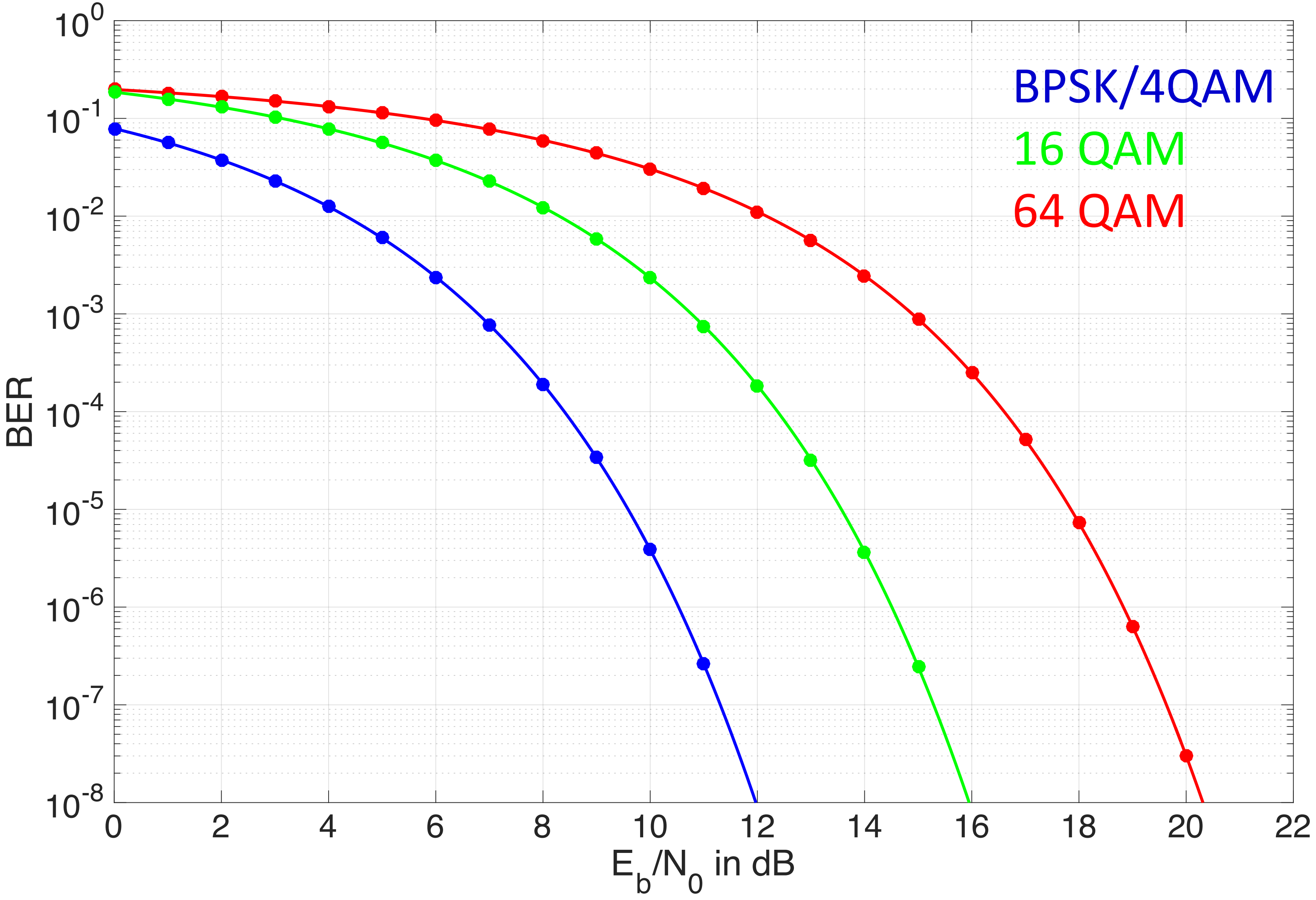
Bit Error Rate (BER)

	$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$	$BER:\ P_b = \frac{1}{\log_2 M} P_s$
• BPSK	$Q(\sqrt{2E_s/N_0})$	$Q(\sqrt{2E_b/N_0})$
• QPSK/4-QAM	$2\,Q(\sqrt{E_s/N_0})$	$Q(\sqrt{2E_b/N_0})$
• MPAM	$\frac{2(M-1)}{M}\,Q\left(\sqrt{\frac{6E_s/N_0}{M^2-1}}\right)$	$\frac{2(M-1)}{M\log_2 M}\,Q\left(\sqrt{\frac{6\log_2 M\,E_b/N_0}{M^2-1}}\right)$
• MPSK	$2\,Q\left(\sqrt{2E_s/N_0}\sin\left(\frac{\pi}{M}\right)\right)$	$\frac{2}{\log_2 M}\,Q\left(\sqrt{2\log_2 M\,E_b/N_0}\sin\left(\frac{\pi}{M}\right)\right)$
• MQAM	$4\,Q\left(\sqrt{\frac{3E_s/N_0}{M-1}}\right)$	$\frac{4}{\log_2 M}\,Q\left(\sqrt{\frac{3\log_2 M\,E_b/N_0}{M-1}}\right)$

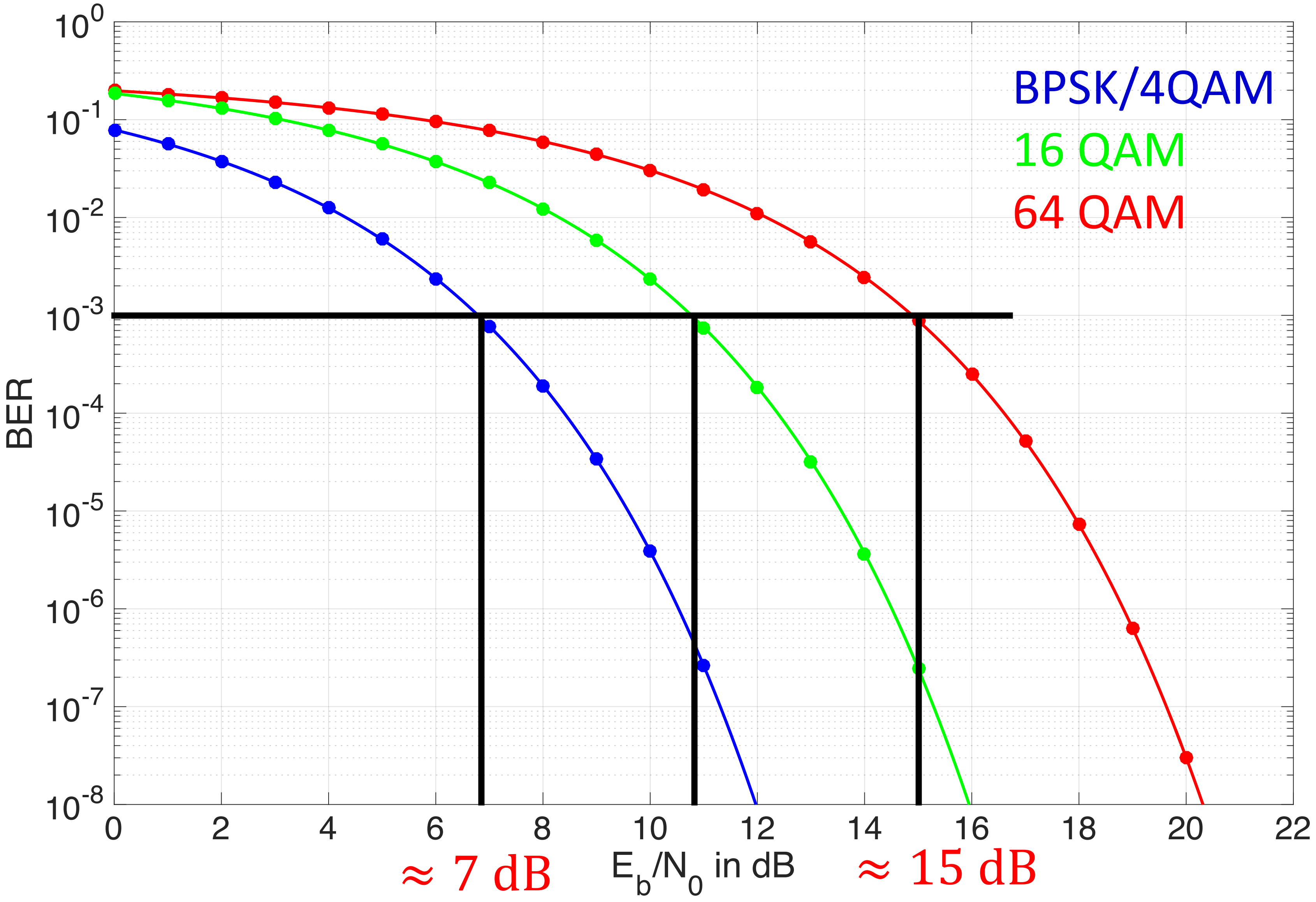
$$\alpha_M Q(\sqrt{\beta_M E_s/N_0}) \qquad \hat{\alpha}_M Q\left(\sqrt{\hat{\beta}_M E_b/N_0}\right)$$

$$\hat{\alpha}_M = \frac{\alpha_M}{\log_2 M}, \qquad \hat{\beta}_M = \beta_M \log_2 M\,, \qquad \frac{E_b}{N_0} = \frac{1}{\log_2 M}\frac{E_s}{N_0}$$

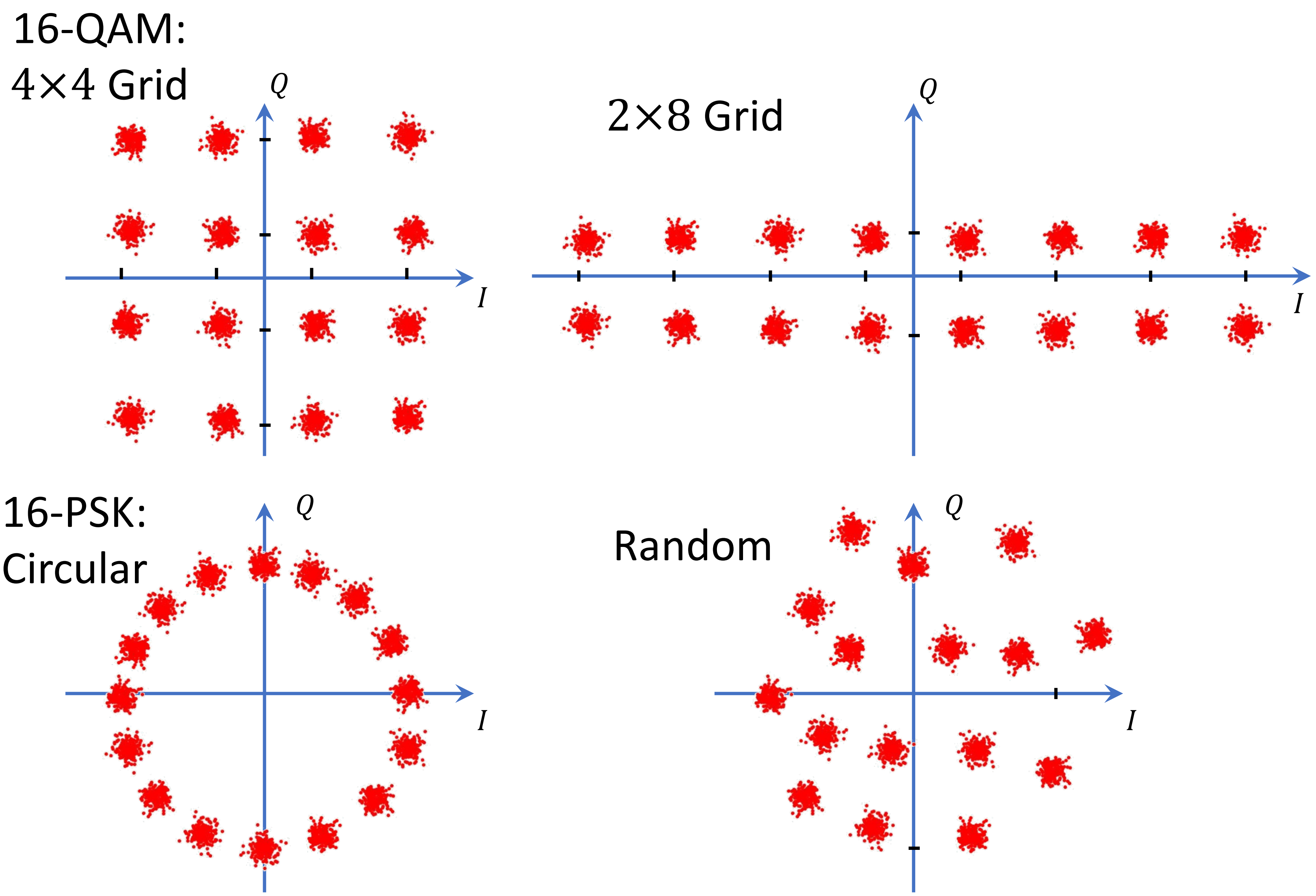
Bit Error Rate (BER)



Bit Error Rate (BER)



Shape of the Constellation (e.g. $M = 16$)

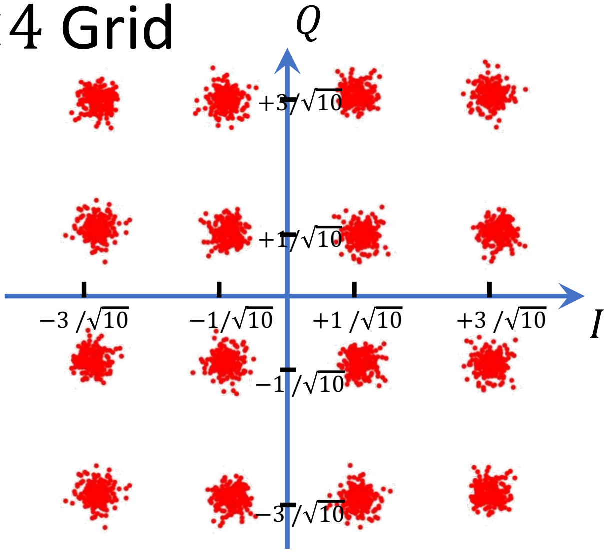


Shape of the Constellation (e.g. $M = 16$)

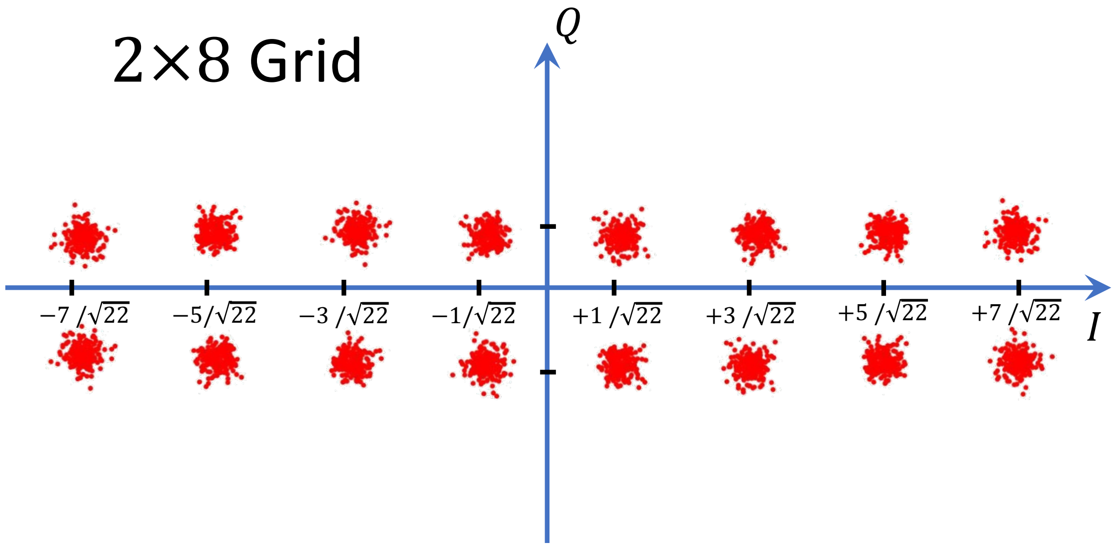
SER: $P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

Goal: Maximize d_{min} for the same overall power

4×4 Grid



2×8 Grid



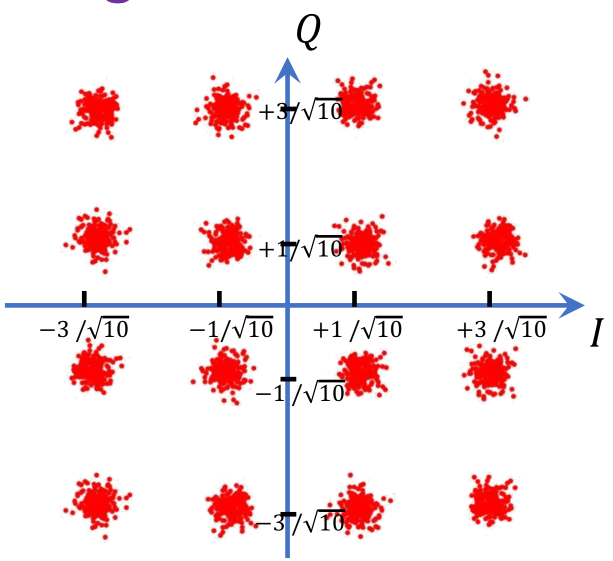
$$d_{min} = \frac{2}{\sqrt{10}}\sqrt{E_s} \gg d_{min} = \frac{2}{\sqrt{22}}\sqrt{E_s}$$

Shape of the Constellation (e.g. $M = 16$)

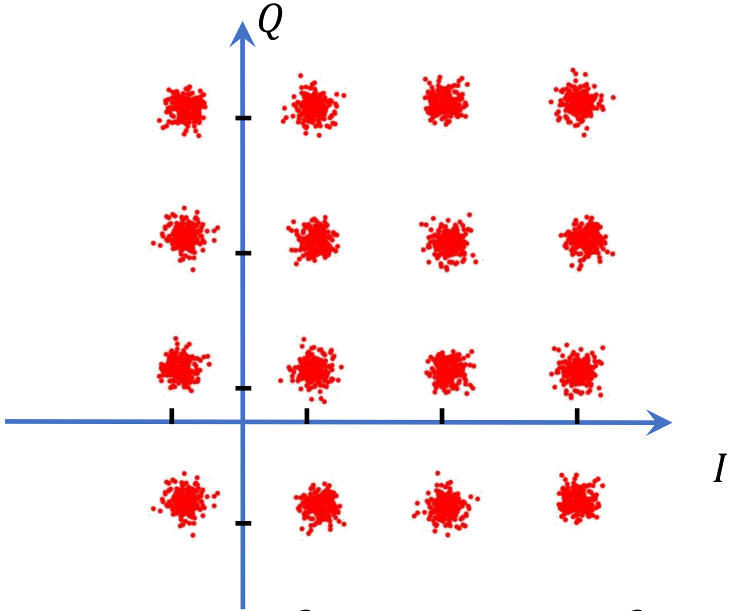
$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.



$E[|x(t)|^2] = 1$



$E[|x(t)|^2] = 1 + |E[x(t)]|^2$

If Average is not zero, we have higher TX Power for the same d_{min} or smaller d_{min} for same TX power.

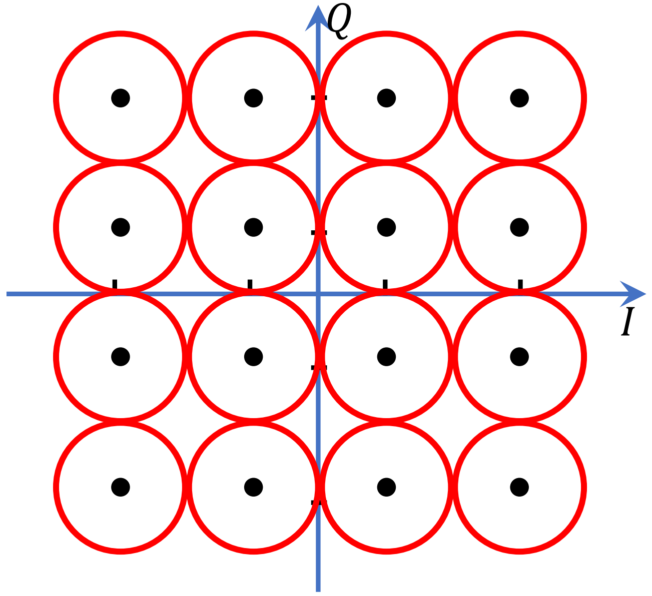
Shape of the Constellation (e.g. $M = 16$)

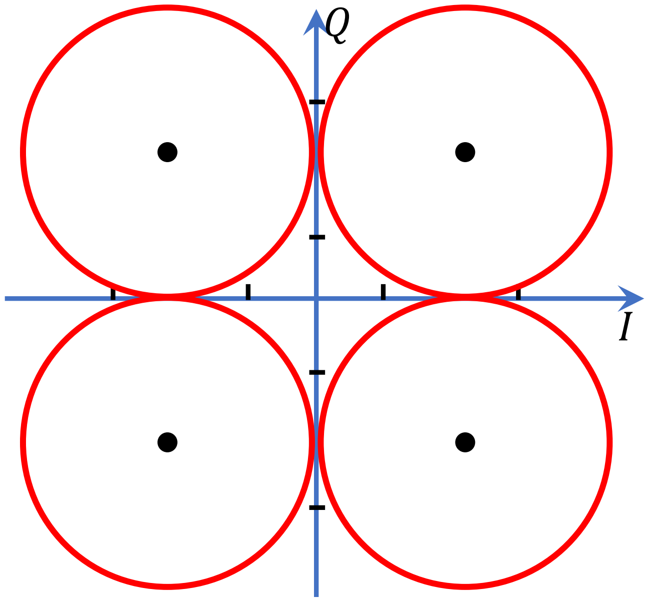
$$SER: P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.

Geometric packing problem.





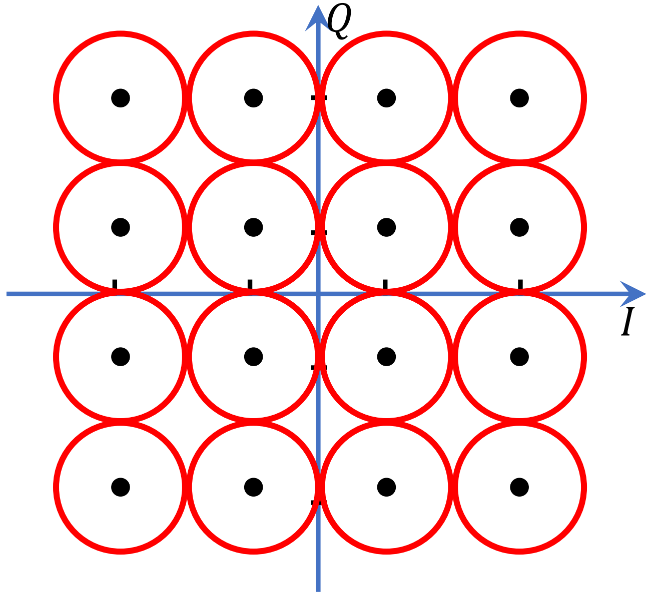
Shape of the Constellation (e.g. $M = 16$)

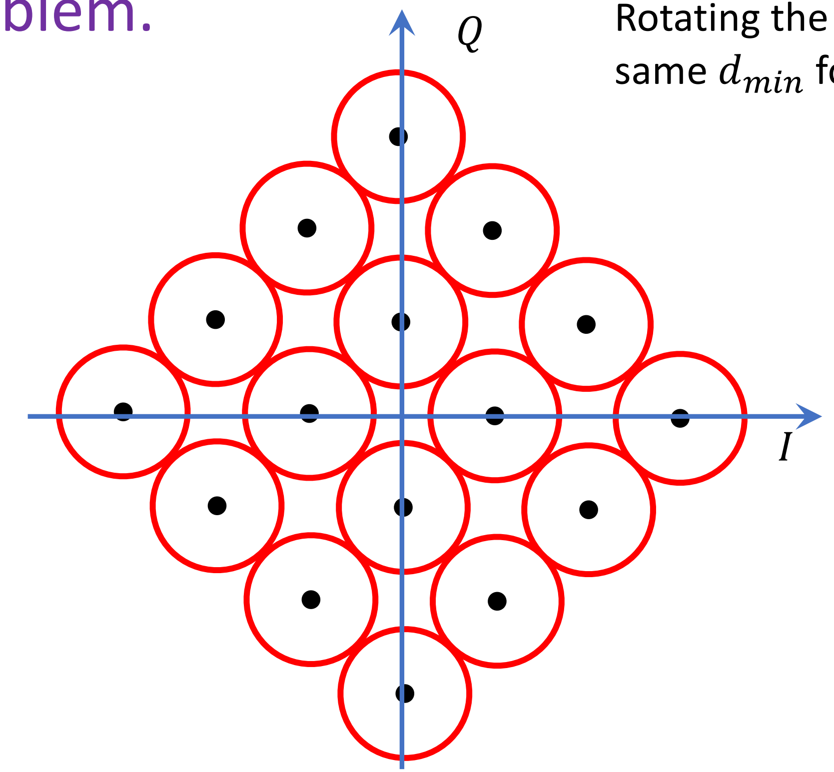
$$SER: P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.

Geometric packing problem.





Rotating the constellation maintains same d_{min} for same overall power

Shape of the Constellation (e.g. $M = 16$)

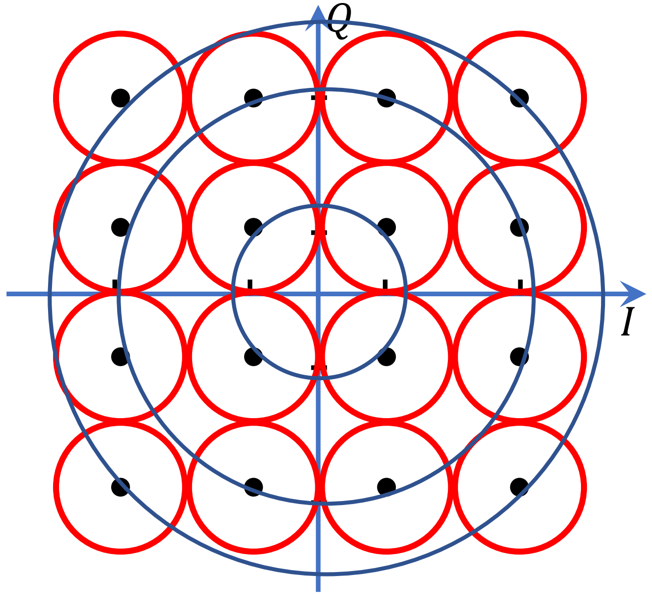
$$SER: P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

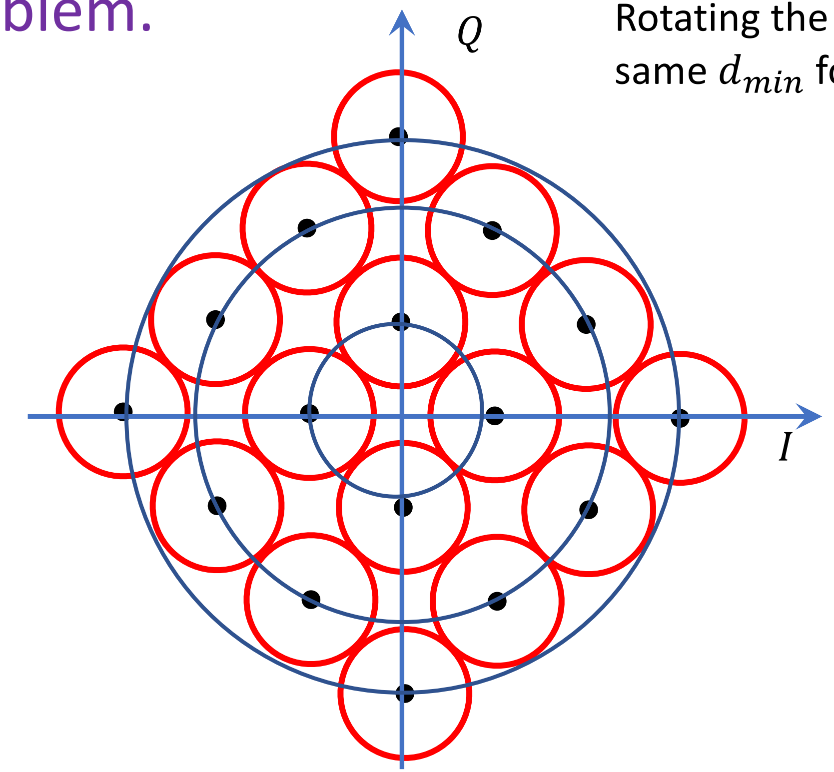
Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.

Geometric packing problem.

Rotating the constellation maintains same d_{min} for same overall power





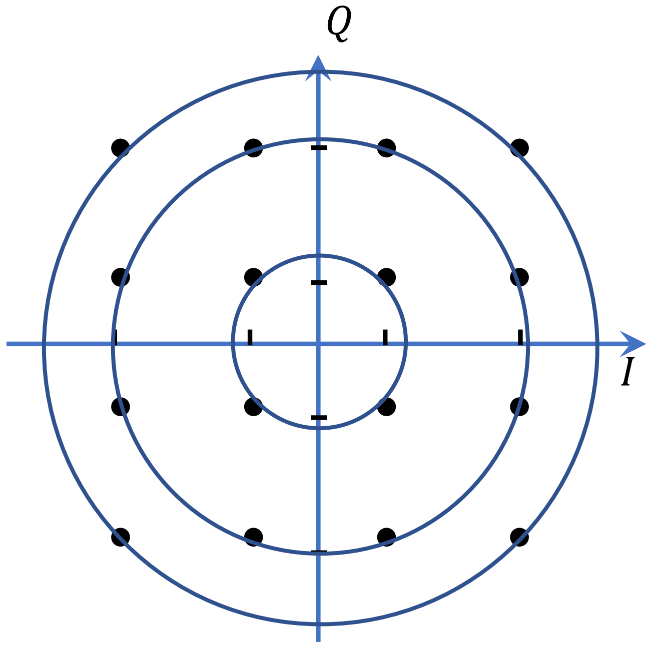
Shape of the Constellation (e.g. $M = 16$)

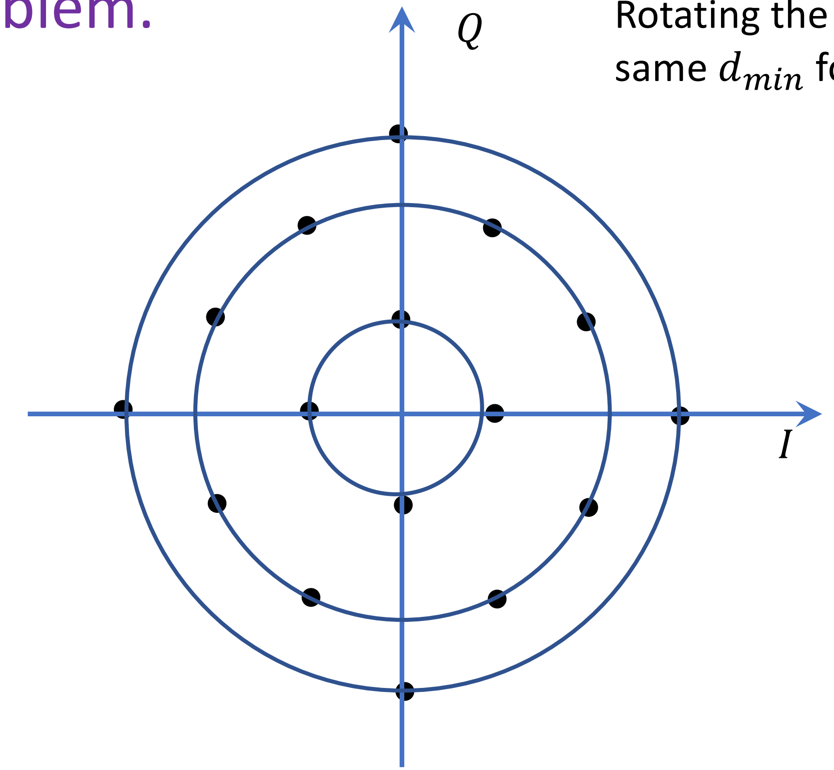
$$SER: P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.

Geometric packing problem.





Rotating the constellation maintains same d_{min} for same overall power

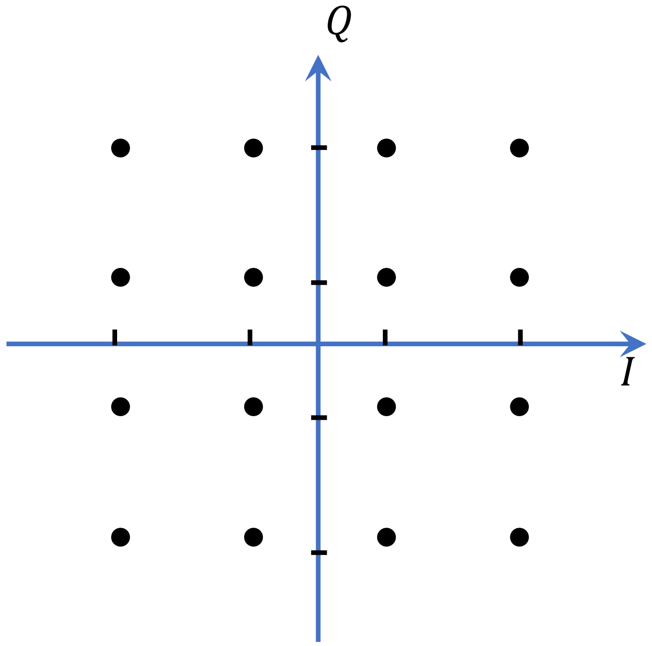
Shape of the Constellation (e.g. $M = 16$)

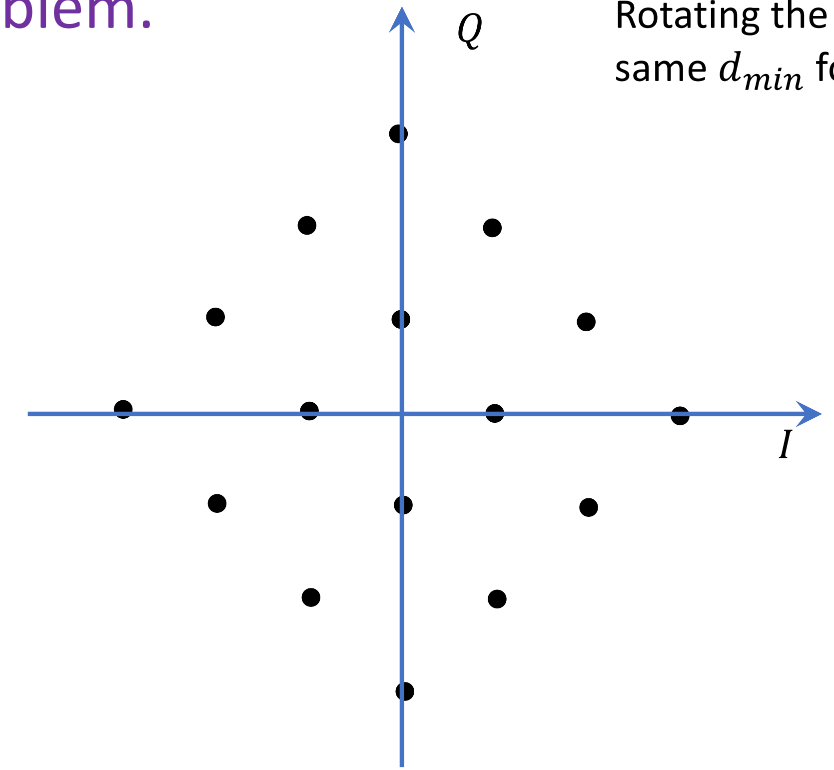
$$SER\colon P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.

Geometric packing problem.





Rotating the constellation maintains same d_{min} for same overall power

QAM Is Popular!

- WiFi

  802.11n: BPSK, QPSK, 16-QAM, 64-QAM

  802.11ac: BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM

  802.11ax (2019): BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM, 1024-QAM

- LTE, 4G, 5G: QPSK, 16-QAM, 64-QAM, 256 QAM

- Digital TV:

  DVB-C: 16-QAM, 64-QAM, 256-QAM

  DVB-C2: 16-QAM, 64-QAM, 256-QAM, 1024-QAM, 4096-QAM

  Next Gen. to include 16364-QAM & 65536-QAM

- Ethernet, Phone lines, Power lines…: 1024-QAM, 4096-QAM

- ADSL: 32768-QAM

65536-QAM

- Number of Constellation Points: $M = 65536$
- Bits per symbol: $\log_2 M = 16$
- QAM Grid: 256×256
- Need 8 bits to represent I and 8 bits to represent Q

What is the problem?

ADCs need to have a large
dynamic range!

Quantization noise kicks in!

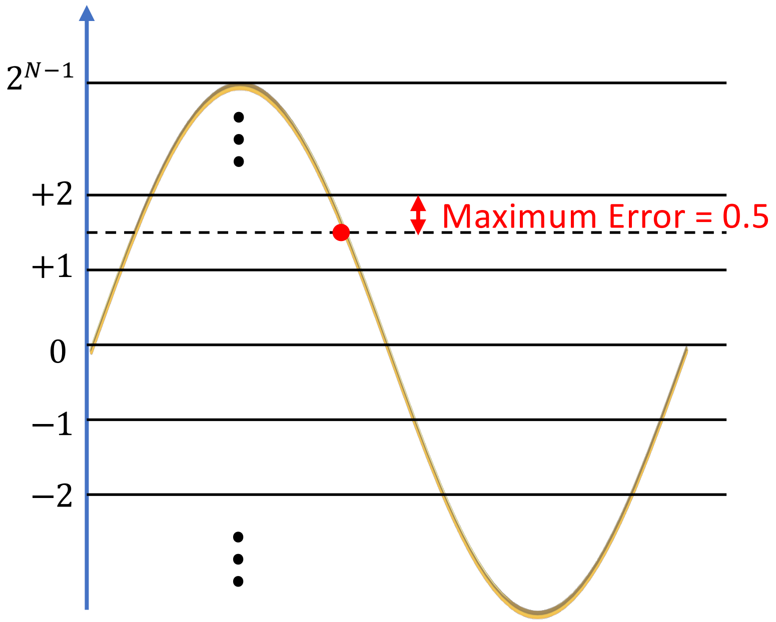
65536-QAM

- Number of Constellation Points: $M = 65536$
- Bits per symbol: $\log_2 M = 16$
- QAM Grid: 256×256
- Need 8 bits to represent I and 8 bits to represent Q
- ADC with K bits typically supports $N < K$ bit samples

ENOB: Effective Number of Bits $N < K$

Quantization Noise

- Consider N bit quantization



$$SNR_{quant.} = \frac{(Signal\ Amplitude)^2}{(Quantization\ Noise)^2}$$

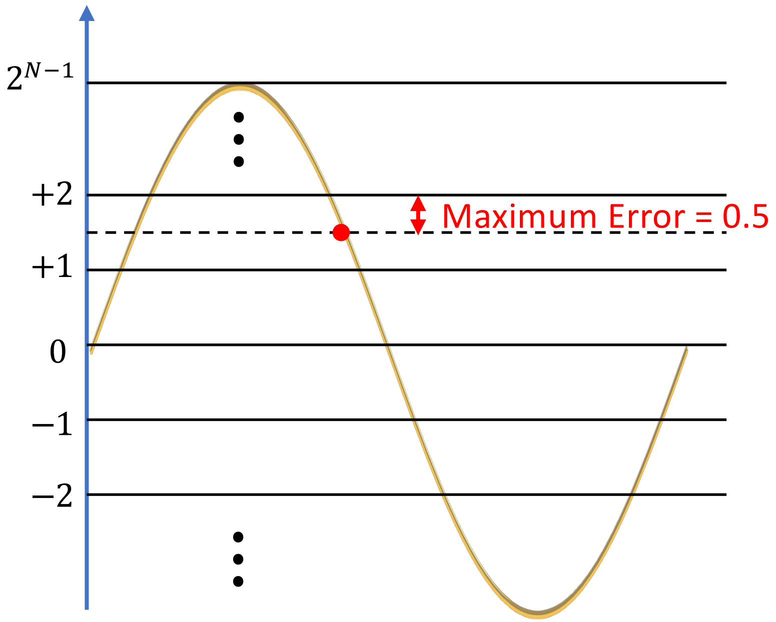
$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\text{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

$\approx$ 6 dB per bit

Quantization Noise

- Consider N bit quantization



$$SNR_{quant.} = \frac{(Signal\ Amplitude)^2}{(Quantization\ Noise)^2}$$

$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\mathrm{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

$\approx$ 6 dB per bit

Need: $SNR_{quant.} > SNR_{thermal}$

65536-QAM: $BER = \frac{1}{4} Q\left(\sqrt{\frac{3\ SNR}{65535}}\right)$ ➡ Need: $SNR > 52$ dB to achieve $BER < 10^{-3}$

➡ Need: $N > 9$ bits

65536-QAM

- Number of Constellation Points: $M = 65536$
- Bits per symbol: $\log_2 M = 16$
- QAM Grid: 256×256
- Need 8 bits to represent I and 8 bits to represent Q
- ADC with K bits typically supports $N < K$ bit samples

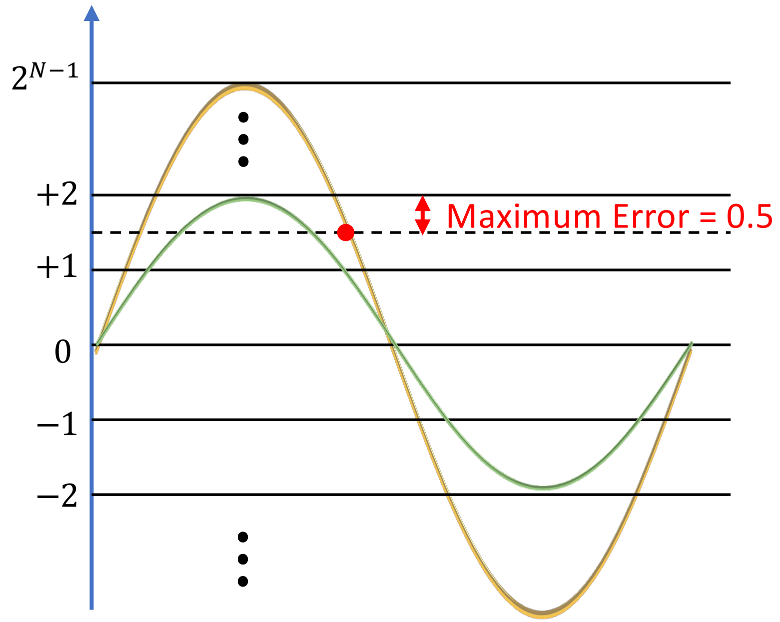
ENOB: Effective Number of Bits $N < K$

- Need very high SNR to achieve reasonable BER.
- Need to minimize quantization noise: $N \geq 12$

Need at least a 14 bit or 16 bit ADC

Quantization Noise

- Consider N bit quantization



$$SNR_{quant.} = \frac{(Signal\ Amplitude)^2}{(Quantization\ Noise)^2}$$

$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\text{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

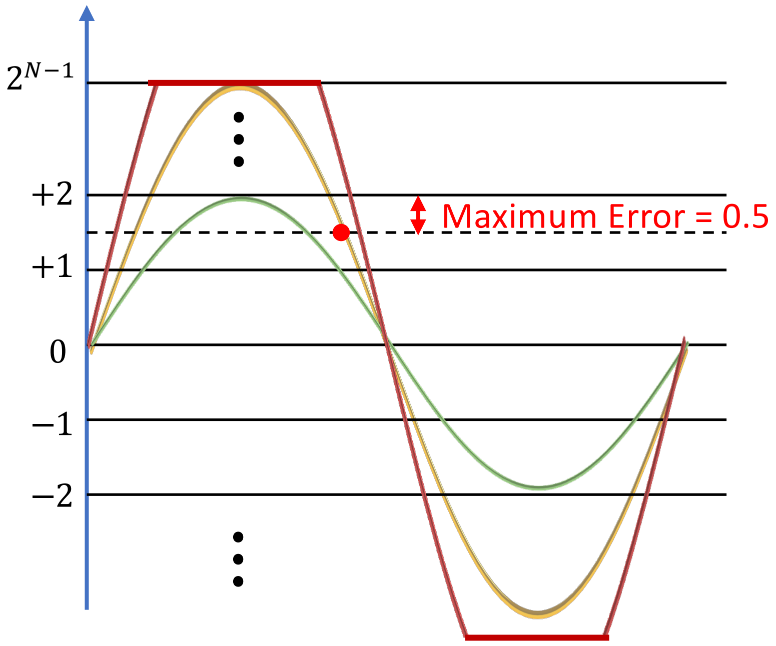
$\approx$ 6 dB per bit

Assumes signal amplitude fills maximum quantization level.

Signals arrives too attenuated → fill smaller quantization levels → Low $SNR_{quant.}$

Quantization Noise

- Consider N bit quantization



$$SNR_{quant.} = \frac{(Signal\ Amplitude)^2}{(Quantization\ Noise)^2}$$

$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\text{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

$\approx$ 6 dB per bit

Assumes signal amplitude fills maximum quantization level.

Signals arrives too attenuated → fill smaller quantization levels → Low $SNR_{quant.}$

Signals arrives too amplified → Clipping

AGC: Automatic Gain Control

- Consists of Variable Gain Amplifier and Control Circuit
- Adjust the gain to minimize quantization noise & avoid clipping.
- Receiver Circuit: